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10m Prototype Interferometer

Reflectance of Coated Mirrors

Most gravitational wave detectors use interferometry to detect the waves. Thus, these detectors use many mirrors. These mirrors are coated with a multilayer stack of alternating materials to obtain a very high reflectance. It can be helpful in designing a detector to know exactly how much of the light these mirrors will reflect at different incidence angles. It is also helpful to know the dependence of the reflectance on the number of layers and thickness of the layers as a way to help cut down coating thermal noise.

There are programs that are already in existence that calculate this, however they are expensive. Instead of buying one of these programs, my project was to create a Matlab code that calculates the reflectance of a mirror at different angles. It was important that this would work for any number of layers on the mirror. The goal was to be able to specify the indices of refraction and thicknesses for each layer as well as the wavelength of the light and have the code return the reflectance for different angles.

I tried to go about solving this problem in two ways, and unfortunately was unable to finish by the end of my time in Germany. I was able to create a code that worked for one layer, and will continue working on the project at home to finish. The approach that I finally decided on was a layer by layer analysis.

My first attempt to solve this problem was using the transfer-matrix method. I tried to come up with a matrix that would relate the light waves on each side of an interface. I then multiplied together all of the transfer-matrices to find out how much of the light would be reflected by the mirror as a whole.

Unfortunately, I was not able to get this method to work properly for more than one layer. When I tried to input multiple layers, the calculated reflectance was much less than what was expected. Because I didn't have much luck with this method, I decided to spend more time working on perfecting the other method.

My first attempt at a layer by layer approach was not correct. I simply added the Fresnel reflectance coefficient for each layer, multiplying each one by a phase factor that took into account the extra distance that the light traveled through each layer at different angles. I then found out that the phase was already accounted for in Fresnel's equations, and that I needed to take into account multiple reflections in each layer.

Therefore I got rid of my phase factor, and began to take into account the multiple reflections in each layer. The light could be reflected an infinite number of times within each layer, as shown in figure 1.
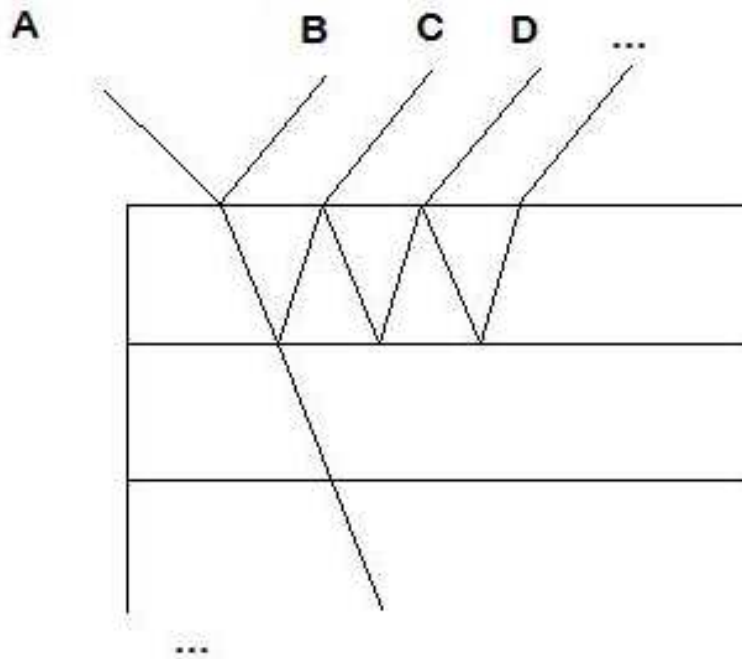

Figure 1- Light in the first layer

If A is a plane wave that hits the first interface, part of it will be reflected and part will be transmitted. Fresnel's equations can be used to calculate how much of A is reflected and how much is transmitted. B, the light reflected at the first interface is simply $B = r_{12}$, where r12 is the Fresnel equation for reflectance at the first interface.

If the light is transmitted, then it becomes t12, the Fresnel equation for transmission at the first interface. We can calculate the phase change, and find that C, a wave that is transmitted through the first interface, reflected at the second, and then transmitted through the first interface again is simply $C = t_{21}t_{12}r_{23}e^{2ip}$, where p is the phase change. Similarly D, a wave that is reflected twice in the first layer is $D = t_{21}t_{12}r_{21}r_{23}^2e^{4ip}$. Then the j-th wave would be $t_{21}t_{12}r_{21}^j r_{23}^{j+1}e^{2ijp}$.

Thus the wave reflected by the first layer would be the sum of each of these partial waves. In this way I decided to try and calculate the reflectance of the mirror as a whole. In order to find the total reflectance, each new layer would be treated as a new reflecting surface. We would then simply repeat the method for the single layer. In this case we would replace the coefficients of the rear surface with those just calculated for the previous layer. Unfortunately due to my unfamiliarity with Matlab, I didn't have the time to fully complete this before I left.

I have created a Matlab function that calculates the reflection of s polarized light on a substrate with one layer. My goal is to expand this to calculate for more than one layer and for p polarized light. I don't have Matlab at home, but I am planning on using a free program called Scilab that is compatible with Matlab to finish up the project.

I learned a lot through this experience. I had only taken a general waves class and we only touched on the subject of optics, so I previously didn't know much about reflectance. I had also never used Matlab or done any programming before. It was exciting to beginning learning

such an important skill in the world of physics. I have also been brought into the global physics community. I was in Germany, but I met physicists from all over the world.