APPLICATIONS OF NOISE IN PHYSICAL SYSTEMS: MEASURING THE LOSS ANGLE OF FUSED SILICA FIBERS AND INVESTIGATING TRANSITIONS FROM LINEAR TO NONLINEAR SYSTEMS IN VIBRATION ENERGY HARVESTING DEVICE

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INTRODUCTION

Gravitational waves are perturbations in space-time created when masses accelerate. Originally predicted by Einstein as a consequence of general relativity, gravitational waves travel at the speed of light, expanding and contracting the space around them in directions orthogonal to their velocity. Theoretically perceptible contractions on the order of 10^{-21}m are expected for accelerated masses on a cosmic scale.[1] Indeed, there has been indirect observation of gravitational radiation, but to this point no direct measurement of these fluctuations has been made.[2]

Today there exist several large-scale interferometric gravitational wave detectors whose aim is to observe these small oscillations of space-time directly. Due to the scale of the distortion created by gravitational waves, noise reduction has become an important obstacle in increasing the sensitivity of these detectors. After seismic noise has been filtered, the predominant limit to the precision of these detectors between 10 and 50Hz – a range where a number of candidate sources are expected – is the thermal noise of interferometric optics and their suspensions.[3] It is for this reason that an effort has been put forth to develop materials with low dissipation.

This paper outlines the fundamental sources of thermal noise in the VIRGO test mass suspension system as well as why and how the author realized a measurement of the pendulum mode of a fused silica fiber supporting a 3kg test mass.

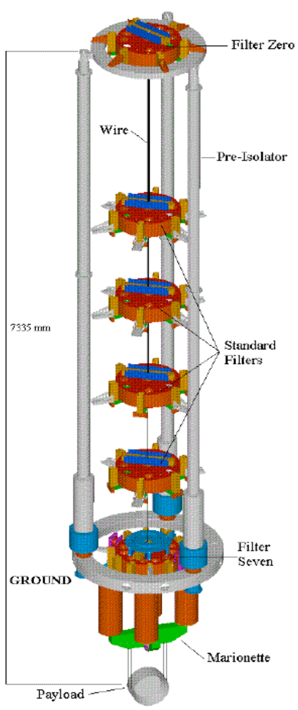

Figure 1: The VIRGO Suspension System. This figure shows the general layout of the VIRGO suspension system. Today the suspension system is altered with the mirror supported by two fused silica fibers instead of the traditional cradling method of suspension.[4]

THEORY

Thermal noise is the name commonly given to the random fluctuations of a macroscopic system in thermal equilibrium with its surroundings.[5] As it is an intrinsic property of all materials, thermal noise provides a fundamental limit to the sensitivity of mechanical instruments. It follows closely that the sensitivity of interferometric gravitational wave detectors is likewise limited by the presence of thermal noise, most notably in the form of suspension thermal noise and the internal thermal noise of the test masses.[6] Regarding the former source of noise in VIRGO, it has been noted that the optimum material for reducing the suspension thermal noise is one with a high breaking stress and low internal friction.[7]

It has been shown that thermal noise can be described by a simple application of the equipartition theorem. Each degree of freedom of a given material contributes an average energy kT to the system where k is Boltzmann's constant and T is the temperature when the system is at equilibrium. This energy can be observed as noise detected in measurements made of the system.[8]

The presence of internal friction originates from the relationship between stress and strain in materials. If a periodic stress is applied to a medium, that substance will respond with a strain that is inevitably out of phase with the original force. The difference in phase angle between stress and strain, known as the *loss angle* ϕ of the material, is a measurement of internal friction.[9] The lower the loss angle of the material, the lower the level of energy dissipated per oscillation. On a more fundamental level, for fused silica the relaxation and repositioning of strained Si-O-Si bonds after the application of mechanical strain is what leads to internal dissipation.[10]

The Fluctuation-Dissipation Theorem describes how fluctuations in a system will be dissipated as the system returns to equilibrium. This theorem has been applied to describe the dynamics of monolithic suspension systems of interferometric GW detectors, as well those that describe the internal friction from test masses.[11] The Fluctuation-Dissipation describes how a system dissipates energy as

The quality factor Q of a pendulum is inversely proportional to its loss angle ϕ and given by the expression

$$Q = \phi^{-1} = \frac{\omega_o}{\Delta\omega} \qquad [1]$$

where ω_o is the resonant frequency of the pendulum mode and $\Delta\omega$ is the bandwidth. In reference to noise reduction in interferometric GW detectors, note that one may improve the sensitivity by constructing a suspension system with a very high mechanical *Q*. This way, one is able to concentrate the thermal noise of the pendulum into a very narrow frequency band surrounding resonance.[12] Quality factors for fused silica have been recorded up to $5x10^{-7}$, making it an excellent candidate for a suspension material.[13] In addition, the loss angle of silica is lower than most candidate materials across a wide range of frequencies, including the frequency bands over which VIRGO is expected to be the most sensitive.[14]

Recently, the VIRGO group at Perugia measured the quality factor of a full-scale model of the VIRGO suspension system. A dummy mass weighing 22 kg was hung from two fused silica fibers attached to an A-shaped steel frame with a low-loss clamping mechanism. The measured value for the pendulum's *Q* was an order of magnitude lower than expected ($3x10^{6}$ versus the theoretical $4x10^{7}$). The pendulum *Q* of this system is dependent on several factors, including losses from the upper, lower clamping system, and recoil losses. After the clamping mechanisms were ruled out as a predominant source of error, it became necessary to investigate the relationship between amount of mass suspended from the silica fibers and the noise introduced by recoil losses.

The limiting Q due to recoil losses be modeled with the equation describing a mass *M* suspended from a pendulum with wires of length *l*:

$$Q_r = \frac{kl}{Mg\theta} \qquad [2]$$

where *k* is the spring constant of the structure and θ is the phase angle between the force applied to the structure and its displacement. We can see from this equation that *Q* is expected to scale linearly with the suspended mass *M*. Using the same suspension system as the Perugia group's full-scale model but with only one fiber (see the Nov. 2009 VIRGO report for more structural details), the quality factor for the same fused silica sample with varying weights attached was measured.[15] These measurements were all taken at a vacuum of $2.0x10^{-6}$ Torr.

The measurement that I will make is but one in a series investigating the dependence on the mechanical quality factor of fused silica to the mass of the suspended by the pendulum. If the measurements are in accord with our model for recoil losses, then the pendulum *Q* should scale linearly with the mass. Setting the pendulum into motion and monitoring its oscillations as they decay will give us a characteristic ring-down time τ for that sample. By using the equation:

$$Q = \pi f_n \tau_{acq} \qquad [3]$$

where f_n is the frequency of the *nth* mode, we can calculate the pendulum mode Q of the sample.

So far, two measurements of the pendulum Q_r with varying masses have been made; first using a 22 kg mass and secondly using a 1 kg mass. So far, the ratio between the two quality factors has been proportional to the inverse ratio of the two masses, in agreement with equation [2]. Next, three more measurements, one for a 2kg mass, one for a 3 kg mass, and a last for a 5 kg mass, shall be made. One measurement of Q has already been taken, but more data points are needed for this weight to secure the accuracy of these results. My project will be to set the stage for and take the measurement of the Q_r of a fused silica fiber pendulum suspending a 3 kg mass.

PROCEDURE

The goal of this procedure is to set up an apparatus that will have the ability to monitor the ringdown time of a mechanical pendulum composed of a fused silica fiber with supporting a 3 kg mass. The fiber is a fused silica bar of 1.5 mm diameter welded to a silica block inserted into a steel box. The fiber was clamped using a similar mechanism into a large A-shaped steel structure, as seen in **Figure 2**. The photodiodes were suspended around the fiber by clamping them to two steel rods positioned orthogonally to each other. Each photodiode was supplied with 8.2 volts throughout the 3 kg trails. All data presented was collected at a vacuum of 2.0×10^{-6} Torr.

The motion of fiber was detected by using two shadowmeters, each comprised of a light emitting diode paired with a split photodiode. The shadowmeters were placed orthogonally around the fiber so that its movement in either direction could be logged. The differential signal from the photodiodes' two windows were amplified and then analyzed via Labview. The analytic signal, that is, the peak-to-peak amplitude of our

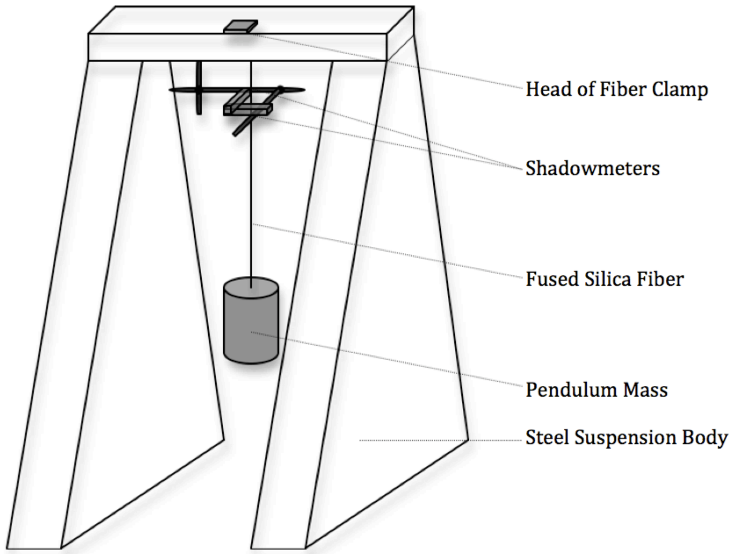

Figure 2: Experimental Setup of *Q* Measurements. This diagram shows the experimental setup used in the measurement of mechanical *Q* for the pendulum. A 1.5mm diamter fused silica fiber suspends a 3 kg mass using a low loss clamping mechanism. The wire is suspended from a large A-shaped steel frame. Photodiodes clamped to steel rods around the top of the fiber were positioned orthogonally to each other in order to track the fiber's oscillations.

signal over a designated sample period, and the signal's power spectrum were also given a place in the Labview analysis. Using these three pieces of information we were able to track the movement of the fiber as its oscillations decayed.

RESULTS & DISCUSSION

There was a strange behavior observed in the analytic signal: instead of a simple sinusoid there was a more complex periodic function observed. According to other reports by the Perugia group with an identical clamping mechanism, the contribution of the upper clamps do not contribute significantly to the overall dissipation of the monolithic pendulum, so another mechanism must have been at play. After several periods were observed it was clear that this behavior was nothing temporary, so screenshots over time of the raw signal were taken to investigate this phenomenon. The screenshots revealed nothing unexpected about the behavior of the signal, as it simply mimicked the information given in the analytic signal with not.

Having not received any new information from the raw data, I chose to vary the sampling rate to see if this would affect the readout. As the sampling rate as heightened, the period of the function became dramatically shorter. I changed the sampling rate several more times, and with each change of the sampling rate, the period of the analytic signal would change as well. With this information, it became clear that what we were

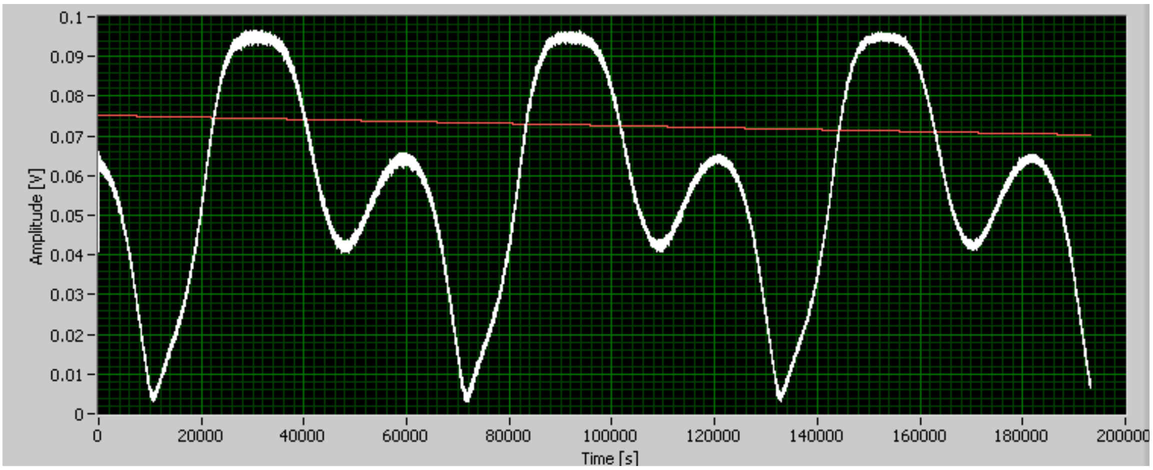

Figure 3: Observed Analytic Signal of Fused Silica Fiber Oscillations. This figure shows the analytic signal – that is, the peak-to-peak amplitude of the fused silica fiber oscillation – in the first 200,000 seconds of data collection. Each 600-second section of the curve the graph represents the peak-to-peak amplitude of the silica fiber's oscillation during this time at a rate of 20 samples/second. By observing the period of the signal change as the sample rate was varied, it became clear that the unusual periodic motion observed in the signal was a consequence of aliasing.

viewing was due to aliasing, as the sampling rate was out of phase with the rate of the real sinusoidal signal.

The signals that we receive from our shadowmeters are not simply that of the pendulum mode of our fiber. Instead, they are the result of combined oscillations of all modes of the pendulum as well as other factors such as distortion introduced by the clamping system, procession of the pendulum about its vertical axis, torsion of the fiber, and perhaps still other distorting components. All of these forces work to complicated our analytic readout and make extracting the pendulum Q from our given signal very complicated. However, if we refer back to the ring down time expressed in equation [3], we can see that, as long as we are able to calculate the ring down time of the sample accurately, we should be able to calculate the overall Q.

II. EXPERIMENT 2: EXPLORING CONCEPTS IN ENERGY HARVESTING: Determining the Power Output of a Nonlinear Oscillator Exposed to Stochastic Noise as Δ Varies

INTRODUCTION

The reduction of noise in the VIRGO suspension system is one of many objectives that the Noise in Physical Systems laboratory is working towards. Another one of these goals is to develop technologies that harvest energy by making use of ambient sources of energy, be them mechanical vibrations, light, or heat. To do this, the NiPS laboratory explores the theoretical possibilities for harvesting through modeling and then realizes them into devices that use either the power of capacitance, inductance, or piezoelectricity to harvest energy. As I was scheduled to attend the Noise in Physical Systems' Summer School on the topic of energy harvesting, it became apparent that a second project investigating this topic would be both timely and instructive.

THEORY

The need for autonomous energy-harvesting devices for small-scale electronics has been a growing topic of research worldwide. Traditionally, these oscillators have been constructed such that their resonance frequencies matched that of the source to which they were exposed. This system is ideal for those vibration sources that are very specific in their vibrating frequency, however these resonating oscillators are not ideal in situations where the frequency range of the source is widely distributed.[16]

When harvesting noise from real systems, the linear (resonant) approach has several disadvantages. Firstly, a considerable difficulty lies in tuning a linear oscillator's resonant frequency to the predominant frequency of the source of energy to be harvested. The resonant frequency of an oscillator is a product of its physical characteristics, including its geometry and the medium out of which it is made. The second problem with resonating oscillators with a high quality factor is that the frequency spectrum provided by noise sources is in fact never in a narrow frequency band, but instead spread out over a wide range of frequencies.[17] This means that for a mechanical oscillator with a very high quality factor much of the energy available from the noise is not being used.

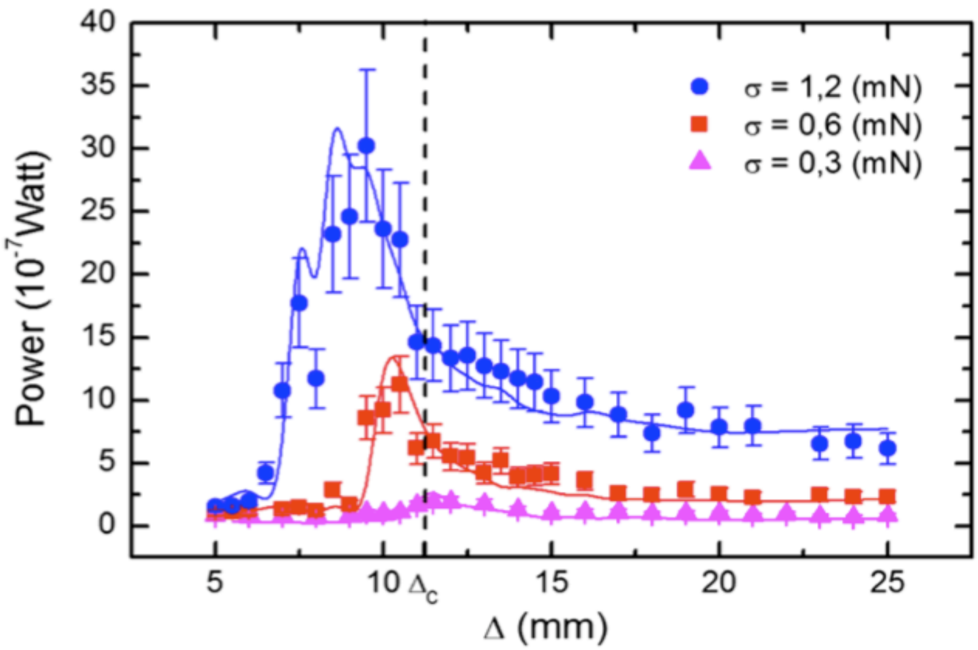

Figure 4: Theoretical Power Output for an energy harvesting oscillator transitioning from monostable to bistable potentials. This graph shows the theoretical and actual results of an experiment conducted by the NiPS laboratory in 2009 from three different trials of noise standard deviation σ. In the experiment a piezoelectric oscillator conceptually similar to the one described in this paper was exposed to a stochastic force with a Gaussian profile. The voltage across a load resistance was recorded as the distance Δ was increased from 5mm to 25mm. The shape of the curves from this experiment is what we expected to see in our piezoelectric spring as the spring makes the transition from a linear to a nonlinear system. Adapted from Vocca et al. http://www.nipslab.org/files/PhysRevLett_102_080601.pdf.

It has been shown that a non-linear oscillator may outperform a linear oscillator in a realistic noisy environment, i.e. when exposed to a stochastic force. In **Figure 4** we can see the predicted curve shape for a piezoelectric oscillator as it makes its transition from linear to nonlinear regimes. The average electrical power P is calculated from the root mean squared $V(t)/R_l$ where R_L is the load resistance. The vertical dotted line marks the approximate transition between linear and non-linear behaviors. We can see that the power harvested from this devices goes from very low when the magnets are very close to each other, peaks at a certain point during the non-linear regime, and then decays back down again as the oscillator makes its transition into a linear regime with a large Δ. As we look to our procedure, we predict a similar graph shape for our piezoelectric oscillator. The goal of this procedure is to realize a piezoelectric spring oscillator whose power can be recorded as it transitions from the non-linear into the linear regime.

PROCEDURE

The goal of this procedure is to investigate the relationship between the distance between magnets Δ and the root-mean-squared *V(t)* voltage drop across a 15kΩ load resistance produced as the oscillator is exposed to a set of realistic stochastic vibrations. The oscillator is a four-layer, 3cm x 3 cm piezoelectric spring composed of lead zirconate titanate (PZT) with two small magnets attached to either side of its innermost loop. The magnets were shielded with a 1 mm thick Teflon square to keep them from sticking to the ferromagnetic iron screws to which they were attracted (see **Figure 5**). Iron screws acted as polar opposing magnets and were placed into the frame a variable distance Δ from the magnets. The piezoelectric spring was firmly clamped into a rigid frame that was connected to a shaker.

For each set distance, the screws were positioned a certain distance Δ from the magnets. Ensuring that both screws were the same position away from both magnets was one of the most challenging components of this experiment, as the adjustments were on the order of millimeters and made manually.

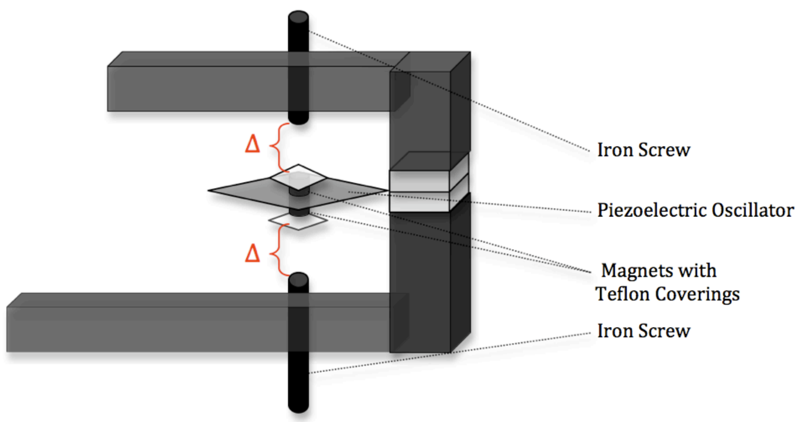

Figure 5: Experimental Setup of *V(t)* measurement of a piezoelectric oscillator. This illustration shows a piezoelectric oscillator built to harvest energy in both linear and non-linear regimes. The piezoelectric oscillator is carved in the form of a spring, and has attached to it two magnets with Teflon coverings. The magnets are a set distance Δ away from iron screws. As Δ is decreased from a very large distance to a much smaller one, the oscillator is expected to transition from a linear (resonant) oscillator to a non-linear (bistable) oscillator.

For each trial, the frame was exposed to a stochastic noise with a Gaussian distribution, having a unitary standard deviation, a zero mean, and an exponential autocorrelation function with correlation time $\tau = 0.0016$s. The root-mean-squared for the voltage drop *V(t)* across the load resistance R_L was recorded after approximately 8 minutes of exposure to this stochastic force. Voltages were recorded as distances were varied from 0.40cm to

0.54cm and voltage readouts were digitized and analyzed via Labview. The power *P* from each trial was then computed as $P=V(t)^2/R_L$ and plotted against Δ in **Figure 6**.

RESULTS & DISCUSSION

The results obtained from this experiment did not correlate significantly with the theoretical model presented in **Figure 4**. It is reasonable to conclude that the error introduced to the system was weighty enough to overpower the voltage difference expected as the oscillator transition from nonlinear to linear that may have been observed. **Figure 6** displays the power *P* as a function of distance *Δ* for all trials of the oscillator.

With a lack of significant results, we may examine the sources of error closely in this experiment and suggest improvement for similar measurements taken in the future. One major source of error in these results was likely to be the deformation of the piezoelectric spring as trials continued. The amplitude of the vibrations was such that there may have been some permanent change in the piezoelectric string caused by the stress of the vibrations. There were several instances when adjusting the small distances between the magnets and the ferromagnetic screws that one of the magnets attached to the spring would stick to a screw and place a large amount of strain in one direction. Another factor that contributed to the spring's deformity was the fact that the system was positioned horizontally, allowing the weight of the magnets on the innermost coil of the spring to pull them continually downwards.

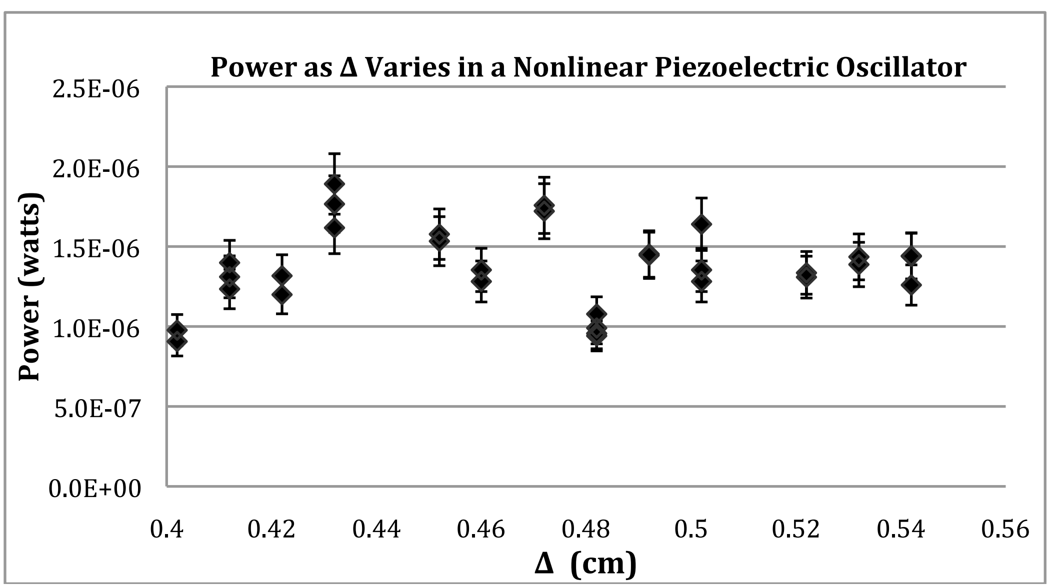

Figure 6: Computed Power for Energy Harvester: This graph shows the computer power $P=V(t)^2/R$ where *V(t)* is the voltage drop across the load and *R* is the load resistance. Each point on the graph

represents the results from an 8-minute exposure to a stochastic vibration with a Gaussian profile. As we can see, no significant correlation arose between voltage and power as Δ was increased from 0.40cm to 0.54cm.

A second major source of error in this experiment was the disparity between distances between the top magnet's Δ and the bottom magnet's Δ. Because the distances in this experiment were on the order of millimeters and adjusted by hand, it is not farfetched to speculate that there were significant differences between the two set Δs. Another source of error may have been variations in magnet size, as the top magnet in this apparatus appeared to be chipped.

In the future this experimental setup could be made better by introducing a micro-stepper for adjusting the distances between the iron screws and the magnet. Also, positioning the piezoelectric spring vertically so that it was not subject to extra strain vertically. With a more sensitive apparatus, it may be possible to observe the transition from linear to nonlinear using a similar piezoelectric spring device.

III A Note on My Cultural Experiences and the NiPS Summer School

During my stay in Perugia, Italy, I had many opportunities to travel and to immerse myself in the language and culture of Italy. Almost every weekend of our stay in Perugia, my REU partner and I took the opportunity to travel to a different place. These locations included Paris, Rome, Florence, Genoa, as well as local Italian hill towns such as Assisi, Todi, Spello, and Orvieto. Living in Italy was an experience I will never forget, and the experience has me looking for similar international research opportunities for the future.

The Noise in Physical Systems Summer School was an invaluable experience. Not only did I get to hear lectures by some of the most progressive theorists and researchers involved in energy harvesting, but I also had the pleasure of meeting these scientists who are active in the international science community on a personal basis. The Summer School was well attended by physicists and electrical engineers at every level ranging from PhD students to seasoned experts and, even though some of the lecture content was over my head, I enjoyed learning as much as I could about these interesting topics.

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