Quality Factor of a Single Pendulum for Use in Gravitational Wave Interferometry

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Abstract

In this experiment, we worked to determine the quality factor of a single pendulum for use in a gravitational wave interferometer. The quality factor is a measure of the damping of an oscillator. A high quality factor is desired, as it means a low level of thermal noise. Our goal was to achieve a quality factor on the order of 10^4-10^5 under vacuum conditions. Though the mirror we used is only 23 mg, a much smaller scale than those used in actual gravitational wave detectors, we are still able to explore the quality factor of the materials used. Potential sources of damping explored included structure damping, air resistance, and friction at the attachment point. We determined the quality factor of the pendulum mode, yaw (or rotational) mode, and violin modes of the wire. We also compared the quality factors found both in air and under vacuum conditions to determine the extent of air resistance's effect on the motion.

1 Introduction

Due to the required high sensitivity of gravitational wave interferometers, careful studies of the thermal noise of the interferometer components are required. Pendulums are used in these systems as they have less thermal noise than the materials of which they are composed. At the NAOJ, we studied the quality factor of the 23-mg mirror suspension, shown in Fig. 1. This suspension is a small-scale model of the suspension used as the end mirrors in the LIGO and TAMA interferometers. However, using this size mirror will allow us to observe the radiation pressure noise with a less powerful laser, as radiation pressure is proportional to power. Last year, Stuart Reid et al. measured the quality factor for the pendulum mode and and 2nd violin mode of a double pendulum with a clamped middle mass to be 7360 and 11505, respectively. These results are 15 times smaller than the calculated value [1]. At this level, the suspension thermal noise is only 2 times smaller than the radiation pressure noise budget. Though the quality factor does not seem limited by the UV resin, re-measurement is required.

2 Theory

2.1 Quality Factor

The quality factor is a measurement of the strength of the damping of a resonator's oscillations, and can be calculated as the ratio of the resonance frequency over the full width at half power of the resonance [2]. It is a measure of the sharpness of the resonance peak. It can also be defined as the number of radians of oscillation for the amplitude of the free oscillation to fall by 1/√e. Using the latter definition, we calculated the Q-factor as

$$Q = \pi f_0 t_0 \quad (1)$$

In systems with a low Q-factor, the resonance is highly damped and the energy is dissipated quickly. By calculating the quality factor of the pendulum system, we can determine the loss factor, ϕ, and the thermal noise. The loss factor is the lag angle between a force applied to the oscillator and the response of the oscillator [3]. It is related to the quality factor as

$$\phi = 1/Q \quad (2)$$

In the case of structure damping, the loss angle $\phi(f)$ can be taken as independent of frequency [4].

2.2 Thermal Noise

Dissipation sources for the pendulum include thermoelastic damping and structure damping from the suspension wire, friction and deformation at the suspension point, and air resistance [5]. Structure damping arises from internal friction in the materials. There is some lag time between the moment stress is applied to a material, and the response of its internal degrees of freedom [4]. The thermal noise spectrum with respect to frequency is

$$x^2(f) = \frac{4k_B T k\phi}{2\pi f\left[(k - m(2\pi f)^2)^2 + k^2\phi^2\right]} \quad (3)$$

Here, T is temperature, m is the mirror mass, and k_B is Boltzmann's constant. The spring constant k is calculated as $k = m\omega_0^2$, where ω_0 is the fundamental angular frequency of the pendulum. From these relations, it can be seen that the quality factor and thermal noise spectrum are inversely related.

2.3 Resonance Frequencies

We measured the quality factor for three resonance modes of the pendulum: the pendulum mode, the yaw (rotational mode) and the transverse (violin) mode.
The pendulum motion comes from the force of gravity on the mirror. We calculated the pendulum frequency and violin frequencies expected for our pendulum setup. For a single pendulum, this frequency is calculated as

$$f_{pend} = \left(\frac{1}{2\pi}\right)\sqrt{\frac{g}{L}} \quad (4),$$

where L is the length of the suspension fiber and g is the acceleration due to gravity. In our case, the pendulum mode frequency is 4.98 Hz.

The yaw mode is the frequency of the rotational motion of the mirror about the vertical axis (parallel to the fiber). The calculation of the yaw motion frequency is complex, and we determined it experimentally.

The violin modes come from the transverse motion of the fiber. These occur as a harmonic series. For a thin wire, the damping mechanism for these modes is independent of frequency [6]. The potential energy of the wire's bending along its length is negligible compared to the bending energy at the fixed points. This energy is constant for all harmonics. The predicted violin modes of a single fiber suspension are

$$f_n = \left(\frac{n}{2L}\right)\sqrt{\frac{Mg}{\rho}} \quad (5)$$

where ρ is the linear density of the fiber, *M* is the mirror mass, and *n* is the violin mode [2]. For our setup, the frequency of the violin modes, f_n, is 1.8056*n kHz.

3 Experimental Setup

3.1 Q-Factor Measurement Procedure

We created a setup to measure the quality factor of the pendulum, yaw and violin modes of the single pendulum. Our general procedure was to excite the resonance frequency of the pendulum using a high voltage wire placed 1-2 mm away from the fiber set with an oscillating current at the desired resonance frequency. Once the oscillation of the pendulum stabilized, we shut off the high voltage, and recorded the decay of the motion's amplitude with an oscilloscope. We measured the decay both in air (with the lid of the chamber on) and under vacuum conditions. When taking measurements, only the laser being recorded was turned on.

When recording the pendulum motion and pitch motion, we excited the motion at the fundamental mode of the pendulum. As such, the quality factors calculated for the pitch motion should match the quality factors of the pendulum motion, as both are recording the decay of the pendulum mode oscillation. For the yaw motion, we excited the motion at the rotational mode resonance frequency. When measuring the violin mode, we used the output of the vacuum-compatible PD.

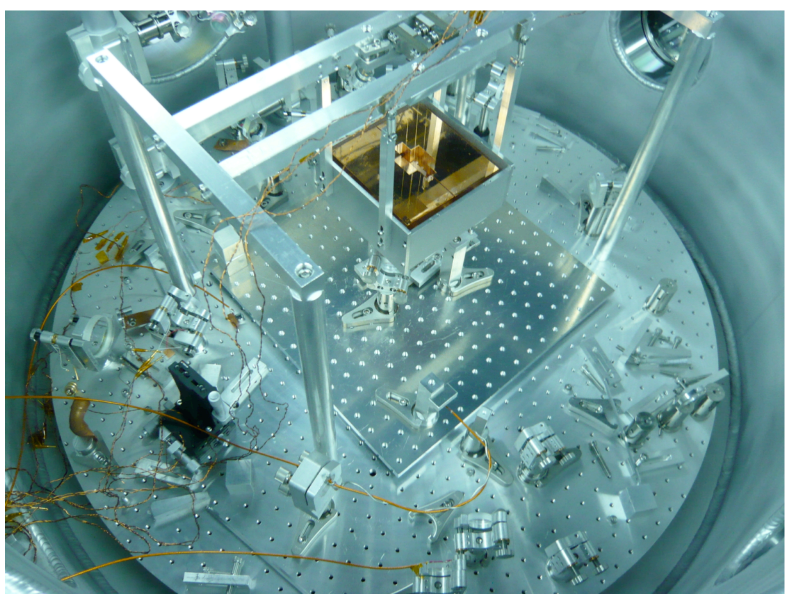

Figure 1. View of experimental setup inside vacuum chamber.

3.2 Suspension

For this experiment, we first had to build our pendulum setup. Rather than clamping the middle mass of the existing double pendulum, we chose to build a new single pendulum. It is difficult to clamp the middle mass without breaking the fiber. Also, the friction of the fiber moving in the clamp would create another source of noise. Using a single pendulum simplified our experimental setup and decreased the possible sources of noise.

To make the silica fiber, we stretched a quartz pillar (1 cm long, 2 mm in diameter) heated by an oxyhydrogen flame to create a fiber 100 μm in diameter. Then, these fibers were heated and stretched with tweezers to create a group of candidates for the desired 10 μm diameter fiber. After measuring their diameter using the Keyence VH-8000 Digital HF Microscope, looking for consistent thickness, we chose a fiber with a thickness of 10 μm +/- 3 μm. Next, we assembled the pendulum using UV resin (ThreeBond 3030®). We placed a small drop of the resin on the mirror using tweezers, and lowered the fiber's tip into the resin, making sure not to bend the fiber. We then cured the resin with UV light for 30 seconds. Next, we repeated this procedure to attach the free end of the fiber to the suspension system. We performed this procedure while viewing the attachment points with a microscopic camera for precision. We then secured the new pendulum (see Figure 2) to the large suspension to minimize the influence of seismic noise.

Figure 2. 23mg mirror suspended by 1 cm long, 10 μm diameter silica fiber.

3.3 Photodiodes

Photodiodes are used in interferometers to measure the amount of light that strikes them. They are able to convert oncoming photons to an outgoing voltage.

We assembled a vacuum compatible photo diode to place behind the suspension to record the oscillation of light scattered by the wire. Two PDB-C609-2 (Advanced Photonix Inc. ®) photodiodes were soldered together. They were then attached to a mirror with UV cured resin. Once placed inside the chamber, these photodiodes recorded the motion of the visible laser light scattered by the fiber.

To measure the optical level, we placed a QPD (quadrant photo detector) outside the chamber. This QPD has the capability to measure both the horizontal (yaw) motion and the vertical (pitch) motion of the IR beam spot reflected off the suspended mirror. It also measures the power of the beam spot.

3.4 Lasers

To measure the optical level, we used an infrared (IR) laser, the Innolight Mephisto OEM with a power of 200mW and a wavelength of 1064nm. This laser was placed outside the chamber and directed at the suspended mirror by a series of mirrors.

To measure the transverse motion of the fiber and the pendulum motion, we used a visible laser. Using a visible laser allowed us to align the system with greater ease and to see the scattered light on the wall of the chamber (Figure 3).

3.5 High Voltage Wire

A high voltage wire was placed 1-2 mm away from the fiber in order to excite the resonance modes of the pendulum (Figure 4). An oscillating current excites the motion. As the placement of the wire has a large effect on the fiber's motion, we moved the wire carefully to obtain large amplitude of oscillation without having the fiber touch the wire when in motion.

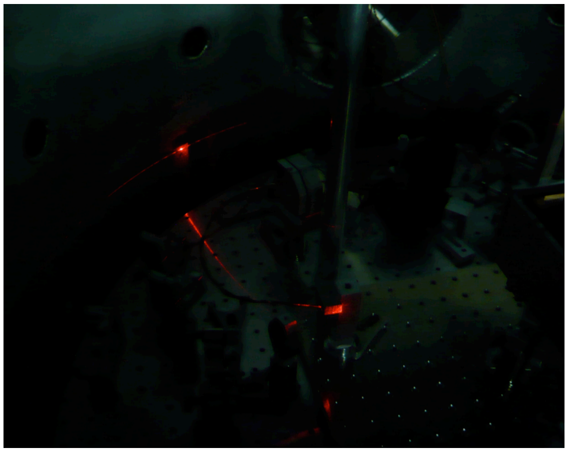

Figure 3. Experimental setup in vacuum chamber with visible laser on. Note the scattered light visible on chamber wall.

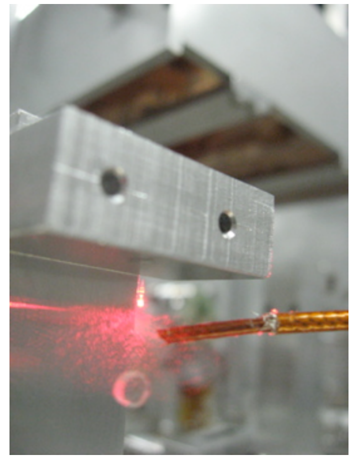

Figure 4. High voltage wire placed 1-2 mm away from the silica fiber.

3.6 Oscilloscope

To record the amplitude of the pendulum's free decay, we used the Tektronix TDS3014B for data taken in air and the Yokogawa DL1640 Digital Oscilloscope. The DL1640 was required for the vacuum measurements as it allows for a longer recording time than the TDS3014B. We measured the DC signal of the PD output.

3.7 Lock-in Amplifier

When measuring the free decay of the oscillation, we sometimes encountered large amounts of noise. In these cases, we ran the PD signal through a LI5640 Multifunction Digital Lock-in Amplifier (NF Corporation) before sending it to the oscilloscope. We set the lock-in amplifier's reference frequency to the resonance frequency. The lock-in amplifier attenuates all signals that are not at the reference signal to zero, leaving only the desired signal. By setting the internal reference of the lock-in amplifier to the resonance frequency, we were able to avoid the noise added when an external reference signal (function generator signal) was connected to this lock-in amplifier and the high-voltage amplifier at the same time.

4. Results

4.1 Resonance Frequencies

Before we could excite the pendulum motion, we first had to determine the actual resonance frequencies for the pendulum, yaw, and violin modes. Using an Agilent 35670A Dynamic Signal Analyzer, we excited the pendulum motion over a range of frequencies and recorded the amplitude of the motion from the vacuum-compatible PD. We then looked for resonance peaks at the calculated frequencies. We recorded a peak at 5.125Hz, close to the expected pendulum mode resonance of 4.98Hz.

We then looked for the yaw motion resonance peak, analyzing the output of the exterior PD with the signal analyzer. We found a peak at 2.77 Hz.

We were unable to observe a peak for the violin mode frequencies in air. Under vacuum conditions, we recorded a peak at 1.502 kHz. We compared the peaks present without excitation by the high voltage wire to the peaks present while the pendulum was shaken. A peak at 1.500 kHz is only present when the pendulum is shaken, indicating a resonance frequency (Figure 4). This value is close to the previously measured value of 1.567 kHz for a double pendulum with a clamped middle mass, though off from the calculated 1.806 kHz [7]. However, we were unable to see peaks for any of the higher violin modes.

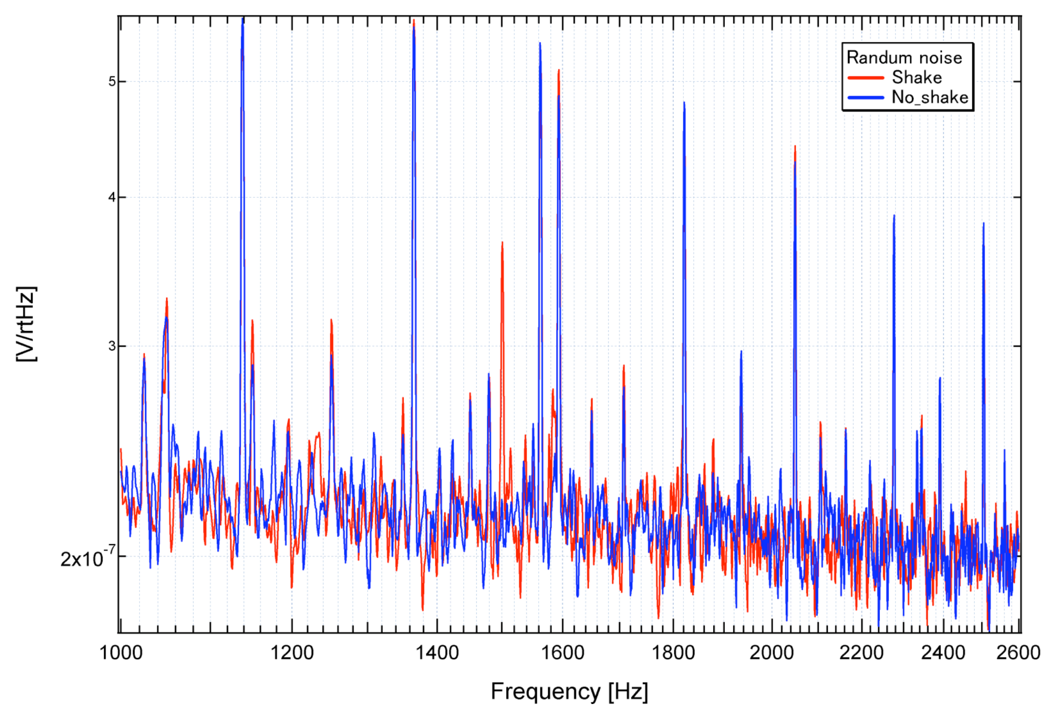

Figure 4. Analysis of Resonance Peaks in Vacuum. Note peak at approximately 1500 Hz visible only during shaking with HV wire.

4.2 Quality Factor Calculation

We fit an exponential curve to the decay envelope using the following equation.

$$A(t) = ae^{\frac{-t}{t_0}} \quad (6)$$

Using the value of t_0 derived from the fit, we used equation 1 to find the Q-factor.

4.3 Observed Quality Factors

In air, we found quality factors on the order of 10^2. The pendulum mode quality factors found from the pitch motion and pendulum motion are close in magnitude. The yaw motion quality factor is approximately one third the pendulum mode quality factor.

Motion		*Run 1*		*Run 2*		
	f_0 (Hz)	*t_0 (s)*	*Q*	*t_0 (s)*	*Q*	*Avg. Q*
Pendulum	5.195	19	**308**	17	**275**	**292**
Pitch	5.195	32	**514**	22	**351**	**433**
Yaw	2.77	12	**101**	11	**98**	**100**

Table 1. Measurements of Q-Factor in air.
Measured with Tektronix TDS3014B Digital Oscilloscope.

In the vacuum, we found that the quality factor of the pendulum motion increased by approximately one order of magnitude. The yaw motion quality factor only tripled compared to its magnitude in air. This suggests that air resistance had a small effect on the yaw motion.

	Run 1, 7/27/10		*Run 2, 7/28/10**			*Run 3, 7/29/10**		
f_0 (Hz)	*t_0 (s)*	*Q*		*t_0 (s)*	*Q*	*t_0 (s)*	*Q*	*Avg. Q*
5.195	140	**2292**	*Input*	77	1257	98	1599	
			Output	94	1534	102	1665	
			Avg.	86	**1403**	100	**1632**	**1776**

Table 2. Pendulum mode Q-factor measurements under vacuum conditions.
Measured with Yokogawa DL1640 Digital Oscilloscope.
*Measurements taken with LI5640 lock-in amplifier.

		Run 1, 7/27/10	*Run 2, 7/28/10*	*Run 3, 7/29/10**	

	f_0 (Hz)	t_0 (s)	Q	t_0 (s)	Q		t_0 (s)	Q	Avg. Q
Pitch	5.195	135	**2202**	64	**1046**	*Input*	112	1828	1708
						Output	118	1926	
						Avg.	115	**1877**	
Yaw	2.77	42	**326**	25	**218**	*Input*	33	287	280
						Output	35	305	
						Avg.	34	**296**	

Table 3. Pitch and yaw mode Q-factor measurements under vacuum conditions.
Measured with Yokogawa DL1640 Digital Oscilloscope.
* Measurements taken with LI5640 lock-in amplifier.

	Run 1, 7/28/10		*Run 2, 7/28/10*		*Run 3, 7/29/10***		*Run 4, 7/29/10***		
f_0 (Hz)	t_0 (s)	Q	t_0 (s)	Q	t_0 (s)	Q	t_0 (s)	Q	Avg. Q
1502	0.589	**2775**	0.153	**719**	0.373	**1759**	0.222	**1046**	**1575**

Table 4. Violin mode Q-factor measurements under vacuum conditions.
Measured with Yokogawa DL1640 Digital Oscilloscope.
**Output of the LI5640 Lock-In Amplifier.

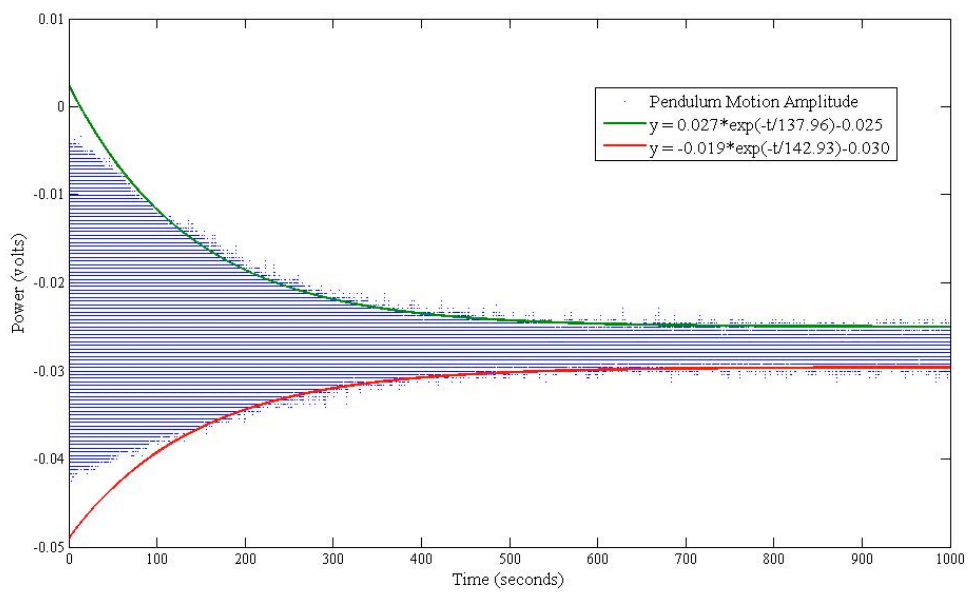

Figure 6. Pendulum mode decay at 5.195 Hz in vacuum, with exponential fits. Run 1, 7/27/10. A similar procedure was followed for all fits.

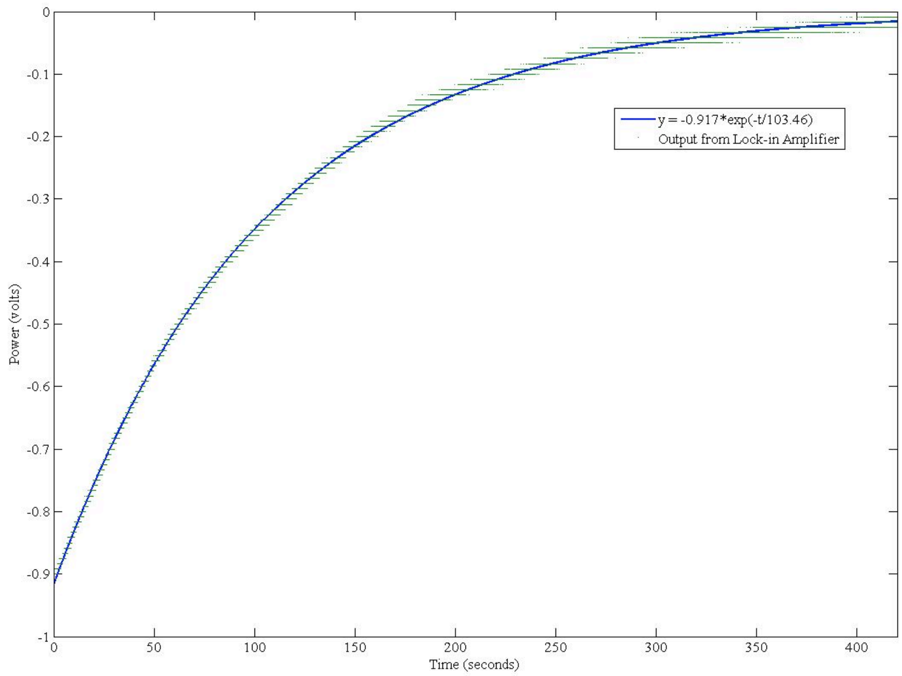

Figure 7. Lock-in amplifier output for pendulum mode decay at 5.195 Hz in vacuum. Run 3, 7/29/10.

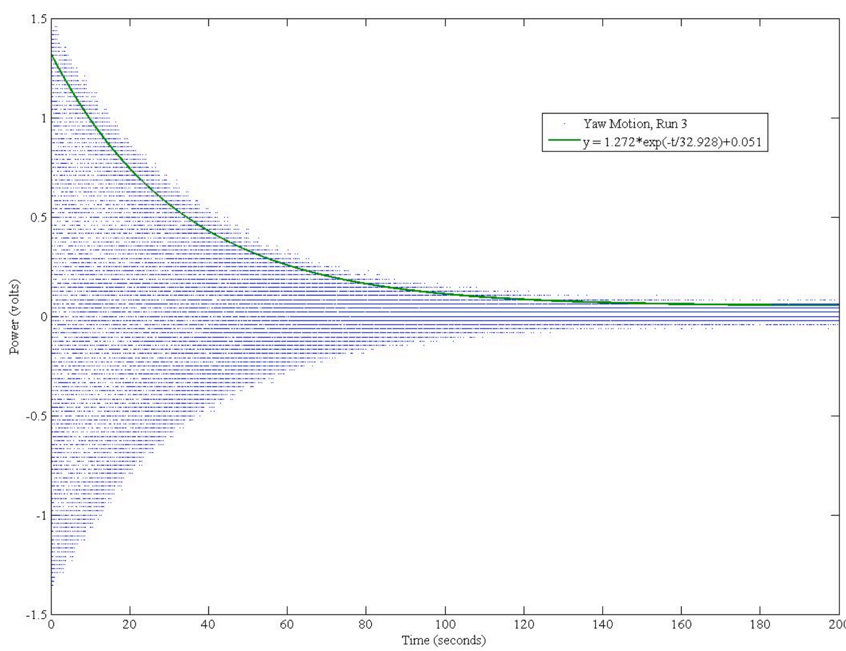

Figure 8. Yaw motion decay at 2.77 Hz in vacuum. Run 3, 7/29/10.

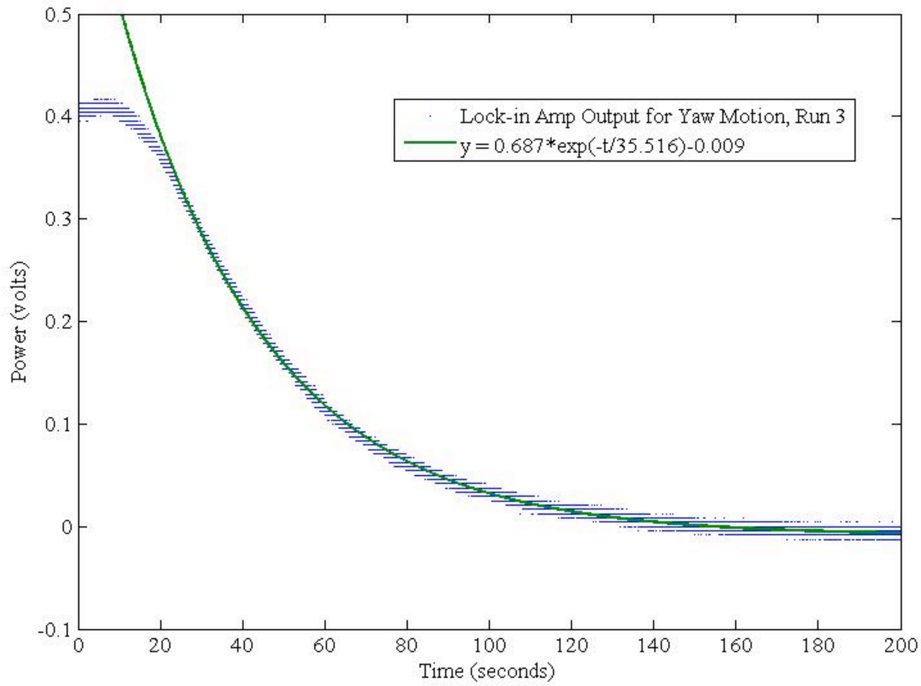

Figure 9. Lock-in amplifier output for yaw motion decay
at 2.77 Hz in vacuum. Run 3, 7/29/10.

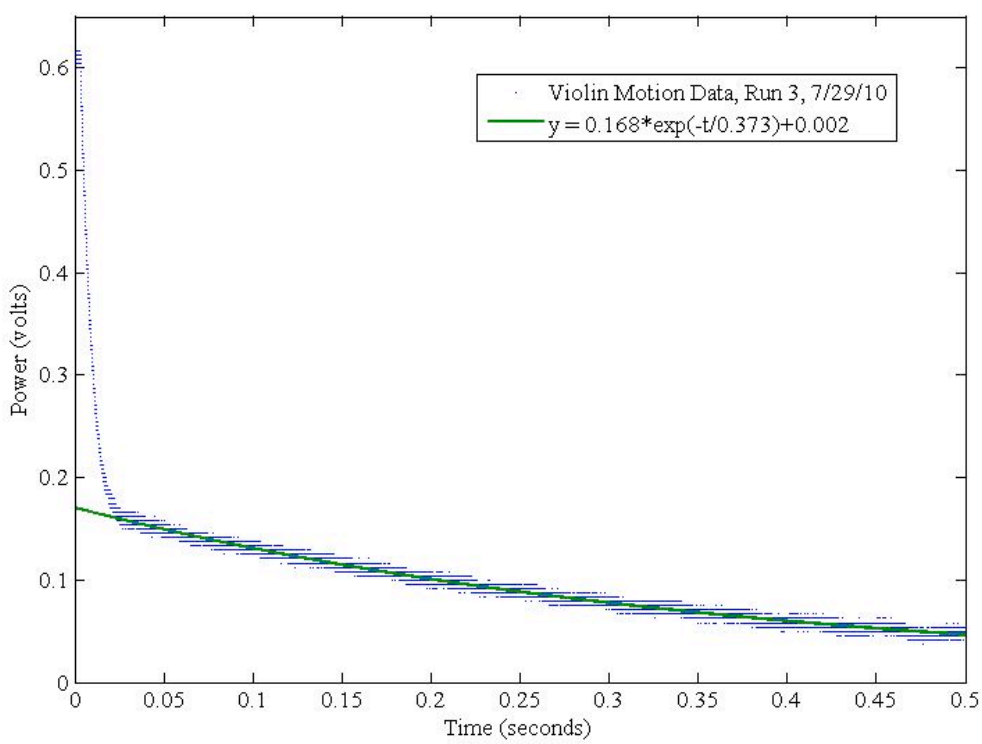

Figure 10. Lock-in amplifier output for violin mode decay
at 1502 Hz in vacuum. Run 3, 7/29/10.

5.2 Comparison of Observed Q Factors

The pendulum setup has a smaller loss than the suspension wire material. The restoring force for the pendulum mode is dominated by gravity, which is lossless, rather than the by the flexing of the wire [2]. The quality factors of the yaw motion and pendulum motion are related by a dilution factor as

$$Q_{pend} = \frac{Q_{yaw}}{D_{dil}} \quad (7)$$

The dilution factor is

$$D_{dil} = N\frac{\sqrt{TEI}}{2MgL} \quad (8)$$

where N is the number of wires suspending the mass, T is the tension in each wire, E is the Young's modulus for the wire material, I is the area moment of inertia of the wire, L is the length of the suspension fiber, and M is the mirror mass [2]. Using the following experimental parameters, we find D_{dil} = 0.0197

Parameter	**Symbol**	**Value**
Tension	T	$2.25*10^{-4}$ N
Young's Modulus for fused silica	E	71.7 GPa
Mirror Mass	M	$23*10^{-6}$ kg
Fiber Length	L	0.01 m
Area Moment of Inertia of Fiber Cross-Section	I	$4.909*10^{-22}$ m^4

Table 5. Experimental parameters for the single pendulum suspension.

Using this dilution factor and the average value for the quality factor of the yaw mode in air (Q_{yaw} = 99), we find an expected value for the quality factor of the pendulum mode in air to be 5045. This value is over 17 times greater than the measured quality factor of 292 for the pendulum mode. We find a similar situation when we compare the yaw and pendulum quality factors in the vacuum using the dilution factor. Using $Q_{yaw} = 280$, we find an expected $Q_{pendulum}$ in the vacuum of 14213. This expected value is 8 times greater than the measured $Q_{pendulum}$ of 1776. This discrepancy in the expected relation suggests that the potential energy of the fiber is dominated by the elasticity of the fiber rather than gravity.

We can also compare the Q factor of the violin mode to that of the pendulum mode. The loss angle of the pendulum mode is ½ that of the violin modes [8].

$$\phi_{pendulum} = \frac{1}{2}\phi_{violin} \quad (9)$$

Combining the above relation with equation 2, we find the following relation between their Q factors:

$$Q_{pendulum} = 2Q_{violin} \quad (10)$$

Comparing the average values for $Q_{pendulum}$ and Q_{violin} in the vacuum, we see that $Q_{pendulum}$ (1776) is only 1.13 times larger than Q_{violin} (1575). However, as seen in Table 4, there is a large variance in the measured value of Q_{violin}. With more trials, we may have been able to achieve a more accurate result for Q_{violin}.

5.3 Thermal Noise

With the quality factor, we can calculate the thermal noise spectrum of the oscillator with respect to frequency using equation 4 (Figure 11). In our setup, one source of thermal noise is structure damping. This damping comes from the anelasticity of the fiber. As the quality factors in air compared to those in the vacuum differ by only 1 order of magnitude or less, it is clear that air resistance is not the main damping mechanism. In the case of gravitational wave interferometry, we are interested in the thermal noise at a frequency of 1 kHz. Using equation 3, the thermal noise for the pendulum mode at 1kHz is $6.57*10^{-18}$ m/√Hz (Figure 10).

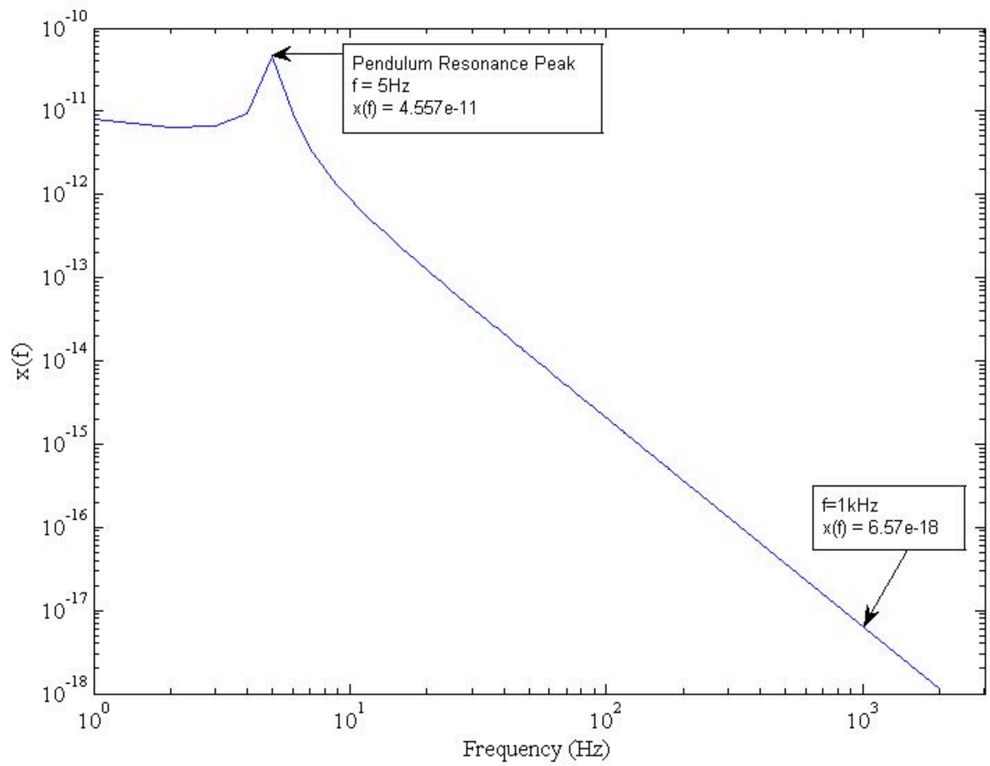

Figure 10. Thermal noise spectrum for the pendulum mode measured in vacuum, using Q = 1776.

6 Discussion

6.1 Sources of Error and Suggestion for Further Experiments

Overall, we found a Q factor on the order of 10^3 for the pendulum mode in the vacuum, and Q factors on the order of 10^2 for the yaw and first violin modes. These values lead to thermal noise on the order of 10^{-18} m/√Hz, only one order of magnitude below the desired sensitivity of 10^{-17} m/√Hz at 1 kHz. A thermal noise level of 10^{-19} is desired at this frequency.

One possible contributor to these low Q factors is electrostatic loss from the fiber. Another possible cause is too much resin on the fiber. This would cause dissipation at the attachment points due to friction. Finally, the motion could be damped by the fiber touching the high voltage wire while in motion. However, it seems unlikely this would occur without the suspension breaking, as the fiber is extremely delicate.

The low quality factors we measured suggest an unaccounted source of damping. A more detailed study of the effect of the UV cured resin on the quality factor would help determine this source of damping. By building a few suspensions with varying amounts of UV resin, we could better see the UV resin's effect on quality factor.

As we set the high voltage wire by hand, it was impossible to move the wire once the vacuum tank was sealed. By clamping the wire to a remotely controlled motor, we could easily see the effect of the wire's placement on the quality factor.

6.2 Conclusion

In this study, we found the quality factor for the pendulum, yaw, and first violin modes of a single pendulum. The measured quality factors in the vacuum were on the order of 10^3, a full order of magnitude below the expected results. Though less than expected, our results were on the same order of magnitude as those from the previous measurement taken by Stuart Reid et al. [1]. This level of thermal noise would obscure the signals we are hoping to detect in gravitational wave interferometers. Our results suggest the difficulty in building a very low noise suspension. However, more attachment methods should be explored, as in our case it seems probable that the quality factor was limited by the UV resin.

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