Gravitational Physics International REU

Research at the Australian National University

Andrews University Danielle Wuchenich *wuchenic@andrews.edu*

Submitted to the University of Florida on 13 August 2007

1. Gravitational Waves & Interferometry Background

The classical Newtonian theory of gravity was reconciled with special relativity when Einstein published his General Theory of Relativity in 1916. Einstein proposed that gravity is not a force but rather a result of the geometry of space-time. Because massive objects curve space-time, an accelerating mass would cause the surrounding curvature to change. These perturbations in space-time can cause ripples to propagate away from curvature-changing events, and these ripples are called gravitational waves.

Until recently, the area of gravitational physics was dominated by theorists. Experiments to test theories of gravity and the existence of gravitational waves were rare due to a lack of technology. The area of experimental gravitational physics, however, has grown significantly with the development of new technology, faster computers, and sensitive instruments allowing for small-scale measurements.

One very promising setup intending to directly measure gravitational waves utilizes a Michelson interferometer configuration. A simple Michelson interferometer consists of a light source split into two beams by a beam splitter. Each beam travels a distance down the "arm" of the interferometer before being reflected back by a mirror at each arm's end. The two reflected beams recombine at the beam splitter and the resulting intensity is measured by an instrument called a photodetector. A schematic of a simple Michelson interferometer is given in Figure 1. Thus an interferometer compares the amount of time it

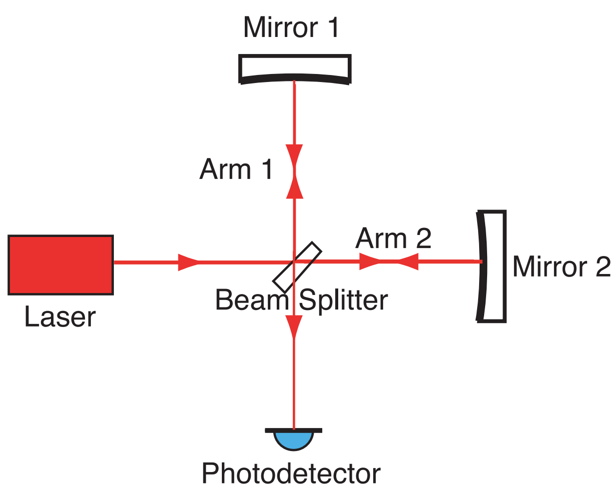

Figure 1. A simple Michelson interferometer. After being reflected by mirrors at the end of each arm, the light beams interfere at a beam splitter and are added according to the superposition principle.

takes for the light beams in the two arms to complete their paths. When a gravitational wave passes by, it distorts the space-time around it, causing small changes in the lengths of an interferometer's arms. The Laser Interferometer Gravitational-Wave Observatory (LIGO) operates three interferometers, two having 4-km arms and one having 2-km arms. An international collaboration between LIGO and other groups in gravitational physics has been established, and the area of gravitational wave detection continues to grow. By having a network of interferometers in different locations, the positions of gravitational wave sources can be determined by a method called triangulation.

Large amounts of energy carried in a gravitational wave only distort the surrounding space by a small amount, and thus the length changes in the interferometer arms are very minimal. Coalescing black holes, binary neutron stars, and other astronomical events are likely to be the most probable source of gravitational waves that current interferometers will be able to detect [2]. The strain h of an interferometer is defined to be:

$$h = \frac{2 \triangle L}{L} \tag{1}$$

where L is the arm length of the interferometer and $\triangle L$ is the differential length change in the arms [5]. Gravitational waves are predicted to have strains on the order of $h \approx 10^{-21}$ [2]. For an interferometer having 4-km arms, we can rearrange Equation 1 to solve for the expected differential change in arm length $\triangle L$:

$$\triangle L = \frac{hL}{2} = \frac{(10^{-21})(4000\text{ m})}{2} = 2 \times 10^{-18}\text{ m} \tag{2}$$

This means that to see the effect of gravitational waves, one must be able to measure extremely small changes in length. Notice that $\triangle L$ could be increased by increasing the arm length L of the interferometer. Fabry-Pérot cavities are added in the arms of an interferometer to effectively increase the arm lengths. A Fabry-Pérot cavity is comprised of two partially transmitting mirrors; when light enters the cavity, it bounces back and forth between the mirrors, thus increasing the distance that the light travels before returning to the photodetector.

Lasers are the light sources used in gravitational wave interferometers because the emitted light is coherent and in phase [1]. When an atom is raised into an excited energy state, it can revert to a lower energy state through a process called photon emission. When the atom is stimulated by electromagnetic radiation to release the photon, the mechanism is referred to as stimulated emission. When the latter process is used, the emitted photon propagates in the same direction, has the same polarization, and is in phase with the electromagnetic radiation that triggered the process. When a large percentage of the atoms in a system are excited, the necessary population inversion is achieved. A pump source allows the laser to emit a continuous beam of coherent light.

For the setup in Figure 1, the electric field of the laser can be expressed as $E_{laser} = E_o e^{iwt}$. Assuming none of the laser's energy is lost or absorbed, a 50/50 beam splitter will reflect half of the light into one arm and transmit half of the light down the other arm.

When the reflected beams are recombined at the beamsplitter, the resulting electric field at the photodetector is:

$$E_{PD} = \frac{E_o}{\sqrt{2}} e^{i(\omega t + \phi_1)} + i \frac{E_o}{\sqrt{2}} e^{i(\omega t + \phi_2)} \tag{3}$$

where ϕ_1 and ϕ_2 are the phase shifts resulting from the roundtrip time in arm 1 and arm 2. Thus, denoting E_{PD}^* as the complex conjugate of E_{PD}, the power detected by the photodetector will be:

$$P_{PD} = E_{PD}^* E_{PD} = E_o^2 - E_o^2 sin(\phi_2 - \phi_1). \tag{4}$$

Even with current technology and the most sensitive instruments, achieving the desired performance in gravitational wave detectors is very challenging due to various types of noise. Seismic noise, shot noise, thermal noise, radiation pressure noise, and gravity gradient noise limit the sensitivities of the interferometers [5]. Many feedback mechanisms have been developed to compensate for these types of noise. One common method used in measuring the length noise in a cavity is the Pound-Drever-Hall technique [3]. This technique can be used to stabilize a laser's frequency by locking it to a Fabry-Pérot cavity (or vice versa). By setting up a feedback mechanism, one can measure the noise in the cavity based on the amount of compensation performed by the feedback circuit to keep the system in resonance. Because the photodetector only provides a measurement of the intensity at the output, the phase of the input light needs to be modulated so a proper feedback circuit can be implemented. Modulating the input light allows the system to know which side of resonance it is on and thus how to respond to re-achieve resonance.

2. Digitally Enhanced Heterodyne Interferometry

Obtaining precise measurements of the displacements between mirrors or other optical components would be very useful in providing accurate feedback to a system and maintaining lock. Digitally enhanced heterodyne interferometry [4] is a technique that modulates a Pseudo-Random Noise (PRN) code on the laser to provide realtime information on the relative positions of multiple mirrors.

Heterodyne interferometry involves combining a high-frequency signal with another to produce a lower frequency. An Acousto-Optic Modulator (AOM) uses an acoustic wave to shift the frequency of the light. When an AOM is used in an interferometer's arm to shift the beam by ω_{AOM}, then the electric field of frequency-shifted beam [the Local Oscillator (LO)], can be expressed by:

$$E_{LO} = E_o e^{i[(\omega + \omega_{AOM})t + \phi_{LO}]} \tag{5}$$

When the LO beam combines with the beam from the other arm, the resulting electric field at the photodetector is:

$$E_{PD} = \frac{E_{LO}}{\sqrt{2}} + i\frac{E_o}{\sqrt{2}}e^{i(\omega t+\phi_2)} \tag{6}$$

and the power detected at the photodetector is:

$$P_{PD} = E^*_{PD}E_{PD} = E_o^2 - E_o^2 sin(\omega_{AOM}t + \phi_{LO} - \phi_2). \tag{7}$$

See the appendix for a complete derivation and note how this result differs from Equation 4.

In our experiment, a beam splitter separated the laser into two beams: a Local Oscillator (LO) and a probe beam. Please see Figure 2 for a diagram of the experimental setup. An

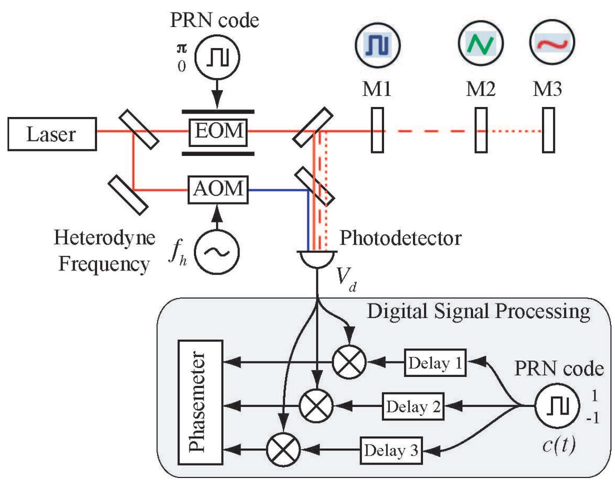

Figure 2. A Digitally Enhanced Interferometer. This was the setup used in our PRN experiments. Illustration with permission from Reference [4].

Electro-Optic Modulator (EOM) was used to modulate the phase of the light. A pseudo-random noise code generator sent a PRN code to the EOM, and the EOM modulated the phase of the light of the probe beam appropriately. The phase was randomly shifted by 0 or π, resulting in the multiplication of the amplitude by ± 1. This modulated light was directed toward and reflected by the three mirrors, combined with the frequency-shifted LO, and observed at the photodetector. Figure 3 shows a simulation of the photodetector output. The original unmodulated signal could be recovered by multiplying the modulated signal (read at the photodetector) by the original signal shifted by an appropriate time delay. A correct time delay needed to isolate reflections from individual optical components equals the optical propagation time of the particular reflection. Figure 4 provides an excellent demonstration of the technique.

To test the effectiveness of this technique, we modulated the first mirror in our experiment with a square wave, the second mirror with sawtooth wave, and the third mirror

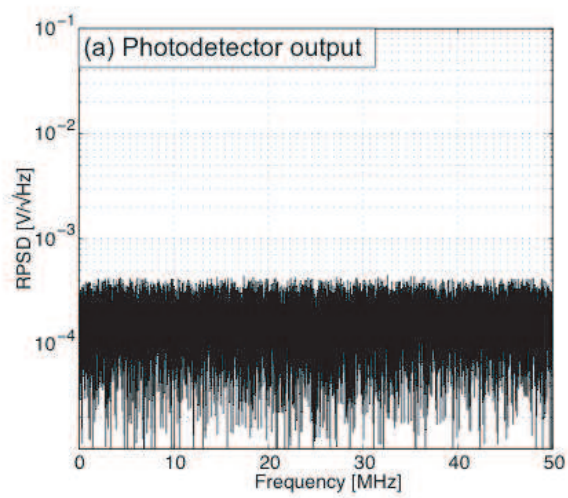

Figure 3. A simulation of the output of the photodetector before demodulation. Illustration with permission from Reference [4].

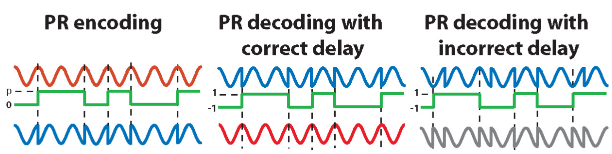

Figure 4. A PRN code modulates the phase of the light by 0 or π. When an appropriate time delay is applied to the original code, the time-shifted code can be multiplied by the photodetector signal to obtain the original signal. When an incorrect time delay is applied, demodulation will not recover the carrier signal.

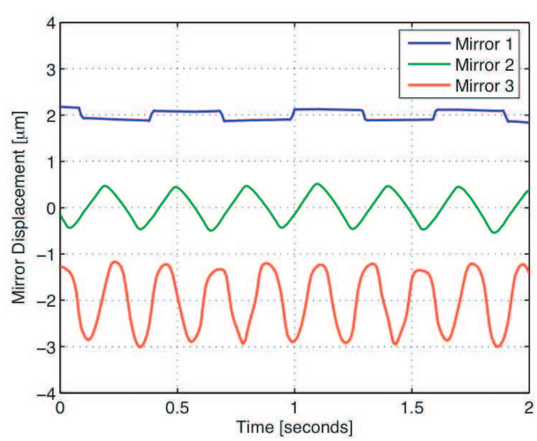

Figure 5. Preliminary experimental demonstration of technique employing digital modulation to track displacement of mirrors.

with a sine wave. Appropriate times delays were applied to the code using LabView™, and we were able to track the movement of each mirror (see Figure 5).

Though the technology and the use of PRN is not new, it is the first time this technique has been applied to gravitational wave detection. This technique may be very useful in lock acquisition in Advanced LIGO and the Laser Interferometer Space Antenna (LISA). Because the signals from each optical component are isolated, the overall sensitivity of the system is increased. Spurious interference is suppressed by $1/(2^N - 1)$, where N is the length of the code [4]. This technique still retains the sensitivity of interferometry, providing a full-range readout (0 to 2π). Applying multiple delays to the same signal, one output signal can be decoded to isolate different reflections. The technique would be especially useful in LISA as it has the ability to track motion over multiple fringes. Further work and experiments will be performed at the Jet Propulsion Laboratory and the Australian National University to optimize the code and improve suppression by using a longer averaging time.

Acknowledgments

This paper was written using LaTeX at the Australian National University in July 2007. A special thanks to David McClelland for being a wonderful host, Daniel Shaddock, David Rabeling, and Glenn de Vine for countless explanations and letting me help with the experiments, and Kirk McKenzie and the other physicists of the Centre for Gravitational Physics at the Australian National University for always being helpful and fun.

Thanks especially to Guido Müeller, Bernard Whiting, and the University of Florida for organizing the REU. The Gravitational Physics International REU was funded by the National Science Foundation.

Appendix

We can express the electric field of a laser by:

$$E_{laser} = E_o e^{i\omega t} \tag{A.1}$$

A beam splitter partitions this light into two beams, a Local Oscillator (LO) beam and a signal (S) beam. When an Acousto-Optic Modulator (AOM) is used to shift the frequency of the LO, the electric field at the photodetector due to the LO is:

$$E_{LO} = E_o e^{i[(\omega+\omega_{AOM})t+\phi_{LO}]} \tag{A.2}$$

The electric field at the photodetector due to the signal beam can be written as a phase-shifted version of the original laser beam:

$$E_S = E_o e^{i(wt+\phi_S)} \tag{A.3}$$

Thus the electric field at the photodetector due to the combined LO and S beams is:

$$E_{PD} = \frac{E_{LO}}{\sqrt{2}} + i\frac{E_S}{\sqrt{2}} \tag{A.4}$$

and the intensity measured at the photodetector is:

$$\begin{aligned} P_{PD} &= E_{PD}^* E_{PD} \\ &= \frac{E_{LO}^*}{\sqrt{2}}\frac{E_{LO}}{\sqrt{2}} + \frac{iE_S^*}{\sqrt{2}}\frac{iE_S}{\sqrt{2}} + \frac{E_{LO}^*}{\sqrt{2}}\frac{iE_S}{\sqrt{2}} + \frac{E_{LO}}{\sqrt{2}}\frac{iE_S^*}{\sqrt{2}} \end{aligned} \tag{A.5}$$

Notice the relationship between the last 2 terms of Equation A.5:

$$\frac{E_{LO}^*}{\sqrt{2}}\frac{iE_S}{\sqrt{2}} = [\frac{E_{LO}}{\sqrt{2}}\frac{iE_S^*}{\sqrt{2}}]^* \tag{A.6}$$

Thus,

$$\begin{aligned} [\frac{E_{LO}}{\sqrt{2}}\frac{iE_S^*}{\sqrt{2}}]^* + \frac{E_{LO}}{\sqrt{2}}i\frac{E_S^*}{\sqrt{2}} &= 2Re[\frac{E_{LO}^*}{\sqrt{2}}\frac{iE_S}{\sqrt{2}}] \\ &= Re[iE_{LO}^* E_S] \\ &= Re[iE_o^2 e^{-i(\omega_{AOM}t+\phi_{LO}-\phi_S}] \\ &= -E_o^2 sin(\omega_{AOM}t + \phi_{LO} - \phi_S) \end{aligned} \tag{A.7}$$

Substituting this value for the last two terms of Equation A.5,

$$\begin{aligned} P_{PD} &= \frac{E_{LO}^*}{\sqrt{2}}\frac{E_{LO}}{\sqrt{2}}) + \frac{iE_S^*}{\sqrt{2}}i\frac{E_S}{\sqrt{2}}) + \frac{E_{LO}^*}{\sqrt{2}}\frac{iE_S}{\sqrt{2}} + \frac{E_{LO}}{\sqrt{2}}\frac{iE_S^*}{\sqrt{2}} \\ &= \frac{E_{LO}^*}{\sqrt{2}}\frac{E_{LO}}{\sqrt{2}} + \frac{iE_S^*}{\sqrt{2}}\frac{iE_S}{\sqrt{2}} - E_o^2 sin(\omega_{AOM}t + \phi_{LO} - \phi_S) \\ &= \frac{1}{2}E_o^2 + \frac{1}{2}E_o^2 - E_o^2 sin(\omega_{AOM}t + \phi_{LO} - \phi_S) \\ &= E_o^2 - E_o^2 sin(\omega_{AOM}t + \phi_{LO} - \phi_S) \end{aligned} \tag{A.8}$$

References

[1] E. Hecht
Optics
Addison-Wesley, Reading, 2 edition, 1987.

[2] P. R. Saulson
Fundamentals of Interferometric Gravitational Wave Detectors
Farrer Road: World Scientific Publishing Co. Pte. Ltd., 1994.

[3] E. Black
"Notes on the Pound-Drever-Hall technique."
4/16/98. http://www.ligo.caltech.edu/

[4] Daniel A. Shaddock
"Digitally Enhanced Heterodyne Interferometry."
Submitted to *Optics Letters* June 2007.

[5] Daniel A. Shaddock
Advanced Interferometery for Gravitational Wave Detectors
PhD thesis, 3 November 2000.
Austalian National University

[6] R. W. P. Drever, J. Hall, F. K. Kowalski, J. Hough, G. M. Ford, A. J.Munley, H. Ward
Laser phase and frequency stabilization using an optical resonator
Applied Physics B **31**
97-105 (1983)

[7] S. O. Kasap
Optoelectronics and Photonics: Principles an Practices
Upper Saddle River: Prentice-Hall, Inc., 2001.