Hough Transform Method and Analysis for the All-Sky Search

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1 Introduction

The group at the Università di Roma—Sapienza is involved in multi-faceted research into gravitational wave detection and analysis. The group carries out searches targeting non-accreting known pulsars both with and without timing data, unknown isolated stars, and accreting stars in both known and unknown binaries. One of the most promising sources of continuous gravitational signals, rapidly rotating neutron stars, is currently being exploited by the group in the all-sky search. Of particular are isolated neutron stars. These stars are expected to emit through deformations due to elastic or magnetic deformations and through free procession around the rotation axis.

The gravitational signal given off by these objects are supposed to be sinusoidal with (possibly) two harmonics and a large number are thought to be in our galaxy. The most likely locations are near the center of the galaxy where more stellar material exists or in globular clusters with the nearest and more detectable sources being isotropic. In general, the amplitude of such signals is

$$h_0 = 1.05 \cdot 10^{-27} \left(\frac{I_3}{10^{38}\, kg \cdot m^2}\right)\left(\frac{10kpc}{r}\right)\left(\frac{v}{100\, Hz}\right)^2\left(\frac{\varepsilon}{10^{-6}}\right)$$

The search for such objects is complicated by computational power limits. In the case of the all-sky search, no information is known about the frequency, spin-down, location, or other parameters governing the neutron stars. The number of templates, N_p, needed to for the entire sky that cover a large frequency band and range of spin-down parameters during a duration T is roughly proportional to T^6. In most cases, it is often not possible to use values of T greater than a few days, even if using existing coherent codes and full computational power.

The computational problem involved in the blind all-sky search is partially remedied by using hierarchal search procedures based on a sequence of increasing resolution steps that alternate between coherent and incoherent. The particular method used by the group in Rome is the Hough Transform.

2 The Hough Transform Method

The Hough Transform is a linear transform used to recognize the parameters of a curve from the position of some points on it. It operates on a peakmap in the time-frequency plane. For each of these peaks the set of bins of a multi-dimensional histogram defined on the parameter space called the Hough Map. The parameter space is defined as $f - \dot{f}$. The position for each source is fixed on the map. The noise detected in the map is defined with parameters N(the number of spectra) and μ (the expected value of the Hough Map) as

$$p = \frac{\mu}{N}$$

The frequency Hough procedure has recently been implemented by the group at Rome. In this transformation the time-observed frequency plane is mapped into the source frequency-spin down plane. With f as the Doppler corrected frequency of the sky direction, f_0 as the intrinsic source frequency, t the time at the detector, and t_0 as the reference time,

$$f = f_0 + \dot{f}\,(t - t_0)$$

and

$$\dot{f} = -\frac{f_0}{t - t_0} + \frac{f}{t - t_0}$$

The points on the input plane are of the form $(t - t_0, f)$ are the peaks on the Doppler-shifted peakmap. This peakmap is then transformed into a straight line in the Hough plane that has points of the form $(f_0, \dot{f})$ with a slope ${}^{-1}/_{t\,-\,t_0}$. Each peak is transformed into a stripe among two parallel straight lines such that

$$-\frac{f_0}{t - t_0} + \frac{f - \Delta f/2}{t - t_0} < \dot{f} < -\frac{f_0}{t - t_0} + \frac{f + \Delta f/2}{t - t_0}$$

For each point in the sky grid, for each coordinate in the input plane, and for each spin-down value the map is increased by one bin in the point

$$f_0 = f - \frac{\Delta f}{2} - \dot{f}(t - t_0)$$

and decreased in by one bin in the point

$$f_0 = f + \frac{\Delta f}{2} - \dot{f}(t - t_0)$$

The spin down step that determines the limits of the Hough Map has been optimized at

$$\frac{\delta v}{T_{obs}} \approx 10^{-11}\ {}^{Hz}/_{s}$$

Here, $\delta v = {}^{1}/_{TFFT}$ with TFFT being the numerical duration (in seconds) of the transform in use. Any values that are less than this will be read as a spin down of approximately 0 in the Hough Map and values that are significantly greater by about one magnitude or more often results in a loss in detection of the signal. This value can also be altered in the Hough code to create a map with more bins and exploit more values, however, this can be computationally expensive.

Once the Hough Transform has been performed on the data, initial candidate are selected with the search this and archived. In the coherent step of the transform, the frequency shift due to the Doppler effect and spin-down are partially corrected and the number of candidates is reduced. Each candidate has a set of parameters that are: the frequency at a certain epoch, the position in the sky, and 2-3 spin-down parameters.

However, a periodic source by nature is permanent—unlike other short-lived sources such as supernovae. It is possible to check the "reality" of a source candidate with the same antenna and different observations and search the data for coincidences between candidates in different periods. If two candidates, n_1 and n_2, survive in two different data analysis sets in the parameter space N_p, then the expected number of coincidences is

$$N_{coin} = \frac{n_1 . n_2}{N_p}$$

A summary of the Hough Transform used by the Rome Group is shown on the next page and taken from *Detection of periodic gravitational wave sources by Hough transform in the f versus $\dot{f}$ plane* (details for the publication are in the bibliography).

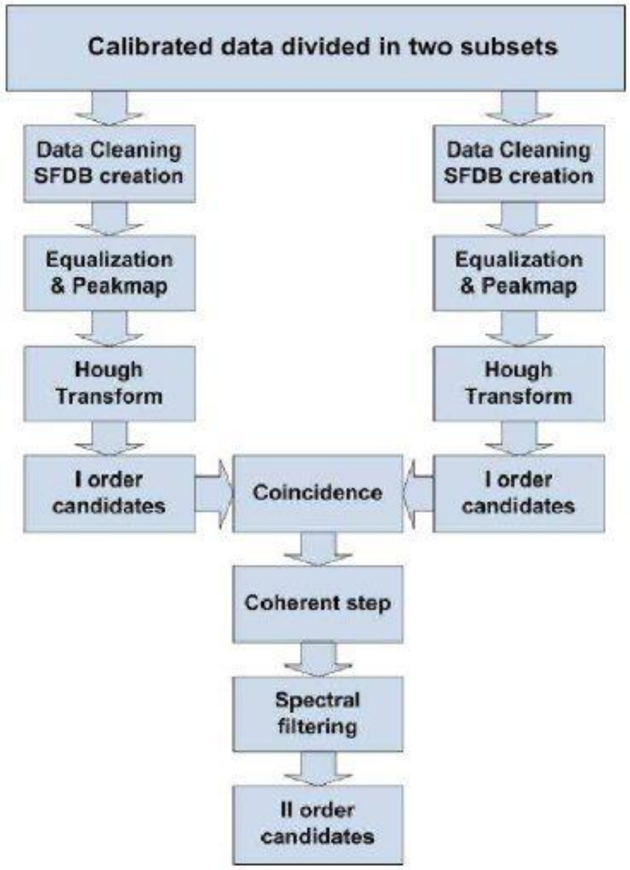


Figure 1. The basic scheme of the hierarchical procedure, used in the Virgo Collaboration.

The efficiency of detection depends upon a number of factors, mainly the amplitude of the source and the position of the source in the sky. Sources with signals at low amplitudes have a low signal to noise ratio and are detected with less clarity than those with higher amplitudes. In the case of sky positions, some positions are better suited to the sky grid than others. In some cases, the source position has coordinates that are (relatively) far outside the bins in the sky grid and this will result in a greater error in detection.

Simulations are carried out using a similar code that uses a software injection to create signals. In the case of simulations, the duration of the injected signal depends on the length of the Fast Fourier Transforms (FFTs) being used. In the Hough simulation code, a parameter exists to allow the user to choose whether a long or short FFT will be sued. For the sort FFTs, the duration is set for 1024 seconds and multiplied by 10,000—which is the number of transforms of that length being carried out. This means that the full length of the transform becomes 10,240,000 seconds or 118.5 days. Similarly, each long FFT has a duration of 8192 seconds and there are 2000 transforms used, so that the duration of the simulated at 16,384,000 seconds or 189.6 days. The frequency range of the injected signal again depends on the transform being used. If the short FFT is used, the detected frequency range is much lower (10-128 Hz) than the long FFT (128-1024 Hz).

I have worked this summer on making the simulation Hough code more "user friendly" and automated. The code allows the user to input initial parameter vales that tells the code (ran in Matlab) whether the software injection will be a long or short FFT, if there will be a second spin down parameter or not, if the transform used is adaptive or non-adaptive, of if a sky grid is to be used, among others. Previously, there was no random generation and all injected values had to first be changed in the code editor program and then ran separately in Matlab. I have designed the simulation program to now prompt the user for input for spin-down values, amplitude values, and frequency values for software injections. These can be either randomly generated or given a range or specific value and, in all cases, have default values so the program runs even if the user does not enter a value for the parameters. This helped to clean up the code as well as lessen the amount of time spent switching between the editor and Matlab interface and made the processes of running simulations go more smoothly.

3 Spin-Down Parameters

In most cases, sources of gravitational waves experience an energy loss or gain which is caused by electromagnetic and gravitational radiation emission. This emission causes a decrease in the rotation emission and thus decreases the emission frequency. It is the derivative of this frequency (its intrinsic decrease or increase) that we call the spin-down parameter. In certain cases, such as those of binary systems which gain energy from their accreting companions, the spin-down detected could be positive. Most of the time, as in the case of isolated neutron stars, the source is losing energy and is the spin-down detected will be negative. In mathematical terms the frequency of the source is determined by

Of particular interest are younger stars which typically have a larger spin-down, meaning they are more energetic and emitting radiation at a greater rate. This makes young, fast neutron stars some of the best candidates for emission of gravitational waves. Up until now, the group at Rome had not used the second spin-down in the all-sky search due to computational limits. In these cases the second spin-down parameter—which is simply the second derivative of the source frequency—needs to be taken into consideration if the sources emissions are to be detected as including this parameter allows for a greater range of exploited frequencies. When the second spin-down is added, the frequency equation becomes

$$\mathrm{f} = \mathrm{f}_0 - \dot{\mathrm{f}}t - \ddot{f}t^2$$

In the Hough Transform, a fixed spin-down step also fixes the minimum age of the source, τ_{min} such that

$$\tau_{min} = \frac{f_0}{\dot{f}}$$

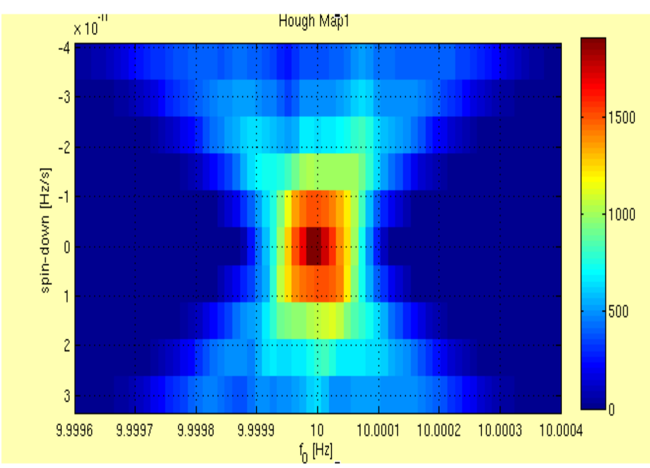
where f_0 is the initial frequency of the source. Typically, τ_{min} is approximately 10,000 years—meaning that the youngest source usually exploited is 10,000 years old. If the detection of younger and faster stars is to be performed, it is important to study the relation of τ_{min} to the second-spin down parameter in order to optimize detection. Recent studies have found that in stars with τ_{min} of about 200 years, this second spin down parameter becomes important for proper detection.

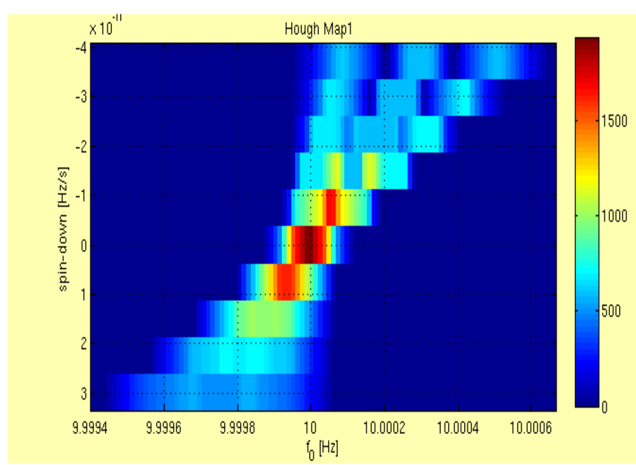
In any case, the use of the second spin down parameter requires much more computer power than those searches performed with only the first spin-down parameter and an all-sky search becomes impractical. However, it is still possible to perform blind, directed searches with the second spin down parameter included. Over the past few months, I have worked with Pia Astone on including the second spin-down of the source as a parameter in the Hough simulation codes for both the adaptive and non-adaptive Hough transforms. In theory, the simulation codes should be altered for use with the parameter, but the second spin-down step still needs to be studied and optimized if it is to be used regularly.

4 Analysis of Beginning and Middle Observation Times

As of now, data analysis and runs have begun at the beginning of the observation for all runs of Virgo and LIGO data. However, there has been recent debate as to whether beginning the Hough transforms at different reference times would be beneficial in the analysis of data and detection of signal. This summer, I worked on an initial analysis of the effects of beginning the long FFT at the middle of the observational period versus beginning the long FFT at the beginning of the observational period.

The initial simulations showed that the slope of the Hough map changes when the transform is started at the middle of the observation period rather than at the beginning.



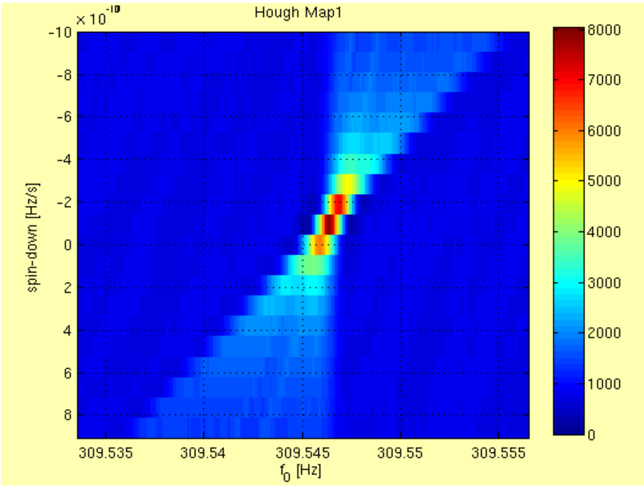


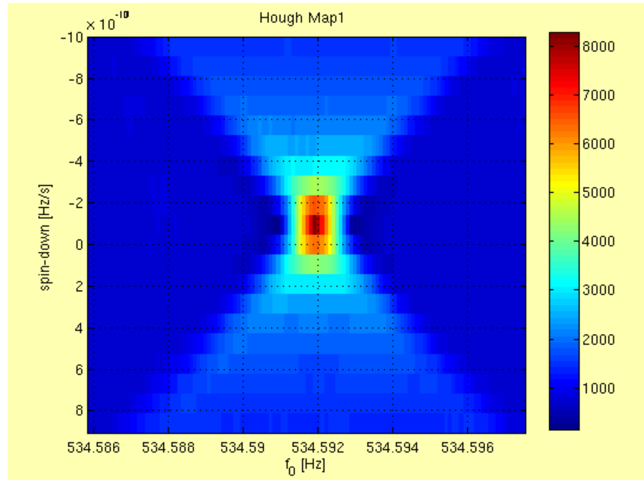
The figures above show simulations that are exactly the same. Each simulation is done with the non-adaptive Hough, a source spin-down value of zero, and a source frequency of 10 Hz. However, the figure on the left was created by starting the Hough Transform at the middle of the observation period while the figure on the right's Hough Transform began at the traditional beginning of the observation period. The figures show noticeable difference, mainly in the slope of the Hough Map and the spread of the injection across the bins. The slope of the Hough is dependent upon the time at the detector (t) and the observation time (t_0) such that

$$Slope = \frac{-1}{t - t_0}$$

From the figures, it is obvious that beginning the transformation at the middle of the observation where the value $t - t_0$ is less than a transform started at the beginning of the observation period creates a map with less of slant due to the difference in the values. This, in turn, injects a signal that is more concentrated (spread through fewer bins). In the above example, the Hough Map's frequency axis for the middle time analysis begins at 9.9996 and extends through 10.0004. In contrast, when the same simulation starts at the beginning of the observation period, the frequency bins span from 9.9994 to 10.0006. The differences are slight, but it seems to be that beginning the analysis at the middle of the observation results in less frequency error and a more concentrated signal detection. Likewise, simulations were run for adaptive and non-adaptive Hough Transformations at various frequencies, spin-downs, amplitudes, and sky position. In all cases, the results of the Hough Map shown above were identical.

In the non-adaptive Hough, lowering the amplitude to around six forced the Hough Transform to detect noise. This simulation is much more likely to be closer to an actual signal detection, however, adding noise into the Hough creates computational problems as extra corrections must be done (i.e. Doppler frequency corrections) in order to detect a signal. This often results in a loss in detection of the signal. In order to study the effects of the middle versus beginning time analysis at low signal-to-noise ratios, twenty simulations of each type were run for amplitudes of 3, 4, 5, 6, 8, and 15. Each separate simulation was run with random spin-down values, frequency, and sky-grid coordinates in order to better simulate real data conditions. Even with the noise, the Hough Map results detailed above remained constant, as shown in the figures below that were both run under the same parameters at an amplitude of 5:





The effects of the noise can be seen in the “fuzziness” of the Hough Maps in comparison to the maps without noise shown on previous pages. However, it was important to look at the exact error in detection in such cases. The tables below show a summarized average for all the runs with the simulations at the beginning time on the left and the simulations of the middle time on the right.

Amplitude	Average Loss	Average Spin Error	Average Frequency Error
3	28.47%	2.38137*10^-11	0.02048
4	32.47%	1.09201*10^-10	0.00049
5	19.61%	1.86462*10^-11	0.000149
6	21.34%	6.36462*10^-11	0.004481
8	21.28%	2.79043*10^-10	0.006729
15	11.98%	4.35785*10^-11	0.000113

Amplitude	Average Loss	Average Spin Error	Average Frequency Error
3	27.18%	1.55*10^-10	0.01423
4	22.46%	2.46*10^-10	0.00157
5	18.64%	3.0588*10^-10	0.00259
6	20.8%	8.45*10^-10	.002466
8	16.14%	1,2554*10^-10	0.002761
15	11.87%	6.5285*10^-11	0.00216

Here, one can see that the average loss in signal detection is slightly better when the transformations are started at the middle of the observation period rather than the beginning. Some of the inconsistencies in the data—for example the average loss for the amplitude of 4 being greater than that of 3 in the beginning analysis—can be explained. While the average loss is expected to be greater the smaller the signal-to-noise ratio is, other factors can contribute to the loss in detection. The two most common factors limiting the detection (besides noise) are spin-down values and position in the sky. In the first case, if a spin-down value is injected that is larger than the spin-down step of the Hough Transform, signal detection is lost due to overcorrection. In the second case, it has been found that certain sky positions are at an advantage over others. If a coordinate location is far away from a bin on the grid constructed during the Hough procedure, a loss in signal will be prevalent. This is usually accounts for a maximum of about 2-3% of the total loss in detection.

5 Conclusion

The all-sky search for gravitational waves has a number of issues to face if it is to be successful in the coming years. Computational limits are among the greatest of these concerns due to the cost of analyzing data in a large range of frequencies, spin-down values, and locations across the sky. Already many results have been obtained and published from the VSR1 data, but major analysis of VSR2 and VSR4 data is still underway.

During the ongoing and upcoming analyses, it might be beneficial to implement the changes made to the Hough simulation code described in this paper. In the case of the second spin-down parameter, better detection of younger, faster neutron stars is possible. Since these stars are thought to emit strong gravitational radiation, better detection could result in an overall better chance of detecting gravitational wave signals in the future. It is also possible that beginning the FFT employed in the Hough Transform at the middle of the observation rather than the beginning might result in better signal detection spread over fewer bins.

Further analysis is still needed in regards to both of the parameters discussed above. The appropriate step to be used in the implementation of the second spin-down parameter need to be determined, as does the relation of this parameter to frequency, amplitude, τ_{min} , computational power, etc. It is also necessary to determine whether the results of the middle time analysis are typical and are similar for the short FFT.

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