

Measuring the Quantum Efficiency of a Photo Diode using Radiation Pressure

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ABSTRACT

The aim of the project is to measure the quantum efficiency of a photodiode with an error less than 1% by utilizing radiation pressure and a Michelson Interferometer. The interferometer has a suspended 23 mg mirror in one of the arms in order to allow a detectable amount of motion caused by the radiation pressure of an intensity-modulated laser. The displacement of the small mirror is related to the power of the laser and thus provides an accurate measure of the number of photons per second entering the photo diode. This in turn is used to calculate the quantum efficiency of the photo diode. At an amplitude modulation frequency of 10Hz, we detected a quantifiable displacement of the mirror caused by radiation pressure which matched the theoretical value. However, the precision of this measurement is not yet sufficient and should be improved in the future to reach the 1% precision target.

1. INTRODUCTION

A photo diodes is a PN junction which converts light into current. A PN junction is made by depositing a P-type silicon over an N-type semiconductor to create a depletion zone which is a nonconducting layer. When a photon strikes the depletion zone, its energy is absorbed creating an electron-hole pair. Because of the electric field due to the reverse bias voltage in the depletion zone, the electron and its corresponding hole are accelerated in opposite directions which results in the production of current. Quantum efficiency describes

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how well a photo diode converts photons to electron-hole pairs and into electric current. More specifically, quantum efficiency (QE) is the ratio of the number of electron per second, N_e , and the number of photons per second, N_p , of the light incident on the photo diode.

$$QE = \frac{N_e}{N_p} \quad (1)$$

The number of electrons per second in the current produced by the photo diode (N_e) is the current (I) divided by the charge of a single electron (q).

$$N_e = \frac{I}{q} \quad (2)$$

The current is determined by measuring voltage of a convertor circuit that drains the current of the photo diode. To eliminate the photodiode dark current effect, the voltage used is the difference of the photo diode's output voltage when the laser is incident on the photo diode ($V_{measured}$) and when no direct light is directed on the photo diode (V_{dark}). By Ohm's law, the current produced by the photo diode is $I = \frac{V_{measured} - V_{dark}}{R}$ where R is the resistance of the convertor circuit.

The number of photons incident on the photo diode per second (N_p) is ratio of the power of the laser (P) and the energy a photon ($h * v$).

$$N_p = \frac{P}{h * v} \quad (3)$$

where h is Plank's constant and v is the frequency.

The power of the laser is determined by measuring the displacement induced on the suspended mirror by the radiation pressure. The displacement as a function of the applied power is measured in a Michelson Interferometer.

To relate the displacement of the mirror and the power of the laser, we must first consider the momentum of a single photon (p), which is the energy of the photon ($h * v$) divided by the speed of light (c).

$$p = \frac{h * v}{c} \quad (4)$$

The change in momentum of a photon reflected by a suspended mirror (Δp) is double the original photon's momentum.

$$\Delta p = \frac{2 * h * v}{c} \quad (5)$$

The force exerted on the mirror is calculated by multiplying the change in momentum and the number of reflected photons per second.

$$F = N_p * \Delta p = \frac{N_p * 2 * h * v}{c} * \cos(\alpha) \quad (6)$$

The $\cos(\alpha)$ is due to the nonorthogonal incident angle of the light. Using equation (4) and Newton's Law of Motion, the force on the mirror is simplified to

$$F = m * \ddot{x} = \frac{2 * P}{c} * \cos(\alpha) \quad (7)$$

where m is the mirror's mass and x is the mirror's displacement along the axis of the photon's propagation. If the power of the laser is modulated at a certain angular frequency, ω , then by taking the Fourier Transform of equation (7), the response of the mirror's displacement is

$$-m * \omega^2 * X = \frac{2 * P}{c} * \cos(\alpha) \quad (8)$$

where X is the displacement of the mirror in the frequency domain.

The Michelson interferometer is used to read out the displacement of the mirror. The output voltage of a photo diode (V_{PD}) on Michelson Interferometer's antisymmetric port is related to the difference of the two arm lengths (X) by

$$V_{PD} = 0.5 * V_{pp} * \sin\left(\frac{4 * \pi * X}{\lambda}\right) + 0.5 * V_{pp} + V_{offset} \quad (9)$$

where $V_{pp} = V_{max} - V_{min}$ for movements longer than one wavelength and λ is the laser's wavelength. By differencing with respect to X , the response of the output voltage to changes in the difference in arm lengths is

$$\frac{dV_{PD}}{dX} = \frac{2 * \pi * V_{pp}}{\lambda} * \cos\left(\frac{4 * \pi * X}{\lambda}\right) \quad (10)$$

Using a feedback control loop, the interferometer can be locked at mid-fringe of the signal so that the response in voltage to changes in X is approximately linear (see Section 3). Thus, the cosine term in equation (10) is held at one and the equation is simplified to

$$\frac{dV_{PD}}{dX} = \frac{2 * \pi * V_{pp}}{\lambda} \quad (11)$$

Thus, the change in difference in arm lengths is calculated directly by measuring the change in voltage: $dX = \frac{\lambda}{2*\pi*V_{pp}} * dV$ (12)

Finally, to relate the power of the laser to the movement of the 23 mg suspended mirror, the Michelson Interferometer was constructed to have a heavier and strongly constrained mirror so that only the small mirror will be moved by the changing radiation pressure of an intensity modulated laser. This means that the difference in arm lengths of equation (12) is the displacement of the small mirror of equation (8). By combining the two equations,

$$P = \frac{m*c*\omega^2*\lambda}{4*\pi*V_{pp}} * dV_{PD} * \cos(\alpha) \quad (13)$$

This connects the power of the laser to the output voltage which in turn allows us to calculate N_p by equation (3).

Note that the photo diodes used to lock the Michelson interferometer are not the photo diodes that we are trying to find the quantum efficiency for. To calculate the quantum efficiency of a photo diode, the N_p must first be measured. Then, the photo diode is placed right in front of the small mirror to measure N_e .

2. EXPERIMENTAL SETUP

An optical table with the laser and various optical devices is set up next to a vacuum chamber that contains the michelson interferometer.

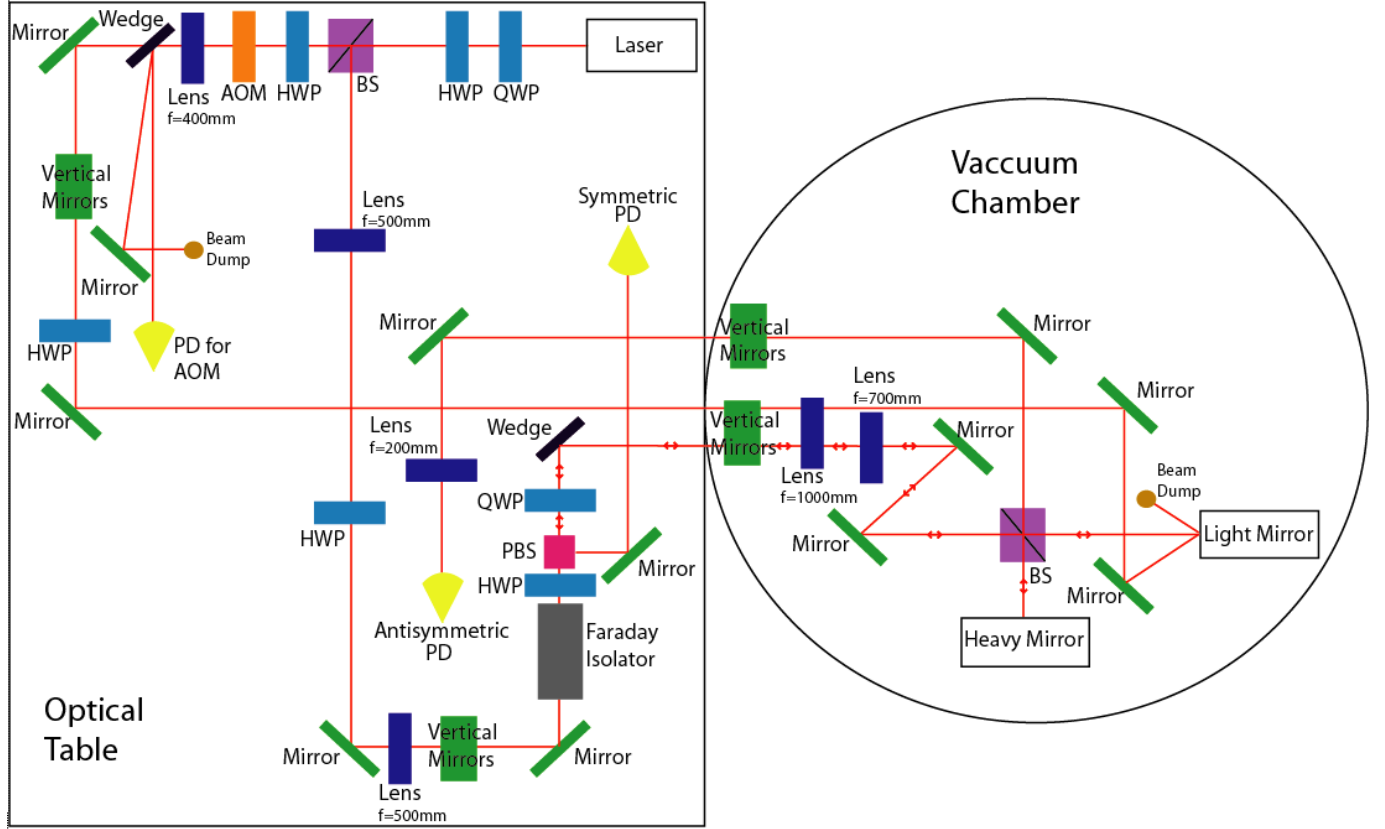


Fig. 1.— The experimental setup of the optical components

Following figure 1, the laser light passes through quarter wave plate, which changes circular polarized light to linear polarized light, and halfwave plate, which changes the direction of linear polarized light. The wave plates are utilized to adjust the power of the laser. Then the light proceeds through a beam splitter, which splits the light into two paths. One of the paths, the light after another half wave plate advances through an Acoustic Optical Modulator (AOM), which is used to modulate the light's intensity. After passing through a lens ($f=400\text{mm}$) to focus the beam, a network of mirrors guides the light down into the chamber in order to reflect it off of the interferometer's small mirror. The labels "vertical mirrors" are periscopes that increase or decrease the height of the laser beam relative to the table. On the other path, the light passes through lens of 500mm , halfwave plate, and another lens of 500mm . It then proceeds through a Faraday isolator, which stops back reflections from returning to the laser. Then, the light, after a half wave plate, advances through the polarizing beam splitter (PBS), which converts light to a well defined polarization and splits the beam. (In this case, PBS is used to deflect the light coming back from the Michelson interferometer.) After a quarter wave plate, the laser light passes a wedge, which splits the

light allowing 10% of the incident light to be transmitted so that 90% of the light is directed into the vacuum chamber. Via mirrors, the light is directed down into the vacuum chamber. After passing two more lens of 1000mm and 700mm, a network of mirrors routes the light into the Michelson Interferometer. The light passes through Michelson's 50/50 Beam splitter, which splits the light into two paths. On one of the paths, the beams advances towards the "heavy mirror" and the other path, the light is directed towards the small mirror. The reflected beam from both mirrors is recombined by the 50/50 beam splitter. The reflected beam (symmetric port) is directed back the same network of mirrors and lens that was used to direct laser light into the interferometer. Due to the quarter wave plate, the returning beam has an orthogonal polarization. It is then deflected by the PBS and focused onto the photo diode. The other beam (antisymmetric) is directed via a different network of mirror and lens to a second photo diode.

The light mirror weighs 23 mg and is 1.5mm thick. A 10 μm silica fiber suspends the mirror in a double suspension setup using a 20 mg middle mass. Since the fiber stiffness is low, the suspension has a high Q-factor. Because the mirror has a small mass, the mirror is easily moved by the intensity modulated laser beam. The image below shows the small mirror suspension.

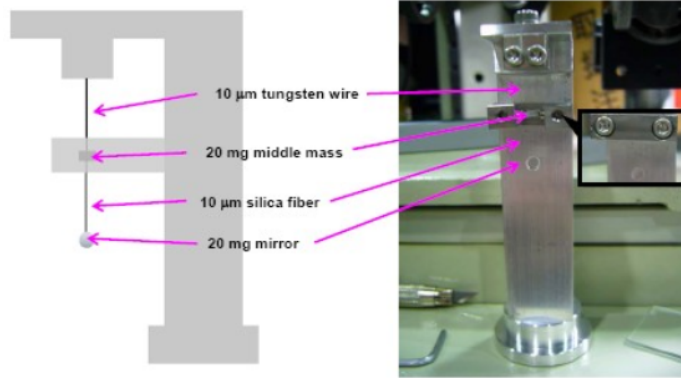


Fig. 2.— Setup of the small mirror

The heavy mirror is a high reflectivity mirror with a diameter of 1 inch. A 50 μm wire suspends the mirror. Eddy-current damping stabilizes the position of the heavy mirror. The interferometer is locked by moving the heavy mirror with coil actuators pushing on four cylindrical magnets that are attached to the back of the mirror. The mirror larger mass keeps the mirror in place so that it is not moved by the intensity modulated laser.

3. LOCKING THE INTERFEROMETER

The signal from the laser (S) is described by

$$S = Ae^{i\omega d} \quad (14)$$

where A is the amplitude, d is the distance travelled, and ω is the frequency. The power of the laser (P) is defined to be absolute value of the signal squared

$$P = |Ae^{i\omega d}|^2 \quad (15)$$

When the light passes through the 50/50 beam splitter of the interferometer, the power of the laser is evenly split into the two paths, P_1 and P_2 , such that $P = P_1 + P_2$ where $P_1 = \frac{P}{2} = A_1^2$ with $A_1 = \frac{A}{\sqrt{2}}$ and $P_2 = \frac{P}{2} = A_2^2$ with $A_2 = \frac{A}{\sqrt{2}}$. Thus, the signal on each path is

$$S_1 = A_1e^{i\omega d} \quad (16) \text{ and } S_2 = A_2e^{i\omega d} \quad (17)$$

respectively. The light travels down the arm of the interferometer and is reflected back towards the BS. Since the light travels a distance of $2L_x$ on one of the path and $2L_y$ on the other path, the signal becomes

$$S_1 = A_1e^{i\omega(d+2L_x)} \quad (18) \text{ and } S_2 = A_2e^{i\omega(d+2L_y)} \quad (19)$$

respectively. The laser light recombines such that the signal read by the antisymmetric photo diode is

$$APD = (A_1e^{i\omega(d+2L_x)} - A_2e^{i\omega(d+2L_y)}) \quad (20)$$

and symmetric photo diode is

$$SPD = (A_1e^{i\omega(d+2L_x)} + A_2e^{i\omega(d+2L_y)}) \quad (21)$$

After simplifications using Euler's formulas, the power is given by

$$P_1 = |APD|^2 = A_1^2 + A_2^2 - 2A_1A_2\sin\left(\frac{4\pi}{\lambda}(L_x - L_y)\right) \quad (22) \text{ and}$$

$$P_2 = |SPD|^2 = A_1^2 + A_2^2 + 2A_1A_2\sin\left(\frac{4\pi}{\lambda}(L_x - L_y)\right) \quad (23)$$

respectively for each port. Locking the interferometer should servo to zero

$$P_n = P_1 - P_2 = 2A^2 \sin\left(\frac{4\pi}{\lambda}(L_x - L_y)\right) \quad (24)$$

The voltage read by a photo diode is proportional to incident power and specifically is given by

$$V = gP_n \quad (25)$$

where $g = R_{hv} \frac{e}{h\nu} QE$ (variables defined in section 1). To connect this to equation 10, note that

$$X = (L_x - L_y) \quad (26) \text{ and } 2gA^2 = \frac{V_{pp}}{2} \quad (27)$$

If we take derivative of equation 25 with respect to X , we arrive at equation 10. At a certain X , the cosine term is equal to 1. This means that a small change in X will results in a small linear change of voltage. By moving the heavy mirror, we lock the mirror at this X .

4. FEEDBACK SERVO AND OPEN LOOP GAIN

The control system of the interferometer includes:

- Pendulum of the heavy mirror
- Driver electronics for the pendulum
- PD read outs with $R=470 \, \Omega$

Below is the picture of the pendulum setup of the heavy mirror

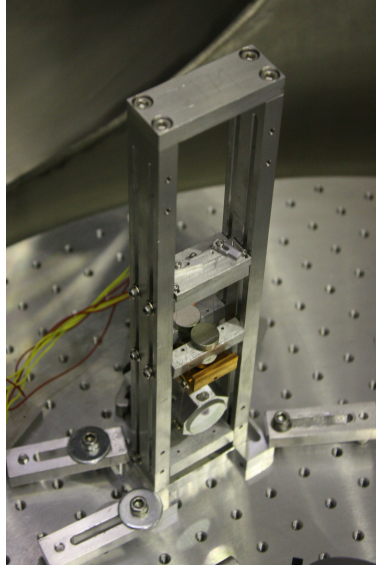


Fig. 3.— Pendulum setup of the heavy mirror

Each of these components has a transfer function. A transfer function (TF) is the relationship between input and output of a system and is given by $TF = \frac{Output}{Input}$ as a function of frequency. Also, transfer functions are represented by bode plots, which are plots that separate the function into its magnitude and phase components. Transfer function are characterized by poles and zeros. A pole at f_p means that the transfer function is equal to $\frac{1}{1+if/f_p} = \frac{1}{1+\frac{i\omega}{w_p}}$. For example, in an electronics system, the pole is given by $f_p = \frac{1}{i\omega RC}$ with $w_p = \frac{1}{RC}$ and $f_p = \frac{1}{2\pi RC}$ where R and C are the resistance and capacitance of the electronic system. A zero at f_z means the transfer function is equal to $1 + \frac{if}{f_z} = 1 + \frac{i\omega}{w_z}$. For example, in an electronics system, the zero is given by $f_z = 1 + i\omega RC$ with $w_z = \frac{1}{RC}$ and $f_z = \frac{1}{2\pi RC}$ where R and C are the resistance and capacitance of the electronic system. The transfer function for the pendulum is a complex pair of poles at the pendulum frequency while the transfer function for the driver electronics and PD is flat. The current setup of the pendulum creates mechanical resonances at 160Hz, 557 Hz, and 700 Hz in the transfer function. The open loop gain (OLG) is the transfer function of the whole control system setup (see Figure 4).

We constructed control servo box in order to satisfy the following requirements:

- stabilize the OLG (avoid OLG=-1)
- two PD inputs
- test points to measure the OLG

Below is a diagram of the final control system setup:

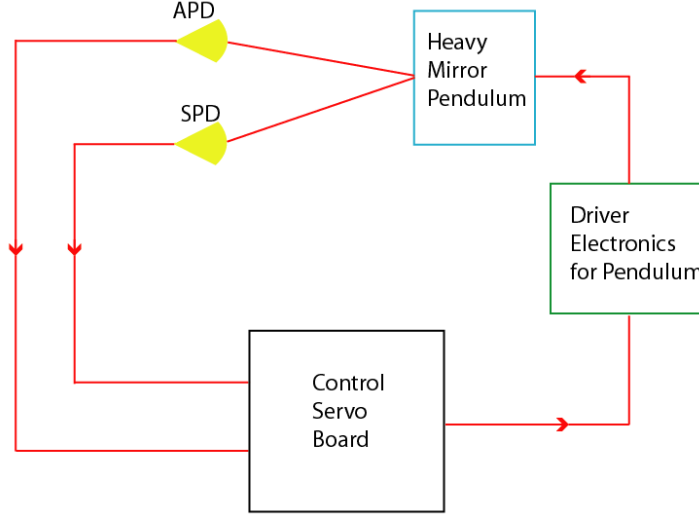


Fig. 4.— The control system setup used to lock the interferometer

Below is a list of the implemented features of the control servo board:

- inverter switches and differential receiver to control and invert the PD signal
- inverting variable gain to adjust the relative gain of the two PD
- summer including offset adjust to minimize the intensity coupling
- Test Exc and two test outputs for open Loop Gain measurement
- Switchable resonant lead filter, consisting of a sallen-key high-pass filter and a straight through path in parallel, to compensate for the first mechanical resonance at 160 Hz
- two zero-pole pairs with variable gain $f_z=66\text{Hz}$ $f_p=420\text{Hz}$ as an additional lead filter
 - one zero-pole was added because the pendulum transfer function dropped as $\frac{1}{f^2}$ while we want the pendulum transfer function to drop as $\frac{1}{f}$
 - the second zero-pole was added for delays and because we needed some extra phase margin around the unity gain frequency.
- switchable integrator $f_p=0\text{Hz}$, $f_z=43.9\text{Hz}$

Below is a schematic of the control servo board

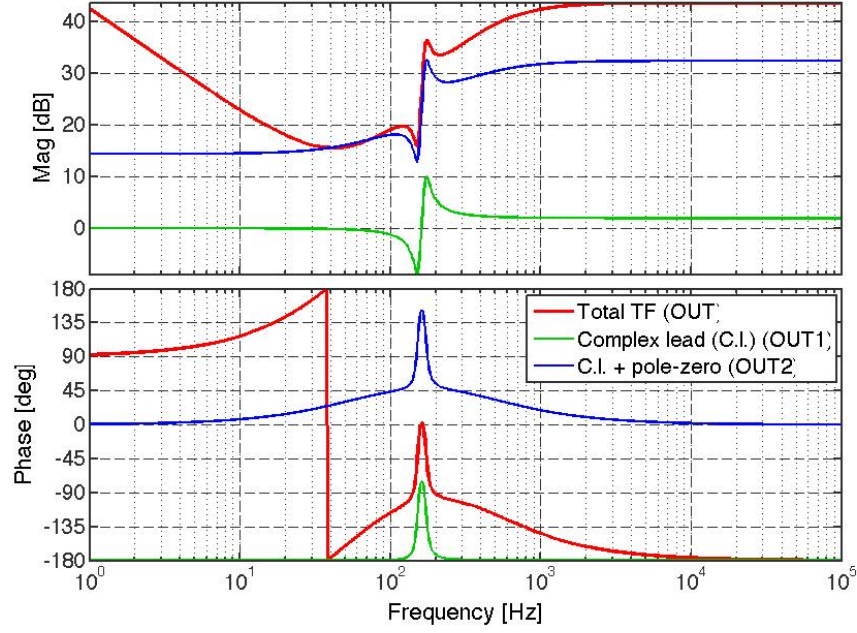


Fig. 7.— Transfer functions of the control servo board. The red line is the total transfer function of the board. The green line is the transfer function of part A as labeled in the schematic (Fig 5). The blue line is the transfer function of part A+B as labeled in the schematic (Fig 5). Note that the transfer function before part A is a flat line.

After implementing the control servo board, the interferometer was locked more stability compared to the first attempt at locking which used three SR560s.

To measure the OLG, the signal analyzer adds a certain signal (Exc), in this case a swept sine wave, to the loop. Swept sine means that the frequency is changed slowly from minimum to maximum. (Note that the signal analyzer is added right before the control servo board). The signal analyzer has two measuring points ($Tst1$ and $Tst2$) before and after the signal Exc so that $Tst2 = Exc + Tst1$ and $Tst1 = -OLG * Tst2$ (see Figure 5). This means that $Tst2 = Exc + (-OLG) * Tst2$ so $Tst2 = \frac{Exc}{1+OLG}$. The closed loop gain (CLG) is defined to be $CLG = \frac{Tst2}{Exc} = \frac{1}{1+OLG}$. CLG is the gain with feedback and it shows how much disturbance the circuits has. Below is the graph of the final OLG and CLG of the overall system of circuits.

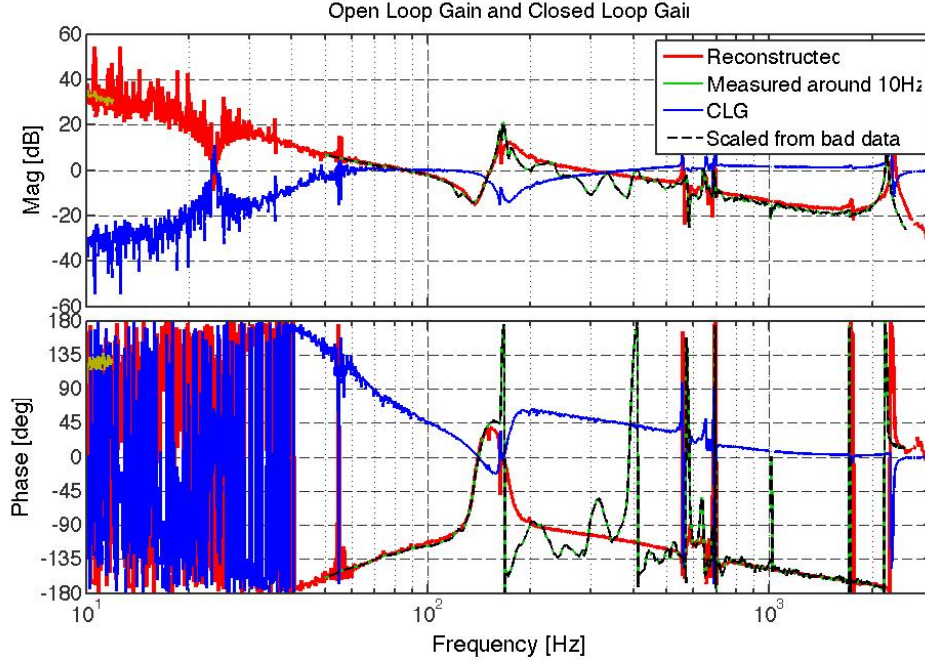


Fig. 8.— Graph of the transfer function of the OLG (red, black, green) and CLG (blue)

As seen in the above graph, there are mechanical resonances at 160Hz, 557 Hz, and 700 Hz with a lead filter at 160Hz.

5. NOISE CALIBRATION

After determining the OLG, the noise spectrum of the Michelson was measured directly from the signal analyzer; however, this noise spectrum needs to be calibrated. By equation 11, $\frac{dX}{dV_{PD}} = \frac{\lambda}{2\pi V_{pp}}$ where V_{pp} is the combined change in voltage from both the symmetric and antisymmetric ports. ($V_{pp} = asymV + symV$ where $asymV$ is the change in voltage of the antisymmetric PD and $symV$ is the change in voltage of the symmetric PD.) Let S_v be the noise measured from the signal analyzer. This measurement determines the noise fluctuations as a function of frequency. If we take $S_v * \frac{\lambda}{2\pi V_{pp}} = S_x$, then S_x is the noise fluctuations as a function of mirror displacement. Moreover, the noise caused by disturbances of the servo feedback loop must be corrected. Thus, the final calibrated noise is $N = \frac{S_x}{CLG}$.

Below is the graph of the calibrated noise:

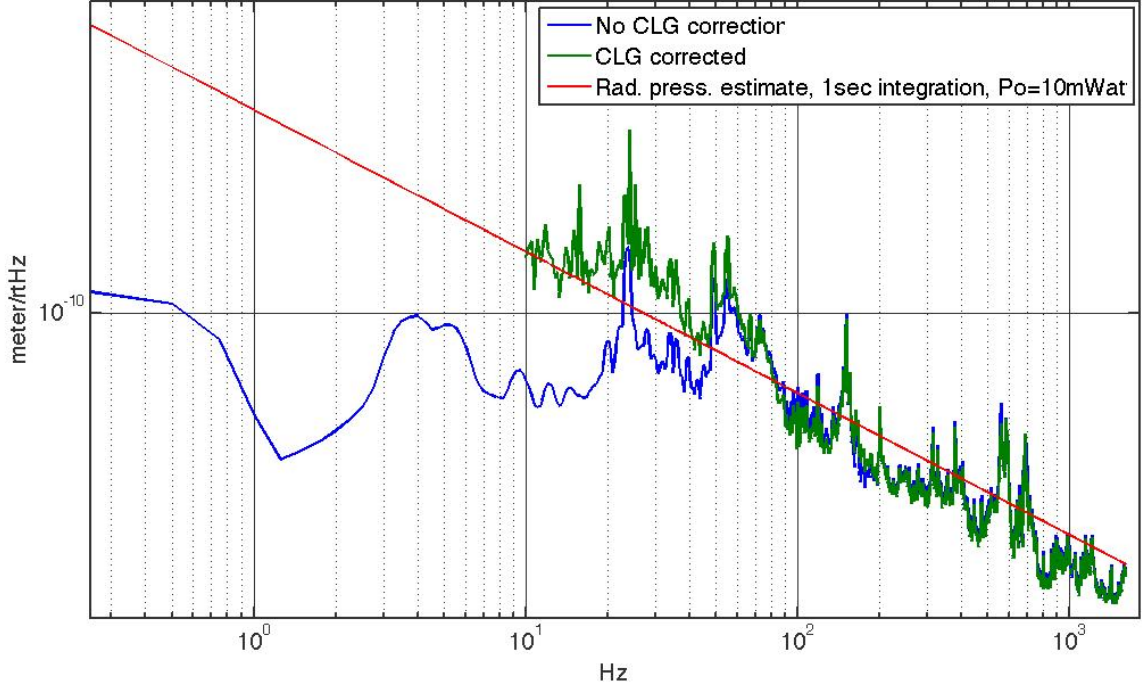


Fig. 9.— Graph of the noise spectrum. The blue line is the noise spectrum not CLG corrected while the green is the calibrated noise spectrum corrected with the appropriate transfer function. The red line is the expected radiation pressure for a 10mWatt standing power and 1sec integration given by equation (8).

The signal from the radiation pressure is close to the noise level. We suspect that the noise between 20Hz and 100Hz is dominated by non-linear upconversion. If so, a higher gain would suppress the upconverted noise. Unfortunately, the applicable gain was limited by the various mechanical resonances in the mirror actuation chain.

6. INTENSITY COUPLING

The power of the laser is affected by intensity fluctuations. Equation 13 in reality has an additional term due to the direct intensity coupling which changes the power signal's amplitude and offset. Thus, Equation 13 becomes $V(I) = \frac{V_{pp} * 4 * \pi * P_m}{\lambda * c * m * \omega^2} * I - o * I$ where I is intensity modulation index and o is an offset. The intensity modulation index is $I = \frac{V_{app}}{2 * V_n}$ where V_n is monitoring photo diode voltage when the AOM is not driven and V_{app} is monitoring photo diode voltage when the AOM is driven at 1.2 Volts.

Initially, the AOM drive signal was on the main interferometer read out beam. However, we observed a large dependence of the intensity coupling on small mirror angular motion and a slightly different PD phase response between symmetric PD and antisymmetric PD. This lead to a limited cancellation efficiency and resulted in a limit for the intensity coupling. The amplitude of the intensity fluctuations (A) divided by equation 11 is the xlimit ($xlimit = \frac{A*\lambda}{2*\pi*V_{pp}}$). Numerically, the $\frac{xlimit}{I}$ is 2.63E-8 m/RIN which dominates the radiation pressure coupling (RIN stands for relative intensity noise). See graph below. However, note that below 6Hz, the pendulum resonance has to be taken into account to estimate the radiation pressure motion.

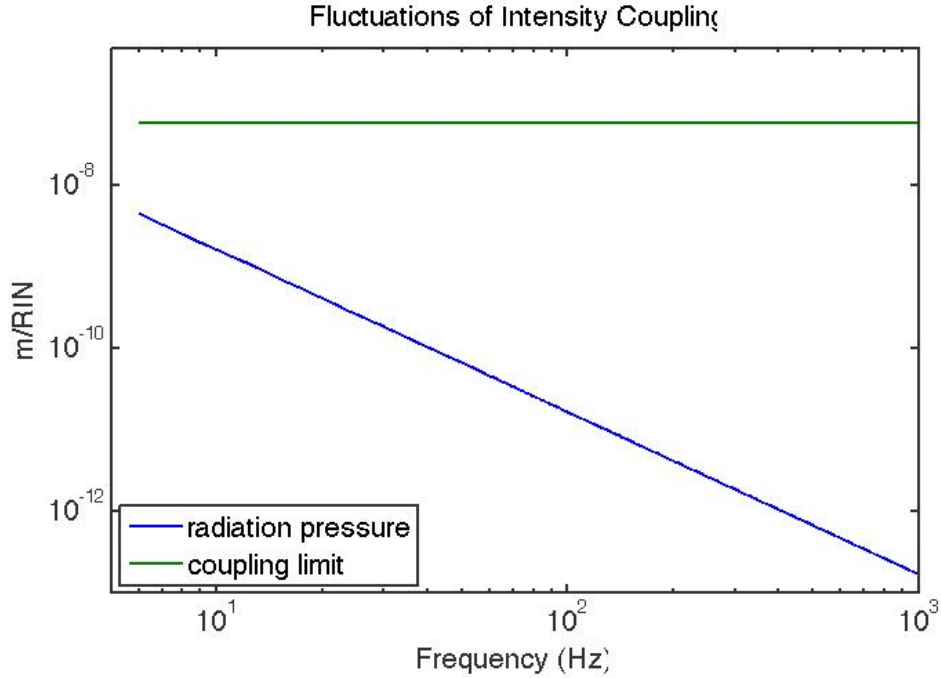


Fig. 10.— Graph of the intensity coupling

In order to solve this problem, we separated the AOM drive from the read out signal. Note that Figure 1 shows the final optical setup.

The electronic setup of the intensity modulation is shown below:

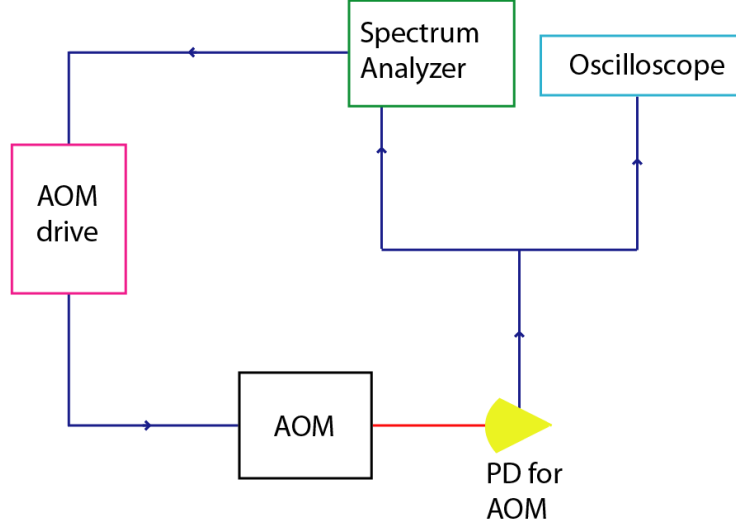


Fig. 11.— Electronic setup for intensity modulation

The raw modulation index for the AOM (a) is $a = \frac{V_{app}}{V_{npp}} = \frac{720}{1100} = 0.65$. However, the PD_{AOM} , PD that reads the AOM output, showed that the intensity modulation was saturating the AOM. Thus, we had to calibrate the actual V_{app} from the read out of the PD_{AOM} . This was done by finding the the transfer function (TFE) from the electrical drive signal to the PD_{AOM} . By multiplying TFE with the electrical drive signal voltage peak to peak (V_{app}), we extract the true $V_{app} = TFE * V_{app} = 0.8855$ at the modulation frequency. Thus the actual modulation index (a) is $\frac{885.5}{1100} = 0.805$.

7. RADIATION PRESSURE

The final step is to modulate the laser's intensity and determine whether the radiation pressure from this modulation causes mirror displacement.

The TF_{up} and TF_{do} , the transfer functions from the AOM drive to the monitoring point (TST1) of the control servo board were measured, but need to be calibrated. The TF_{up} and TF_{do} correspond to inverting polarity of the two PDs of the interferometer. We want to switch the polarities of the two PD because it changes the relative sign of the radiation pressure and scatter coupling. Initially, consider that $TF_{up} = \frac{TST1}{DRV}$. It needs to be corrected by the $TF_{elec} = \frac{PD_{AOM}}{DRV}$ which is the transfer function from the AOM drive to the PD_{AOM} . Thus, $TF_{upcorr} = \frac{TF_{up}}{TF_{elec}} = \frac{TST1}{PD_{AOM}}$. However, both PD_{AOM} and $TST1$ need to be calibrated.

First, to calibrate PD_{AOM} , multiply the power in Watts hitting the small mirror P_m and

the modulation index a so that $P_{mod} = P_m * \frac{a}{2}$ and conclude that $calPD = \frac{P_{mod}}{(V_{pp}/2)}$ (divide by two because we need the amplitude and not peak to peak). This connects the power that hits the small mirror to the voltage read out on the PD_{AOM} .

Second, the calibration of $TST1$, $calTST1$, is done the same way as in the first paragraph of the Noise Calibration section.

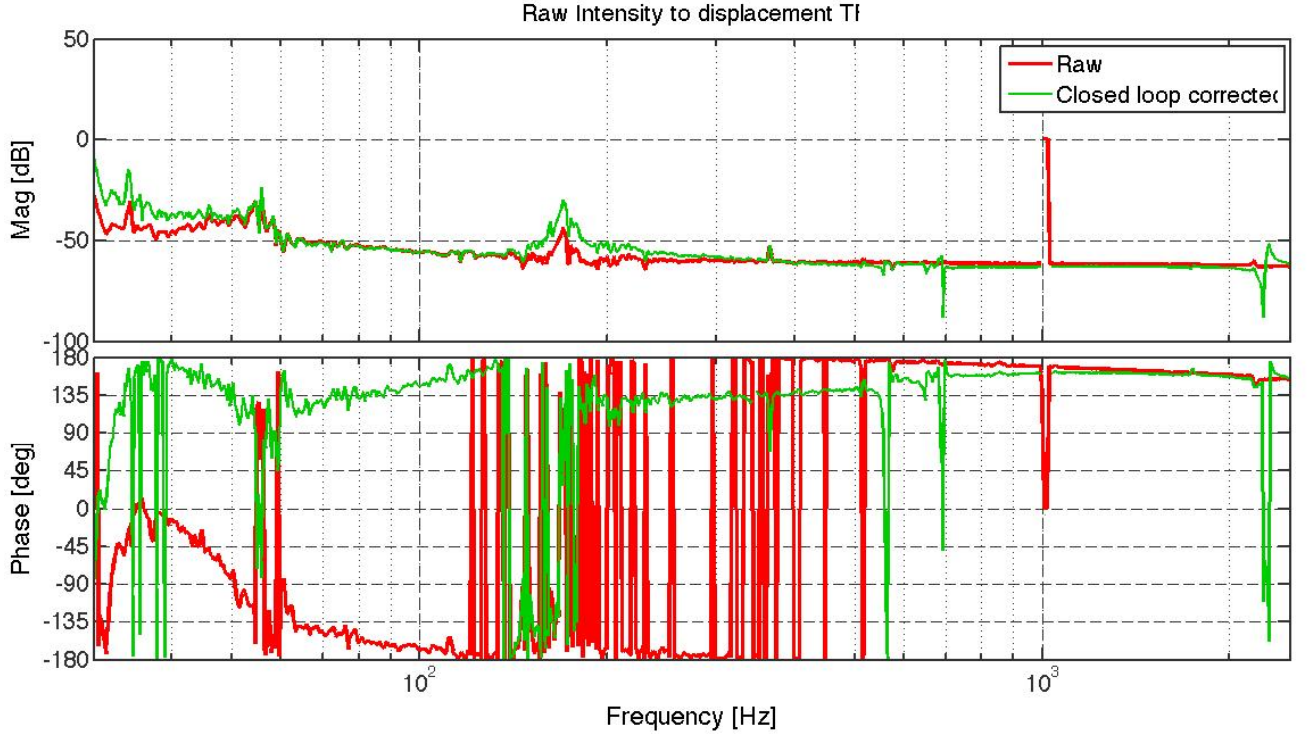


Fig. 12.— Transfer Function with Intensity Modulation.

Finally, the calibrated $TF_{upfinal} = TF_{upcorr} * \frac{calTST1}{calPD}$. The same procedure holds to calibrate the TF_{down} . Numerically at frequency of 10Hz, $|TF_{upfinal}| = 5.5185e-8$ N/m and $|TF_{dofinal}| = 7.2054e-8$ N/m with a combined mean of $6.3475e-8$ N/m. The expected value from radiation pressure is $6.9041e-8$ N/m (using equation (8)). The actual and theoretical values are similar.

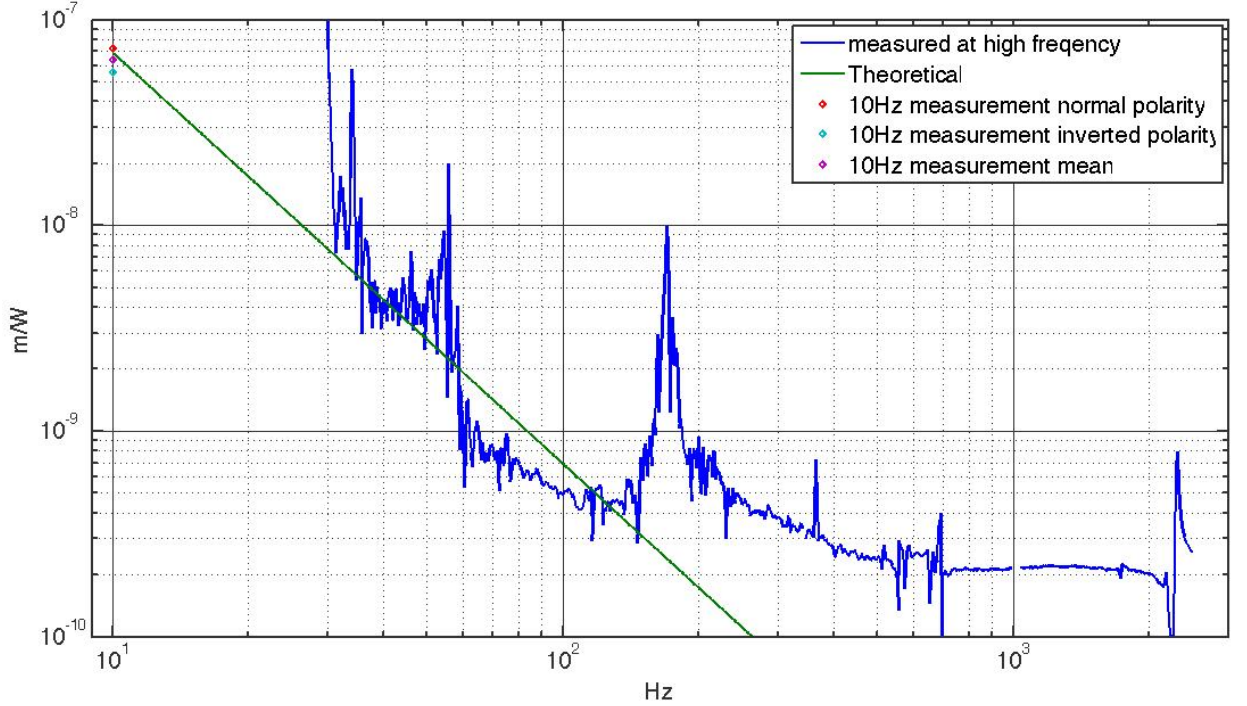


Fig. 13.— Calibrated Transfer Function with Intensity Modulation. The red, light blue, and purple dots are measured 10Hz with normal polarity, inverted polarity, and mean respectively. The green line is the expected radiation pressure. The dark blue is transfer function with intensity modulation at higher frequencies. Note that the integration time for the 10Hz measurement was much longer then the higher frequency measurement. For 10Hz, it is about 128 seconds while at higher frequencies, it was 15 seconds

8. DISCUSSION

At 10Hz, the theoretical and actual measurements of mirror displacement by radiation pressure are very close; however, the measurement needs to be made much more precise. The open loop gain needs to be measured more precisely and also the effect of the switch in polarity at higher frequencies needs to be tested. The calibration can be improved by using the actuation and not the fringe width. A better reduction of the scatter coupling could be accomplished by using the relative gain of the two PDs of the interferometer. Moreover, we need to understand the heavy mirror pendulum. The 160Hz resonance increased when the AOM drive and sensing signal were separated. Note that at this time, the distance of the small mirror and heavy mirror to the BS was also changed. The 160Hz resonance

might be a length to pitch coupling because the spot position changed on the heavy mirror. Since it's a quadruple pendulum, the 160Hz might be where the pendulum changes from swinging all together and separately from the middle mass. Also, more seismic isolation is needed because the interferometer could only lock at night. The beam could also be more accurately focused on the small mirror by adding lens. To decrease the noise, the beam splitter could be suspended. Below is a picture of a prototype suspension for the beam splitter.

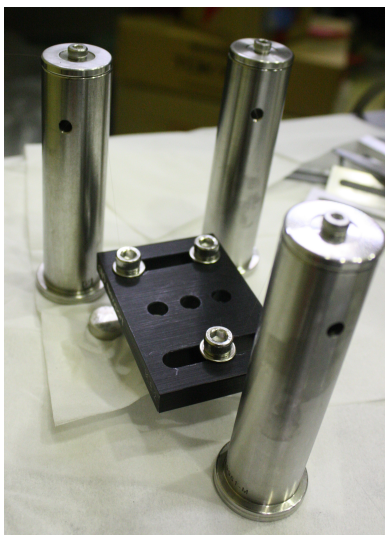


Fig. 14.— Prototype suspension for the beam splitter

REFERENCES

- Hecht, Eugene, and Alfred Zajac. *Optics*. 4th ed. Reading, MA: Addison-Wesley Pub., 1974.
- Rager, Scott. *Measuring Quantum Efficiency of Photo Diode using Radiation Pressure*. University of Florida REU, 2008.