Measuring The Shear Modulus of CO_2 Laser-Drawn Fused Silica Fibers

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Abstract

As advanced gravitational wave detectors work to develop greater sensitivity, thermal noise remains one of the most significant limiting factors. The fluctuation-dissipation theorem shows how thermal displacement within the test masses and suspension system is directly related to dissipation within the fused silica used for the suspension fibers and the test mass substrate. One large source of dissipation is surface loss due to imperfections within the surface layer of the silica. It is important to understand more about the properties of the surface layer in order to further limit thermal noise in future detectors. This experiment looks to investigate the surface of fused silica fibers by measuring the shear modulus of CO_2 laser-drawn fused silica fibers using a torsion balance.

1. Introduction to Gravitational Wave Detection

Albert Einstein's Theory of General Relativity predicts the existence of gravitational waves. These waves are perceived as fluctuating tidal strains that propagate as ripples through space-time and are emitted by asymmetrically accelerating mass [1]. Although gravitational waves have been hypothesized, their detection has not yet been made. A worldwide network of detectors has been developed in hopes of proving the existence of gravitational waves.

Present-day detection efforts for gravitational waves make use of the principles of a Michelson-Morley interferometer as shown in *Figure 1* [2].

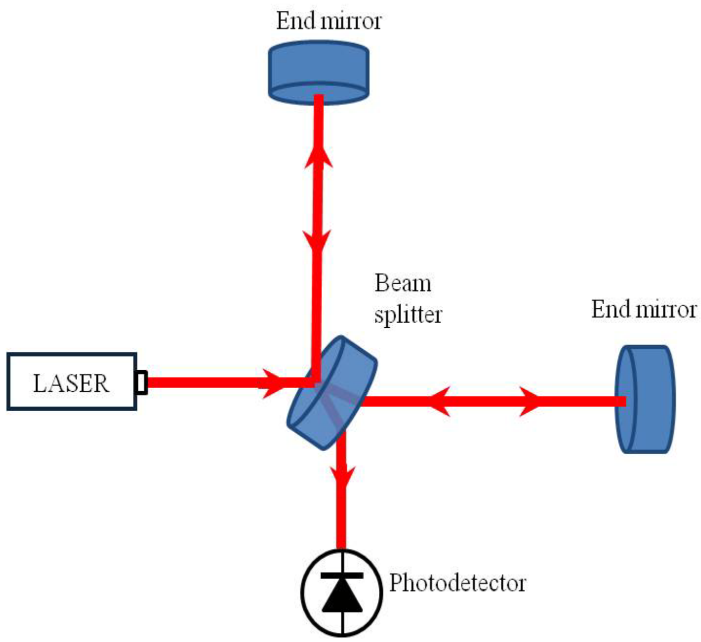

Figure 1: A simplified schematic of a Michelson-Morley Interferometer [2].

A laser emits a beam that is split into two perpendicular beams by a beam splitter. The two split beams travel down long cavities, referred to as the arms of the detector, until being reflected directly back toward the beam splitter by a mirror at the end of the cavity, called the test mass. The two beams will recombine at the photodetector and produce an interference pattern [2].

As gravitational waves move through space, they strain particles via the gravitational force, in accordance with their polarization [3]. If a gravitational wave passes through the interferometer, it will distort the shape of the interferometer, by stretching and squeezing the length of the arms, causing a change in the interference pattern at the photodetector. Strains caused by a gravitational wave are in general very small and can be on the order of $h \approx 10^{-27}$ or less, depending on the source, where

$$h = 2\Delta L / L \tag{1.1}$$

and L is the arm length of the detector [3].

There are several sources of gravitational waves, although astronomical sources are the only ones with a high enough energy density capable of causing a strain within the sensitivity of current detectors [3]. These sources are divided into categories that depend on the length of time for which a wave is emitted. Burst sources, such as coalescing compact binary objects or supernovae, will typically only emit gravitational waves for about a second or less. Continuous sources produce gravitational waves for longer than two weeks; an example of which would be a pulsar. Finally, there is believed to be a stochastic background of gravitational waves in the universe analogous to background cosmic microwave radiation. Both burst and continuous sources emit gravitational waves with different strains depending on the mass of the objects, distance from Earth, the frequency of the wave, and the length of time for which the wave is emitted [3]. The frequency range varies from approximately 10^{-17} Hz to several GHz [1].

There are currently only a few interferometric gravitational wave detectors in the world that are capable of making a detection. Three of these detectors are located in the United States as part of the LIGO (Laser Interferometer Gravitational-Wave Observatory) Network. One of the LIGO detectors is in Livingston Louisiana, while the other two are in Hanford Washington and all have arm lengths of 4km. Initial LIGO was completed in 2006 and is now undergoing an upgrade, aLIGO (Advanced LIGO), to improve sensitivity. aLIGO will be capable of detections across a wide range of frequencies centered around 100Hz and usable down to 10Hz oppose to a 40Hz limit in Initial LIGO [4]. Another major detector is located in Italy with an arm length of 4km called VIRGO, which is also currently undergoing upgrades similar to aLIGO. The is a 600m detector in Germany, GEO-HF, that has already undergone several of the upgrades that will be used in LIGO and VIRGO, although it is unlikely that gravitational wave detections will be made at GEO due to its short arm length [5]. LCGT is a detector currently under construction in Japan and there is also a proposed project to build another 4km LIGO detector in India [2]. Figure 2 shows the network of major gravitational wave detectors.

Figure 2: The worldwide network of major gravitational wave detectors [2].

Having a worldwide network of gravitational wave detectors allows for the ability to rule out noise sources and pinpoint the position in the sky of a gravitational wave source [3].

Due to the incredibly sensitive measurements that these detectors are trying to develop, there are a large number of sources that can create noise and limit the detector's sensitivity [3]. Vibrations in the Earth cause seismic noise and distort the measurements of the detectors. In order to retain the necessary sensitivity of gravitational wave detectors, various methods for isolating the detectors from the Earth have been explored. aLIGO is currently in the process of installing a quadruple stage pendulum, in which the final stage consists of a 40kg fused silica mirror, to be used as the test mass, suspended from another fused silica mass using four silica fibers welded to the sides of the masses

[6]. Although this setup will help isolate the detector from seismic noise, the detector is still prone to many other noise sources including noise within the silica masses and suspension fibers, namely, thermal noise [2].

Thermal noise arises from the fact that a mass is never completely at rest due to Brownian motion. The atoms within the fused silica of the test masses and suspension system have energy of $\frac{1}{2}k_BT$ per degree of freedom. The random fluctuation of particles within the suspension fibers of the pendulum and the mirrors in gravitational wave detectors can excite resonant modes leading to displacement within the mirrors [3]. Thermal noise is one of the most significant sources of noise at frequencies from 10Hz to a few hundred Hz [4].

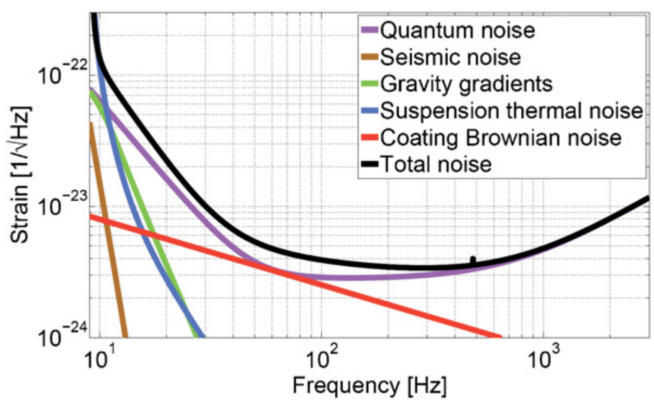

Figure 3: A plot of the anticipated noise contributions for aLIGO sensitivity [4].

2. Thermal Noise And Loss in Silica Fibers

Thermal noise appears within gravitational wave detectors in two forms: through Brownian motion of atoms and molecules and through random temperature fluctuations that both arise within the test masses and suspension fibers [1]. These temperature fluctuations can excite resonant modes and cause displacement of the mirrors [3]. The power spectral density of thermal displacement can be expressed as

$$S_x(\omega) = (4k_BT / \omega^2)\, \boldsymbol{R}(Y(\omega)) \tag{2.1}$$

where k_B is the Boltzman constant, T is the temperature, and $\boldsymbol{R}(Y(\omega))$ is the real (dissipative) part of the mechanical admittance. The mechanical admittance is the inverse of the impedance, Z(ω), which is defined as $Z = F/v$, where v is the resulting velocity caused by the external force, F. Equation 2.1 is referred to as the fluctuation-dissipation theorem [2]. When dealing with thermal noise calculations, the resonant modes of the test masses and suspensions are modeled using a damped pendulum described by the equation

$$F = m\ddot{x} + b\dot{x} + kx \tag{2.2}$$

where F is the external force that excites oscillatory motion, m is the mass, k is the spring constant, and b is the damping term [7]. Using the definition of impedance, and taking $x = i\omega x$, the mechanical impedance can be expressed as,

$$Z(\omega) = b + i[m\omega - (k / \omega)] \qquad [2] \quad (2.3)$$

Combing Equation 2.3 with Equation 2.1, the power spectral density of thermal displacement will now take the form of

$$S_x = (4k_BTb) / (\pi^2 f^2 (b^2 + (\omega m - (k / \omega))^2)) \qquad [2] \quad (2.4)$$

The ideal pendulum exhibits no dissipation and responds with perfect elasticity, however this is not the case with a real pendulum due to the effect of internal friction inside the material. Within any material there exists some degree of dissipation due to the anelasticity of the material [7]. Hooke's law, $F = -kx$, predicts that a material will respond instantaneously to an external force, although in practice there will be a tiny phase lag, which causes dissipation [7]. The lag is typically expressed in radians by the function $\o(\omega)$. We can now let $b = \o(\omega)$, assuming $\o(\omega) << 1$, so that Equation 2.4 can be expressed in terms of $\o(\omega)$ as

$$S_x = (4k_B T\omega_0\, ø(\omega)) \,/\, (2\pi^3\omega m[(\omega_0{}^2 - \omega^2)^2 + \omega_0{}^4 ø^2(\omega)]) \tag{2.5}$$

It can be seen that minimizing the dissipation of the material would reduce the thermal displacement [2]. Because of this, gravitational wave detectors require materials with high quality factors, Q, where

$$Q = 1 \,/\, ø(\omega) \tag{2.6}$$

Fused silica has a loss term about two orders of magnitude lower than steel and other metals [7]. Materials with high Q factors also store more energy in the resonant modes of the mirrors and suspensions. The thermal excitations that are likely to occur within these materials are far from resonance, thus noise can be minimized [3].

Suspending the test masses also creates a low level of thermal noise by storing most of the energy in gravity [2]. Consider a simple pendulum consisting of a fiber mounted on one end and connected to a test mass on the other. When the mass is displaced horizontally from equilibrium, it will feel a restoring force, which can almost entirely be attributed to gravity. Gravity has a spring constant of $k_g = mg/l$, where m is the mass of the test mass and l is the length of the fiber [2]. The material of the fiber also provides a restoring force with a spring constant

$$k_{material} = ((mgEI)^{1/2}) / 2l^2 \tag{2.7}$$

where E is the modulus of elasticity of the fiber and I is the fiber's cross sectional moment of inertia [7]. While the gravitational field is conservative and lossless, the fiber is dissipative [7]. The total energy stored within the pendulum is

$$E_{Pendulum} = ½(k_g + k_{material})x^2 \tag{2.8}$$

The energy lost after each oscillation is some fraction (β) of the fiber's energy

$$E_{material} = ½\beta\, k_{material} x^2 \tag{2.9}$$

Using the definition of loss, the loss of the pendulum is

$$\o_{Pendulum}(f) = \beta\ k_{material} / (k_g + k_{material}) \tag{2.10}$$

while the loss of the fiber is $\o_{material}\ (f) = \beta$. The loss of the pendulum can now be written as

$$\o_{pendulum}(f) = \o_{material}(f)\ (k_{material} / (k_g + k_{material})) \tag{2.11}$$

Generally $k_g >> k_{material}$, meaning

$$\o_{pendulum}(f) = \o_{material}(f)\ (k_{material} / k_g\ ) \tag{2.12}$$

This equation [7] shows how the material loss is directly related to the loss in the pendulum and also how storing the vast majority of the energy in gravity can reduce the loss of the pendulum. The term $k_{material} / k_g$ is referred to as the "dilution factor" [2].

There are several factors that contribute to material loss. There is thermoelastic loss, which arises from the temperature gradient caused by flexing fibers, bulk loss, which is a frequency dependent loss within the material, weld loss, which comes from welded regions of the fibers, and surface loss [2]. Surface loss is due to imperfections within the surface of the material. The surface loss of a suspension fiber used in aLIGO is related to the surface-to-volume and is expressed as

$$\o_{surface} = 8h\o_s / d \tag{2.13}$$

where d is the diameter of the fiber, h is the depth of the surface of the material and $\o_s$ is the mechanical loss of the material's surface [8]. The value of $h\o_s$ for silica fibers has been measured to be $6.5\text{x}10^{-12}$m [9], where h is the depth of the surface and $\o_s$ is the mechanical loss of the surface. Although the value of $h\o_s$ has been measured, the individual components of this product are unknown. Because of this, it is unknown how deep the surface of a silica fiber really is [5].

3. Measuring The Shear Modulus of Silica Fibers

3.1 Theory

Because of the contribution of surface loss to thermal noise, it is important understand more about the surface layer of fused silica fibers. An experiment was devised to measure the effect the surface layer of silica fibers has on the fiber's shear modulus. Several fibers with different radii would be tested in order to determine whether the shear modulus would change as the surface-to-volume ratio of the fiber was varied. By observing some deviation in the value of the shear modulus as the fiber radius became smaller, it could help indicate the depth of the surface [10].

The shear modulus would be found by creating a torsion pendulum using CO_2 laser-pulled fused silica fibers with an attached test mass of a known moment of inertia. The torsion constant, k, for a given fiber can be found using,

$$k = \pi r^4 \mu / 2l \qquad (3.1)$$

where r is the radius of the fiber, l is the length of the fiber, and μ is the shear modulus [11]. The torsion constant can also be found using

$$k = 4\pi^2 I / T^2 \qquad (3.2)$$

where I is the moment of inertia of the test mass and T is the period of oscillation. If the radius and length of the fiber are known, as well as the moment of inertia of the test mass attached to the fiber, the shear modulus can be calculated, if the period of oscillation is also measured [11].

3.2 Experimental Setup

The silica fibers used in this experiment were made using the University of Glasgow's 100W CO_2 laser pulling machine [12]. In this process, fibers are pulled from a stock piece of silica rod. The stock silica rod undergoes a polishing phase before the fiber is pulled where the laser heats part of the rod and allows it cool again in attempt to heal any surface impurities and improve tensile strength. The torsion pendulum was designed to use 8cm fibers pulled from 3mm stock. The fibers were pulled to the desired diameter from the polished stock leaving approximately 4cm of unpolished stock on either side of the fiber. High precision measurements of the fiber's diameter were made by profiling the fiber using a high-resolution camera and a LabVIEW program to convert pixels into units of distance [5].

The silica fibers were mounted and tested inside a vacuum tank. A platform was mounted onto the rim of the vacuum tank and two 15cm long steel poles were screwed into the platform. A piece of aluminum was used as a crossbar to connect the two poles as shown in *Figure 4*. The crossbar held a rotation stage and a mount to hang the silica fiber that used a nylon grub screw to clamp the unpolished stock at the end of fiber. A test mass that was constructed with the intent of readily being able to vary the moment of inertia was attached to the other end of the fiber. It was made using an aluminum cube with a 25cm piece of M6 threaded rod running through it. The aluminum cube had a hole drilled in the top of it with another nylon grub screw to attach to the other end of the fiber. The mass, and the moment of inertia, of the test mass could be altered by hanging 30mm diameter steel washers on the threaded rod.

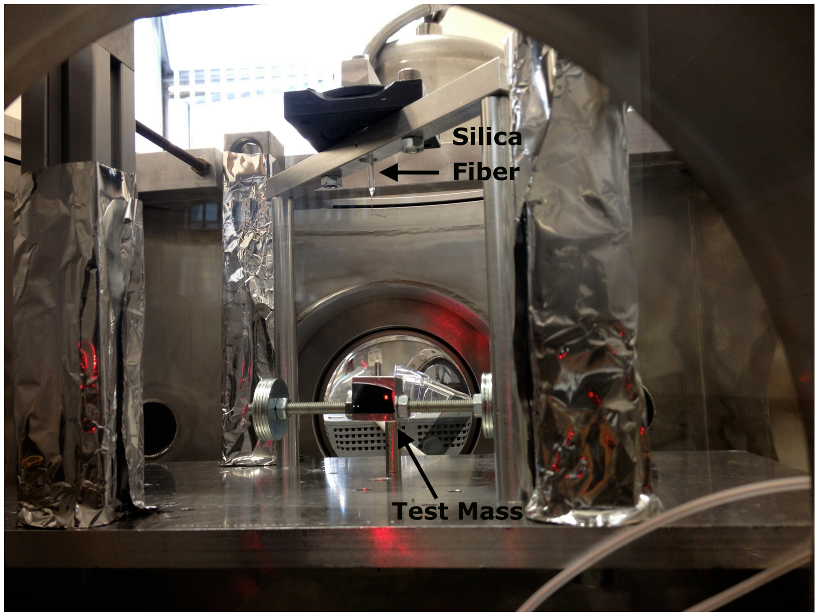

Figure 4: A silica fiber suspended in the vacuum tank with the test mass attached.

Outside one of the viewing windows of the vacuum tank, a Helium-Neon laser was mounted along with a linear photodiode. The laser was positioned so that the beam struck a piece of silicon glued to the test mass, which reflected the beam back toward the viewing window. The period of the pendulum could be found by analyzing the oscillation of the reflected laser.

The Hamamatsu S3932 photodiode used in this experiment has a 1mm x 12mm active area and has an output that is correlated with the position of the laser on the diode. There is both a positive and a negative output on the photodiode, which are associated with the two sides of the diode. If a beam of light is focused directly at the center of the photodiode, the photodiode will emit zero current. As the beam moves away from the center of the diode, the positive or negative output will produce a current depending on whether the beam moves to the positive or negative side. As the beam strays farther from the center of the diode, the current becomes larger. Because of the small size of the photodiode, it was impossible to keep the beam on the photodiode through an entire oscillation. The photodiode was placed so that the reflected laser struck to center of the photodiode at the peak of its oscillation.

The two current outputs from the photodiode were converted into a single voltage signal using a current-to-voltage converter circuit. The two current signals were converted into voltage signals by passing through an operational amplifier circuit with a feedback resistor to set the gain. The op-amps were connected to +8V and -8V power supplies. The two voltage signals then fed into a comparator that was made using an AD8428 op-amp with a fixed gain of 2000 in order to produce one voltage output as well as make sure the voltage signal was at the appropriate magnitude. The single voltage output passed through an NI analog-to-digital converter to a computer that used a LabVIEW program designed to record voltage as a function of time. A simplified circuit diagram for the current-to-voltage converter can be seen in *Figure 5*.

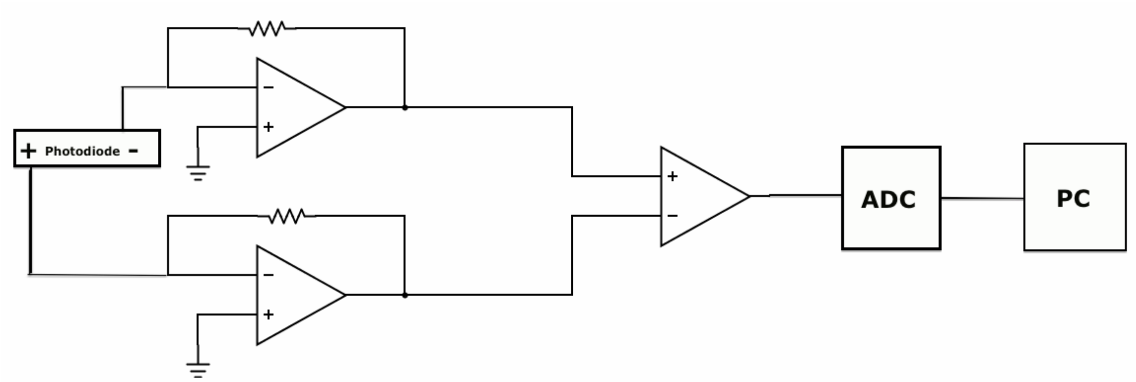

Figure 5: A simplified circuit diagram of the current-to-voltage converter.

3.3 Data Analysis

The data gathered from the photodiode was modified before any analysis was done to determine the period of oscillation. The data collected in LabVIEW appeared as shown in *Figure 6*.

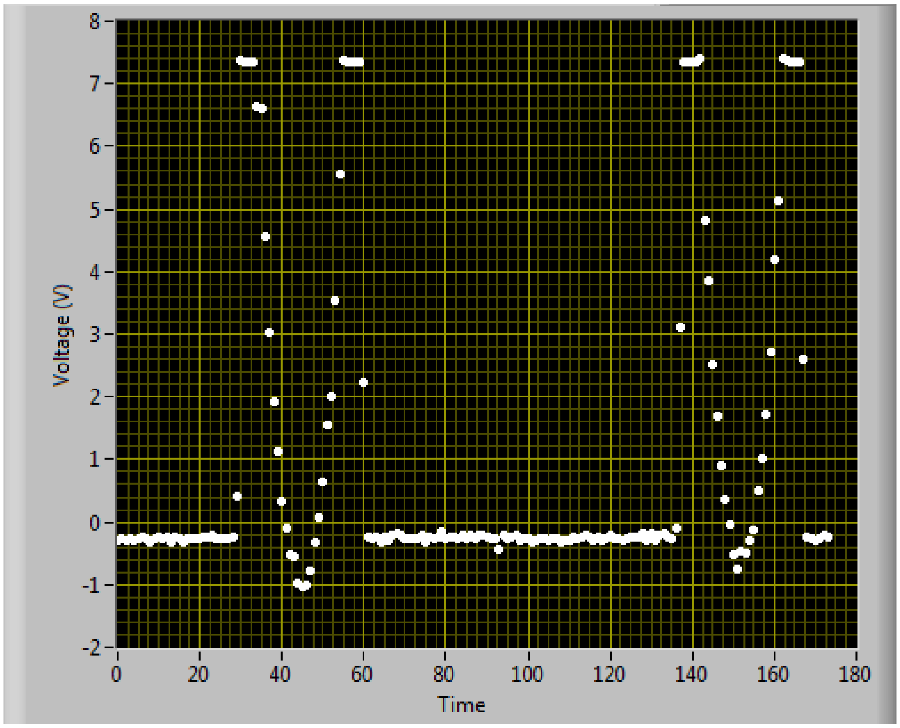

Figure 6: A plot of the LabVIEW readout of the voltage signal received from the photodiode.

The parabolic shapes occur during periods when the laser was on the photodiode and the vertex marks the peak of oscillation. The data points between the parabolas are noise from the time the laser did not strike diode. In order to obtain a dataset more fit for analysis, the Voltage vs. Time plot shown in *Figure 6* was transformed into a square wave function shown in *Figure 7*.

All data points collected in the linear region of the photodiode, between -3.5V and +3.5V, were assigned a value of 1, while all other points were assigned a value of 0. The modified dataset was run through a MatLAB program, which uncovered any periodicity within the data. The program produced a power spectrum, also shown in *Figure 7*, with spikes at the harmonic frequencies of the dataset. The fundamental frequency was taken from this power spectrum as the frequency.

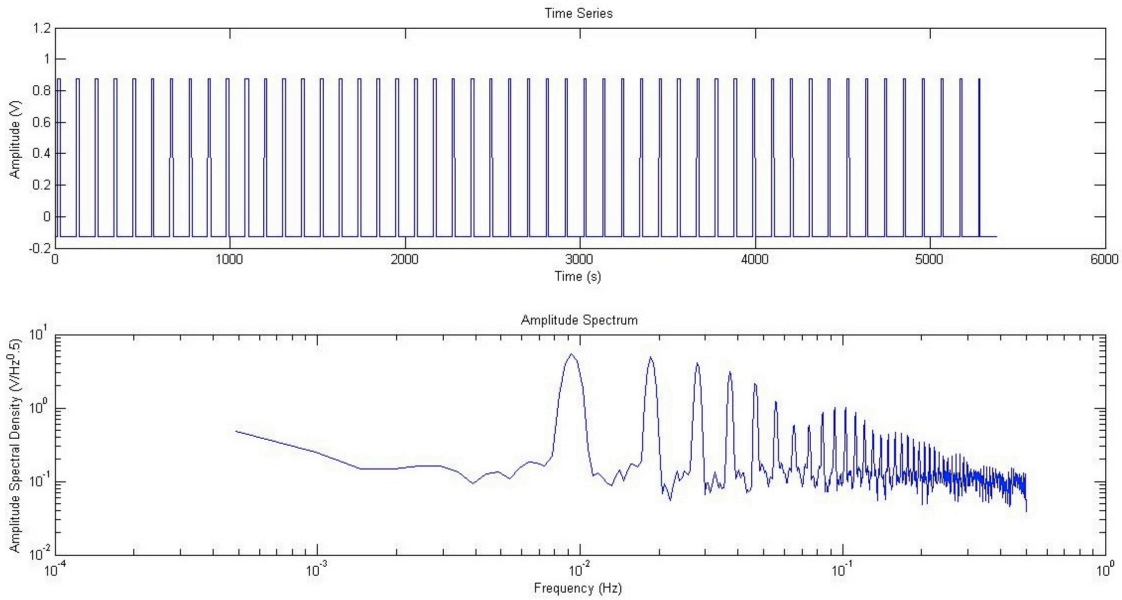

Figure 7: The top plot shows the square wave generated from the photodiode signal. The bottom plot gives the amplitude spectral density of the squarewave. The fundamental frequency can be seen at the first peak.

The moment of inertia of the test mass also had to be found to determine the shear modulus, as shown in Equation 3.2. The test mass was constructed in the Computer-Aided Design program, Solidworks, and the moment of inertia was found using the program. Knowing the moment of inertia as well as the period, allowed for the use of Equation 3.2 to calculate the torsion constant, *k*. With a known value of *k*, and a known radius and length of the fiber obtained from the fiber profile, the shear modulus was solved for using Equation 3.1. Because the radius of any fiber is not constant and often varies significantly between the neck and the middle of the fiber, as seen in *Figure 8*, the shear modulus was solved for using a modified version of Equation 3.1

$$\mu = (2k / \pi) \sum (L_i / r_i) \tag{3.3}$$

where L_i and r_i are the length and radius of the i^{th} section of the fiber.

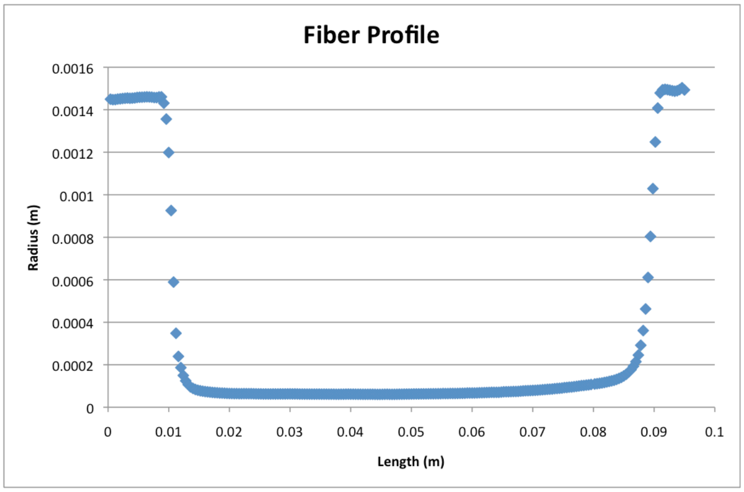

Figure 8: A plot of radius as a function of length of a profiled fiber.

3.4 Results

A total of three different fibers with approximate radii of 200µm, 120µm, and 70µm were tested in this experiment. The 120µm and the 70µm fiber were both tested twice using two different test masses to create different moments of inertia and a different tensile stress. The torsion constant, *k*, and the shear modulus were calculated after each trial and these results can be seen in *Table 1*.

Approximate Radius (µm)	Tensile stress (MPa)	k ($kg*m^2/s^2$)	Shear Modulus (GPa)
200	224	$5.21e^{-5}$	27.4
120	272	$8.77e^{-6}$	19.1
120	228	$8.23e^{-6}$	18.0
70	272	$8.12e^{-7}$	10.3
70	138	$6.03e^{-7}$	7.61

Table 1: The calculated value of k and the shear modulus for each fiber tested with the torsion pendulum.

A noticeable decrease in the shear modulus can be seen as the radius of the fiber decreases and the surface-to-volume ratio increases. Although the two trails done using the 120µm fiber show torsion constants and shear modulus values within 7% of each other, the two trails done with the 70µm fiber differ significantly. In order to gain full confidence in the trend of this data, more trails should be conducted using more fibers of different radii.

4. Conclusion

Although it is hard to draw any firm conclusions from this experiment, the results warrant further exploration. Understanding the surface layer of CO_2 laser-drawn fused silica fibers is a very important step to limiting thermal noise within future gravitational wave detectors. Only the product $h\phi_s$ is known and not the actual depth of the surface layer. If able to determine these two components individually, techniques could be developed in order to remove the surface of fused silica fibers after the fiber is pulled, in order to destroy an important source of dissipation. Reducing, or eliminating, the surface of fused silica fibers could significantly reduce thermal noise.

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