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PULSAR GLITCHES AS A GRAVITATIONAL WAVE SOURCE

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ABSTRACT

This report describes research completed in in Dr. Hyung Mok Lee's group at Seoul National University through the University of Florida's International Research Experience for Undergraduates funded by the National Science Foundation. This project explores the possibility of pulsar glitches as a source of gravitational radiation through hydrodynamic code with applied perturbations that mimic the glitch event. The two perturbations considered mimic the star quake model and angular momentum transfer by vortex unpinning at crust-core interface model. We estimate the characteristic amplitude of the gravitational waves from the evolution of the quadrapole moment of the simulated star.

We use the weak field metric with a generalized source term to Poisson's equation to gain closer agreement with a full general relativistic treatment. This report discusses the validity of this method and some of the testing done to see how well our method agrees with other methods such as linear perturbative analysis. We find that our pseudo-Newtonian method gives better agreement with general relativity than similar methods without a generalized source term.

We find that current detectors are unable to reach the sensitivity to detect gravitational radiation from a glitch phenomena. For both perturbations, strain amplitudes were approximately 10^{-25}. We find that the vortex unpinning model excite the inertial mode more effectively than the star quake model.

1. INTRODUCTION

Neutron stars offer insight into physical states not achievable in any earth based laboratory today. Observations of these exotic stars allow for testing of many modern theories in physics. Unfortunately, their size, distance, and luminosities make it difficult to observe them under most conditions. Fortunately, we can observe neutron stars as pulsars. Neutron stars emit directed radiation along their magnetic axis and, if they are appropriately aligned with Earth, we can observe this radiation as regular pulses due to the rotation of the star.

Pulsars rotate at exceptionally regular rates but are constantly slowing down as they transfer energy to their environment. On occasion, though, pulsars are observed to suddenly speed up without explanation. This increase in rotational velocity if know as a glitch. Since the first pulsar glitch was observed over 100 glitches have been found. Typically, the fractional spin up rates ($\Delta\Omega/\Omega$) of a glitch range from 10^{-9} to 10^{-6} while the largest glitch ever observed had an amplitude of $\sim 3.3\times10^{-5}$(Manchester & Hobbs 2011). Glitches are the result of a sudden internal change of the state of the star and therefore give a unique look into the state of the star. More interestingly, a glitch should excite oscillations within the star which have the potential to emit gravitational radiation.

Observations of gravitational radiation from a glitch would have many benefits scientifically including the possibility of being the very first detection of gravitational radiation. In addition, the state of the star is highly dependent upon the equation of state (EoS) which is largely unknown for extremely dense regions such as those of a neutron star. Therefore, observations of gravitational radiation from pulsar glitches could offer further constraint on the candidates for a neutron star's EoS.

As previously mentioned, the exact mechanism of a pulsar glitch is still widely debated. Currently there exists two major hypotheses. The first is known as the star quake model (Ruderman 1969). In these models the star is treated as two regions, the core and the crust. As the neutron star slows down it adapts a more spherical shape. This model assumes the crust is rigid which causes it to maintain its ellipsoidal shape. The core on the other hand is adapting a spherical shape and is applying pressures on the crust to do the same. When the pressures become great enough, the crust breaks up to become more spherical and the moment of inertia of the star suddenly changes. It is this sudden change in the moment of inertia that leads to the observed change in rotational rate. The second model is vortex unpinning of a superfluid at the crust-core interface (Packard 1972; Anderson and Itoh 1975). The rate at which pulsars slow down is much lower than expected for a rotating normal fluid. For this reason, it is believed that the core of the star is a superfluid. In this model the core is considered a superfluid while the crust is considered a normal fluid. This leads to differential rotation between the core and the crust. Once the difference becomes too great, vortices coupling the core and crust unpin, causing the core to transfer angular momentum to the crust. This transfer of momentum to the crust is the cause of the observed spin up.

The goal of this project is to determine the modes and amplitudes of gravitational radiation resulting from a glitch event. The normal modes of a star can be predicted by linearized methods but these methods cannot predict which modes are excited nor their amplitudes. Therefore the only realizable method for determining this information is through hydrodynamic simulations. In our simulations we create an equilibrium model of the star, apply a perturbation to mimic the glitch model, then follow the hydrodynamic evolution of the star. We use glitch perturbations that are much larger than real glitch magnitudes because of numerical limitations but use this data to extrapolate to real glitch magnitudes. From this information we can infer the maximum values for observed gravitational waves and the frequencies at which they occur.

This report will give some background on gravitational waves and their detection, information on what we know about pulsar glitches and the popular models for them, discuss the formulation used within our simulations and their validity, and finally predict the characteristic strain amplitudes of gravitational waves we might hope to one day see.

2. GRAVITATIONAL WAVES

In Einstein's theory of general relativity, gravity is considered to be the result of the curvature of spacetime due to mass and energy. Since the curvature is dependent on mass and energy any change in the location or density of mass and energy will result in a change in the curvature of spacetime. As one would expect, these changes propagate out from the source at the speed of light. This fact hints to the existence of gravitational waves. Just like electromagnetic radiation, gravitational waves depend on the acceleration of massive object, but, as gravitation is a much weaker force than the electromagnetic force, gravitational waves have very small amplitudes making them difficult to detect. In addition, due to the conservation of momentum, there can be no mass dipole radiation for gravitation unlike electromagnetic radiation. This means that not only does an object have to be very massive, it cannot be changing symmetrically for it to emit gravitational radiation. In other words, gravitational radiation with any chance of measurement must come from non-symmetric, massive systems. Sources commonly suggested for gravitational wave generation are binary systems consisting of neutron stars or black holes.

2.1. *Detection of Gravitational Radiation*

Since a gravitational wave is an actual fluctuation in the curvature of spacetime, when a wave passes through a portion of space, distances between two locations will appear to rhythmically increase and decrease with amplitudes depending on the gravitational wave. Therefore, to detect gravitational waves, one must detect these fluctuations in distance.

Gravitational radiation detectors must measure very precisely the distance between two points in space. Current detectors are all in the form of a laser interferometer. These interferometers take the form of a Michelson interferometer which splits the incoming laser into two different beams along orthogonal paths that reflect the light back to the splitting point and recombines the beams on the path to the detector. Depending on the path lengths of the split beams the detector will either see constructive or destructive interference. It is usually setup such that a dark fringe is registered at the detector normally, and if a gravitational wave were to pass through the detector, it would fluctuate the path length of the detector arms and break the destructive interference of light at the detector.

In addition, this interferometric setup is better utilized if it is placed in multiple locations. By having the similar systems in different locations it allows for a few different benefits. Firstly, it allows to confirm that a gravitational wave signal is real and not just noise in the system. Since the systems are far enough apart it would be uncommon for them to both be effected by the same transient noise signal at the same time. Also, by having largely seperated locations, the interferometers can time the signals when they start and stop. Due to their spatial seperation, the interferometers should observe the signals at slightly different times and allow for localization of the source of the signal.

In concept is a very simple setup but there are many obstacles that an interferometer must overcome to be able to detect the extremely weak spacetime fluctuations caused by a gravitational wave. The sensitivity of a detector must be so great that every source of noise must be minimized and accounted for if one is to properly detect gravitational waves. These sources of noise include things such as power fluctuation of the laser source, seismic noise, photon pressure on the mirror at the end of each arm, thermal noise, and an incredible number of other sources. Locating and removing these sources of noise is the focus of much gravitational wave detectors today. In fact, one the largest gravitational wave detector project today, LIGO, was commissioned under the assumption that it would be very unlikely for it to detect gravitational waves in its initial state (LIGO). Currently, LIGO is an upgrade phase to Advanced LIGO where scientists believe it will likely be able to detect some of the strongest sources of gravitational radiation. During this pivotal point in time, it is important to consider all possible sources of gravitational radiation, what their signals would look like, and how likely it would be to these signals with current detectors.

3. FORMULATION

The overall goal of our hydrodynamic simulations is to measure how the quadrapole moment of a neutron star evolves after a glitch. Our formalism uses the weak-field metric meaning all special relativistic effects are taken into account while general relativistic effects are ignored. This is done because our simulations evolve gravitation with each time step and solving Einstein's equation that many times would be too computationally expensive. Instead, to increase our agreement with a full general relativistic solution, we used a generalized source term to Poisson's equation which takes into account all forms of energy instead of just rest mass density. The following outline of our formulation follows that presented in Kim et. al 2009 and Kim et. al 2012.

Our formalism uses the weak field metric in cylindrical coordinates (R,Z,Φ),

$$\begin{aligned} ds^2 &= g_{\mu\nu}dx^\mu dx^\nu \\ &= -(1+2\Phi)dt^2 + (1+2\Phi)^{-1}\delta_{ij}dx^i dx^j \\ &= -(1+2\Phi)dt^2 + \frac{1}{1+2\Phi}(dR^2 + dZ^2 + R^2 d\phi^2) \end{aligned} \tag{1}$$

As mentioned before, we take a single Newtonian gravitational potential:

$$\nabla^2\Phi = 4\pi\rho_{\text{active}}. \tag{2}$$

Where ρ_{active} is known as the active mass density which was first conceived by Tolman (cite) with the form:

$$\rho_{\text{active}} = T - 2T^0_0 \tag{3}$$

Where T is the trace of the stress-energy tensor. In a full general relativistic treatment, the stress-energy tensor describes the density of mass and energy. For this reason it can be considered the source term for gravity in general relativity. By bringing the stress-energy tensor of a perfect fluid into our formalism and treating all forms of energy as the source to our Poisson's equation, we get significantly better agreement with a full general relativistic treatment than just rest mass density as our source term.

As is standard in most numerical simulations we assume our star to be a perfect fluid. This is both for simplicity as viscosity and other effects of a normal fluid becomes too complex as well as the fact that stress energy tensor for a perfect fluid is easily computed:

$$T^{\mu\nu} = \rho_0 h u^\mu u^\nu + P g^{\mu\nu}, \tag{4}$$

where h is the specific enthalpy, u^μ is the four velocity with respect to a Eulerian observer, and P is the pressure. The specific enthalpy takes the form:

$$h = 1 + \epsilon + \frac{P}{\rho_0}, \tag{5}$$

where ϵ is the internal energy. Using this in combination with the stress energy tensor for a perfect fluid we can find the active mass density to be given by:

$$\begin{aligned} \rho_{\text{active}} &= T - 2T^0_0 \\ &= T^i_i - T^0_0 \\ &= \rho_0 h\frac{1+v^2}{1-v^2} + 2P. \end{aligned} \tag{6}$$

We enforce axisymmetry as well as equatorial symmetry. We define the conservative variables (q) and primitive variables (w) as

$$q = \begin{pmatrix} D \\ S_R \\ S_Z \\ S_\phi \\ \tau \end{pmatrix} = \begin{pmatrix} \rho_0 W \\ \rho_0 h W^2 v_R \\ \rho_0 h W^2 v_Z \\ \rho_0 h W^2 v_\phi \\ \rho_0 h W^2 - P - D \end{pmatrix}, \tag{7}$$

$$w = \begin{pmatrix} \rho_0 \\ v^R \\ v^Z \\ v^\phi \\ P \end{pmatrix}, \tag{8}$$

Which just stand for the conservation of mass and angular momentum. Then we can obtain the hydrodynamic equations:

$$\frac{\partial\left(\sqrt{\gamma}q\right)}{\partial t} + \frac{\partial\left(\sqrt{-g}f^R\right)}{\partial R} + \frac{\partial\left(\sqrt{-g}f^Z\right)}{\partial Z} = \sqrt{-g}\Sigma \tag{9}$$

where

$$f^R = \begin{bmatrix} Dv^R \\ S_R v^R + P \\ S_Z v^R \\ S_\phi v^R \\ \tau v^R + P v^R \end{bmatrix} \tag{10}$$

$$f^Z = \begin{bmatrix} Dv^Z \\ S_R v^Z \\ S_Z v^Z + P \\ S_\phi v^Z \\ \tau v^Z + P v^Z \end{bmatrix} \tag{11}$$

$$\Sigma = \begin{bmatrix} 0 \\ -\frac{\rho_{\text{active}}}{1+2\Phi}\frac{\partial \Phi}{\partial R} + \frac{S_\phi v^\phi}{R} + \frac{P}{R} \\ -\frac{\rho_{\text{active}}}{1+2\Phi}\frac{\partial \Phi}{\partial Z} \\ 0 \\ -\left(S_R \frac{\partial \Phi}{\partial R} + S_Z \frac{\partial \Phi}{\partial Z}\right) \end{bmatrix}. \tag{12}$$

Finally, the gravitational potential in cylindrical coordinates is

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \Phi}{\partial R}\right) + \frac{\partial^2 \Phi}{\partial Z^2} = 4\pi\rho_{\text{active}}. \tag{13}$$

These hydrodynamic formulas are used to evolve the star after the glitch perturbation using a finite volume where each grid enforces the conservation laws. The follow sections discuss how the initial model of the star is generated and the perturbations to the initial model used to mimic a glitch event.

4. INITIAL MODEL

4.1. *Equilibrium Models of Rotating Stars*

For our simulations we use the Hachisu Self Consistent Field method (HSCF) to generate our initial star (Hachisu 1989a). The HSCF method uses an integral form to the hydrostatic equation instead of directly solving the differential equations. This is more computationally efficient and generates stable star systems. The special feature of Hachisu's method is that instead of using a rotational velocity parameter as a condition to generate the star it uses an axis ratio (r_p/r_e) to define the star. This allows one to define the star by its elliptical shape caused by the rotation. This is a more stable way to calculate a stars equilibrium state than just using a velocity which can be more cumbersome as well as fail in generating a stable star if the velocity condition is insufficient.

In addition to this, to complete our formulation for hydrostatic equilibrium we need a relation for the pressure of the star (the equation of state). The equation of state of a neutron star is a highly sought after quantity in physics. The densities of a neutron star cannot be found anywhere in the universe and are not achievable in any laboratory today. Discovering the equation of state of a neutron star would be a huge achievement in the astronomical physics as well as nuclear physics. Today, most scientists use a piecewise polytropic equation of state while enforcing continuity at the boundaries to model the equation of state of a neutron star. This is primarily because a polytropic equation of state can be tuned to produce stars of the proper mass and radius as a neutron star. In our simulations we assume a simple polytropic equation of state:

$$P = \kappa \rho^{1+\frac{1}{N}} \tag{14}$$

where κ and N are the polytropic coefficient and index respectively. For our simulations we use $\kappa = 100$ and $N = 2$ because they produce stars of the appropriate mass and radius.

In addition to our assumption of a simple polytropic equation of state, we assume that the rotation to be a rigid one. In other words, our stars are not rotating differentially. This is valid because neutron stars must turn to uniform rotation relatively quickly after birth due to shear viscosity and magnetic tensions in the star. Using these two assumptions, the integral form of our equilibrium stars becomes

$$\ln\left[(N+1)\kappa\rho_0^{1/N} + 1\right] + \frac{1}{2}\ln(1+2\Phi) + \frac{1}{2}\ln(1-v^2) = C \tag{15}$$

where C is the integration constant to be found based on boundary conditions. A more complete treatment of the methods used to generate our initial stars can be found in Kim et. al 2009 (Kim et al. 2009).

This report takes a case study of a neutron star with the largest mass ever observed, $2.0M_\odot$, at a distance of 1kpc. Since this represents some of the best known conditions for a pulsar in which glitch induced gravitational radiation, the magnitudes of the characteristic strain amplitudes in this report are higher than would be observed from most known pulsars.

5. GLITCH PERTURBATIONS

Pulsars usually slow down at a very slow and predictable pace as they transfer energy to their environment but occasionally without explanation spin up. These spin ups are known as a glitch and are not at all well understood. The first pulsar glitches were observed in 1969, glitches have been observed in over 30 different pulsars. Even 40 years later, the exact mechanism that causes these events is still under debate. Because it is known that a pulsar glitch is caused by internal mechanism, proving the exact mechanism by which a glitch occurs would give valuable information about the state and properties of pulsars. The following section discusses two major glitch mechanism models: the star quake model and the vortex unpinning at crust-core interface model.

The following subsections discuss the two glitch mechanisms we consider in our simulations and the perturbations we apply to our equilibrium star to model these mechanisms. For both perturbations we consider the effective crust depth to be 10% the radius of the star and ignore all effects of tension and other complex properties the star might have. In addition we hold the conservation laws, the conservation of baryonic mass (M_0) as well as total angular momentum (J):

$$\begin{aligned} M_0 &= \int \rho_0 W dV \\ &= 2\pi \int \frac{\rho_0 W}{(1+2\Phi)^{3/2}} R dR dZ \end{aligned} \tag{16}$$

$$\begin{aligned} J &= \int S_\phi dV \\ &= 2\pi \int \frac{\rho_0 W^2 R^2 \Omega}{(1+2\Phi)^{3/2}} R dR dZ \end{aligned} \tag{17}$$

5.1. *Star Quake Model*

After the first pulsar glitch was observed in 1969 Ruderman developed a mechanism by which a glitch could occur due to a change in the moment of inertia of the star. This model assumes the star to be two components, the crust and the core. The core is assumed to be fluid while the crust has some rigidity to it. As the pulsar slows down it begins to adapt a more spherical shape but since the crust is rigid it resists this change. While this occurs the core is becoming more spherical and is placing stress on the crust. Once the stresses become strong enough the crust is forced to adapt to the core and this causes a change in the moment of inertia of the star. This change in moment of inertia, due to the conversation of angular momentum, causes the star to spin up and we observe a glitch.

This model has been widely explored and it has been found that even just a vertical surface motion of more than a centimeter would generate a glitch magnitude of $10-6$. However this model has problems explaining pulsars with multiple glitches in short time periods. It was found by Baym and Pines that a significant amount of energy must be transferred per glitch (cite). So much that it would not be possible for a pulsar to build up enough energy for a glitches to occur in a single pulsar as have been observed. Nevertheless, we consider this mechanism for one of our perturbation for comparison with our second mechanism, vortex unpinning at the crust-core interface.

The above figure shows the perturbation we apply for the star quake model. The stellar surface is shrunken by the asymmetric gaussian formula,

$$\frac{\delta\rho_0}{\rho_0} = \begin{cases} \lambda \ \mathrm{e}^{-\left(\frac{r-c}{\sigma_1}\right)^2} & \text{if } r \le c \\ (1+\lambda)\ \mathrm{e}^{-\left(\frac{r-c}{\sigma_2}\right)^2} - 1 & \text{if } r > c, \end{cases} \tag{18}$$

where $\delta\rho_0$ is the perturbation to the rest mass density, and c is the center of the Gaussian profile, which is not necessarily the crust-core interface. σ_1 and σ_2 represent the width of the Gaussian and are chosen from the conservation of mass. λ is the perturbation magnitude that we apply to the system and should not be confused with the observed glitch magnitude we are simulating. The perturbation to the angular velocity follows from this perturbation by the conservation of angular momentum. From the resulting angular momentum perturbation we measure the simulated glitch magnitude.

5.2. *Vortex Unpinning at the Crust-Core Interface Model*

As the problems with the star quake model were realized, Packard (Packard 1972) and soon after Anderson and Itoh (Anderson & Itoh 1975). Considered if the superfluidity of the core could be responsible for the glitch mechanism. As previously mentioned, the interior of a neutron star is considered to be a superfluid due to the slow rate at which they spin down. Because of this, if the core is considered a superfluid while the crust is a normal fluid, the core can be considered an angular momentum reservoir. The basic consideration of this model is that vortices at the crust-core interface couple the crust and the core. As the star spins down, the crust spins down faster than the core and differential rotation develops. At the point when this difference in velocity becomes too great, the vortices unpin and results in the core transferring momentum to the crust. This transfer causes the observed spin up during a pulsar glitch. This theory is generally the most accepted today but has yet to be set in stone.

The second perturbation, for the vortex unpinning model, gives only a change in rotational velocity while enforcing no perturbation in the mass density:

$$\begin{aligned} \frac{\delta\rho_0}{\rho_0} &= 0, \\ \frac{\delta\Omega}{\Omega} &= \begin{cases} -\lambda_J \ \mathrm{e}^{-\left(\frac{r-c}{\sigma}\right)^2} & \text{if } r \leq c \\ \lambda & \text{if } r > c \end{cases} \end{aligned} \tag{19}$$

In this case, λ is both the perturbation and glitch magnitude since we directly perturb the angular velocity. While λ_J is how much the angular momentum of the interior is reduced, and σ is a fitting parameter. Both values are determined by the conservation laws.

6. RADIAL PULSATION MODE TEST

6.1. *Stellar Pulsation*

To test the validity of our formulation and simulation methods we preformed a radial pulsation mode test. In theory, if our stars are in equilibrium, we should not observe any oscillations within the star. Yet, as with all numerical simulations, there are intrinsic errors that cause the star to oscillate including our resolution, floor values, truncation and limitations in describing the stellar surface (Kim et al. 2012). For this test we generate a stationary (non-rotating), spherical star. All stars have normal modes of oscillation depending on their state and our goal in this test is to compare the radial modes measured in our simulations to another method for generating the normal mode frequencies, linear perturbative analysis.

Although numerical simulations will cause oscillations within our modeled star, we apply a radial perturbation to our system to further excite these modes,

$$\delta\rho_0 = B \sin\left(\pi \frac{r}{r_s}\right). \tag{20}$$

This radial perturbation disturbs the steady, equilibrium state of the star and causes the star to oscillate when we evolve it dynamically. We measure the oscillation of the density in the star by measuring the central density of the star at each time step. These oscillation are usually complex and a superposition of the fundamental mode and its overtones. To analyze which modes are excited we use the standard of all oscillation analysis, Fourier analysis.

6.2. *Fast Fourier Transform*

For our discrete Fourier transform calculation we utilize the Fast Fourier Transform of the West algorithm package(Frigo & Johnson 2005). In addition, we use the zero-padding method to increase our sampling in the frequency domain. This adds no new real information to the transform but merely decreases the sample spacing in the frequency domain so that actual maximums are not missed due to insufficient sampling.

In addition to problems due to sampling we also deal with spectral leakage in the frequency domain by multiplying the time domain series by a window function. Spectral leakage can cause significant error in amplitude and frequency of maximum. In addition, weaker components can be complete washed out by spectral leakage. Fast Fourier transform algorithms assume the time domain sample is periodic in natural. Discontinuities in the time domain require many more sinusoidal frequencies to represent such complex features. These discontinuities resulting from the non-periodic nature of a time domain sample are the source of spectral leakage in the frequency domain. To reduce these errors we apply a window function which suppresses the magnitudes of the discontinuities thereby suppressing the spectral leakage in the frequency domain. Specifically we use the Hamming window function:

$$w_j = 0.54 - 0.46 \cos\left(\frac{2\pi j}{N}\right) \tag{21}$$

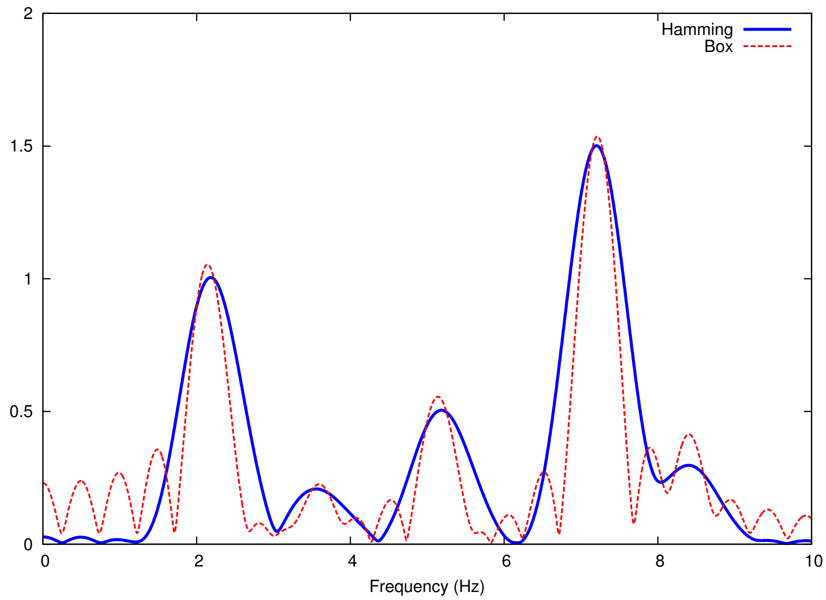

FIG. 1.— This figure shows the effects of a window function on a linear combination of simple sinusoidal waves. The box window is the Fourier transform without any window function. The Hamming window is with a window function modifying the time domain series. It is clear how effectively a window function reduces the error.

Figure one shows the effects of adding a window function to our time domain sample. It is important to note the addition of a window function decreases the overall spectral resolution of sample. So if two modes are located too close to each other, it may be difficult to accurately identify their amplitudes and frequencies.

6.3. *Radial Mode Test Results*

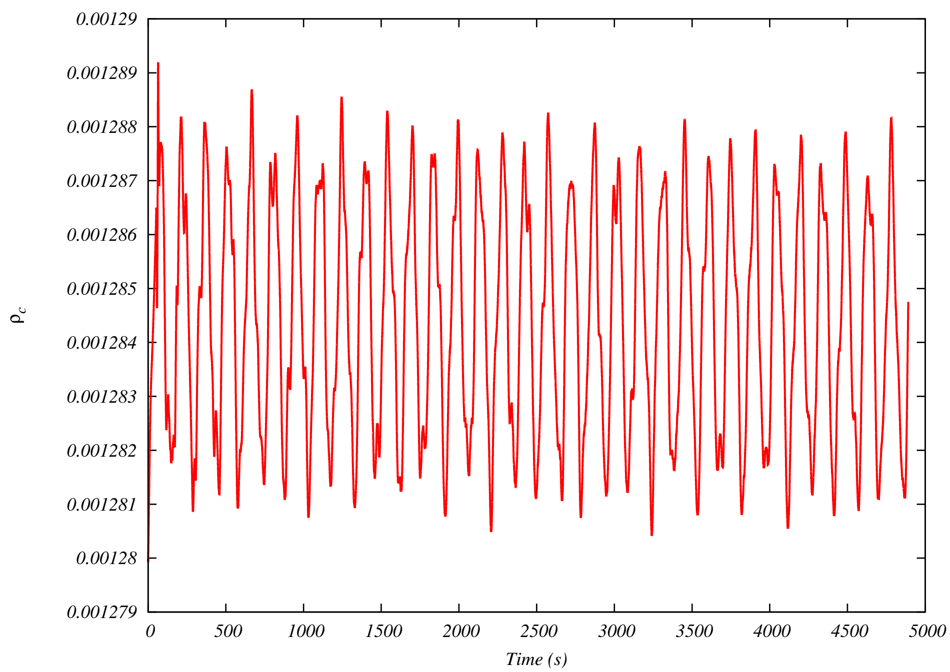

FIG. 2.— The above figure shows the oscillations of the central density of our simulated star in time.

The above figure shows the evolution of the central density of our simulated star in time. The simulation was ran for 100000 time steps to give a strong sample. The central density appears stable over the extent of the run. Below is the Fourier transform of the above central density.

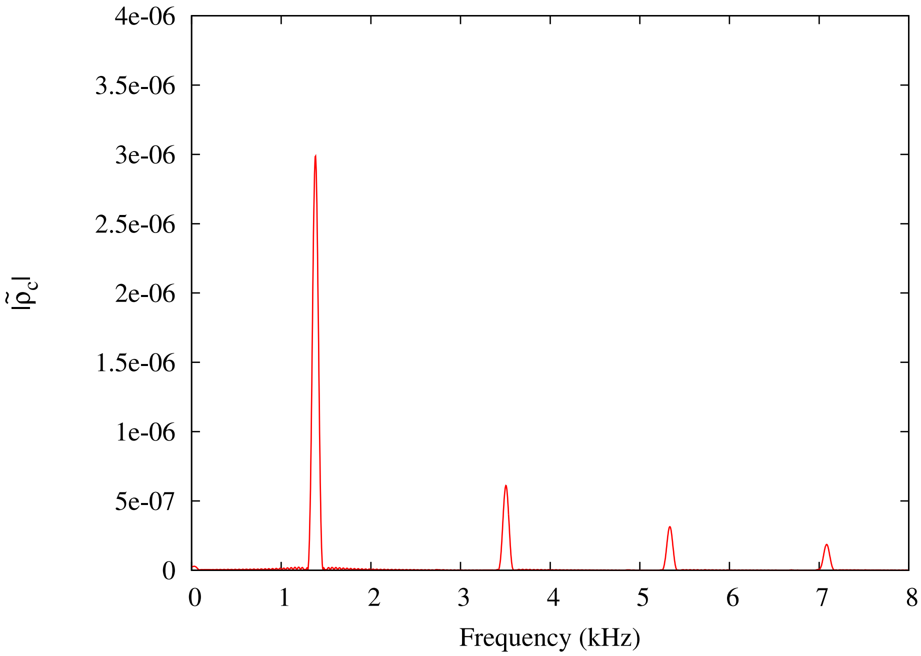

Fig. 3.— The above figure is the Fourier transform of our central density time sample. The transform clearly shows four excited radial modes, the fundamental and its three overtones.

	F	H_1	H_2	H_3
Simulations	1.382	3.505	5.330	7.083
σ	1.383	3.506	5.334	7.083
% Difference	0.07	0.03	0.07	0

The Fourier transform clearly shows four excited modes. We use the change in central density as opposed to the absolute central density so the modes stand out compared to low frequency components. We use a secondary code which utilizes linear perturbative analysis to compare to our simulations results. We find that our results agree very accurately with the linear analysis code.

7. MODE ANALYSIS

The goal of these simulations is to find the modes of gravitational radiation excited by the glitch event. Gravitational waves emitted by the star are related to the quadrapole moment of the star by

$$h_{ij}^{TT} = \frac{d^2 I_{kl}}{dt^2} P_{ik} P_{jl}. \tag{22}$$

Direct differentiation of the quadrapole moment introduces significant error. Instead, we take the Fourier transform of the quadrapole moment which allows us to find the amplitude and frequency of the gravitational radiation at the same time. In this form, the maximum characteristic strain amplitude is

$$h_c = \frac{64\pi^2}{r} f_c^2 \left| \tilde{I}_{xx}(f_c) \right| \tag{23}$$

Therefore, by taking the Fourier transform of quadrapole moment and multiplying by the mode frequency squared we can find the effective amplitude of the gravitational wave modes.

Because real glitch magnitudes are much too small to properly simulate, we run our simulations for many perturbation magnitudes on the same initial model. From this we apply a linear fit to each mode and extrapolate back to real glitch magnitudes. The below plots show the perturbation data used extrapolate back to real magnitudes.

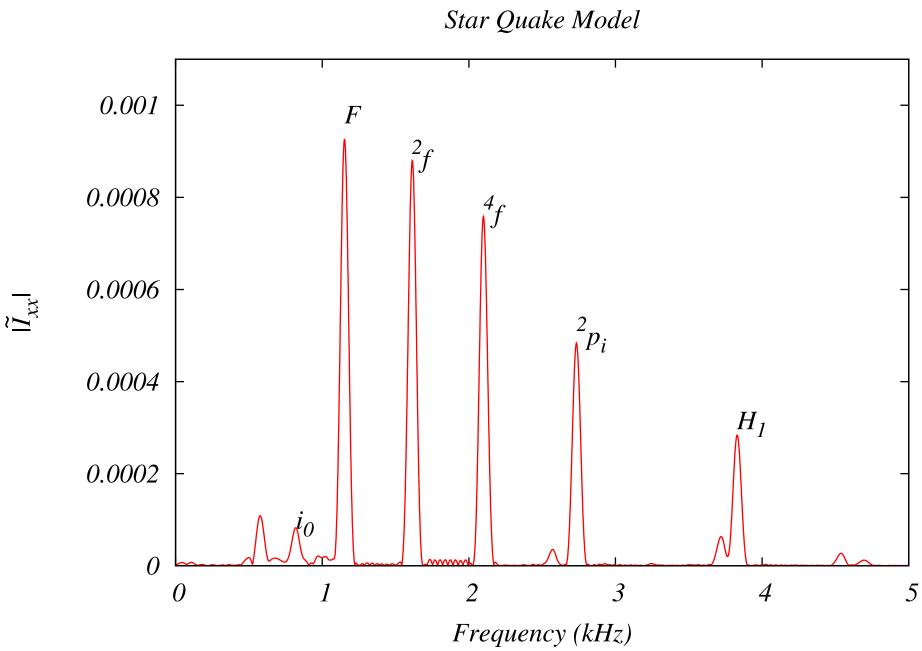

FIG. 4.— This plot is the Fourier transform of the quadrapole moment for a 2.0$M_\odot$star for perturbation one with λ=0.001. The figure shows the modes excited. Although F and H_1 are radial modes, due to the nonspherical shape of the star they have a large quadrapole moment component.

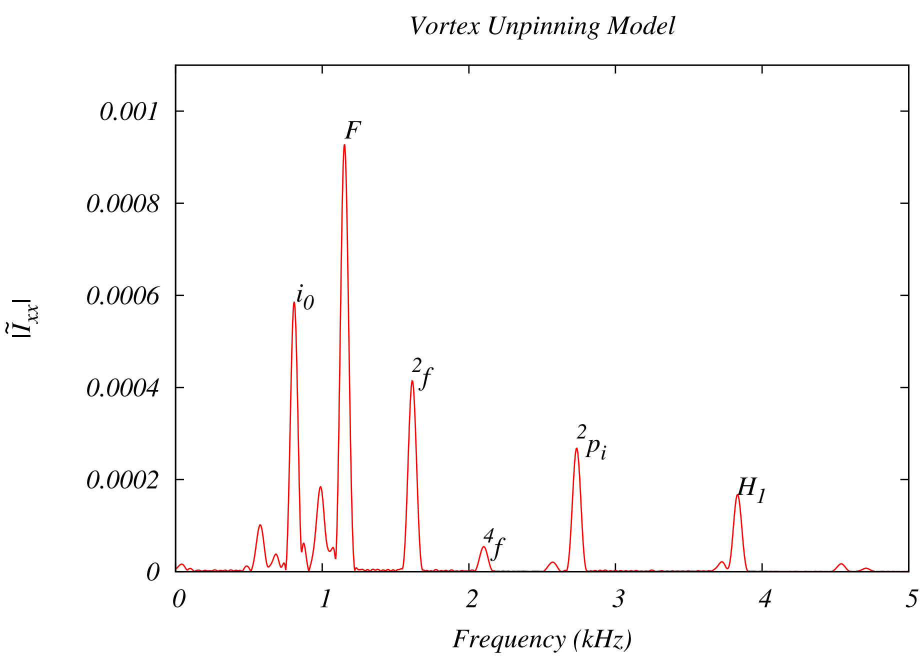

FIG. 5.— This plot is the Fourier transform of the quadrapole moment for a 2.0$M_\odot$star for perturbation two with λ=0.001. The figure shows the modes excited.An interesting feature of this perturbation is that it highly excites the inertia mode of the star. This mode is known to have a long dampening time which can help increase the detect-ability of the modes.

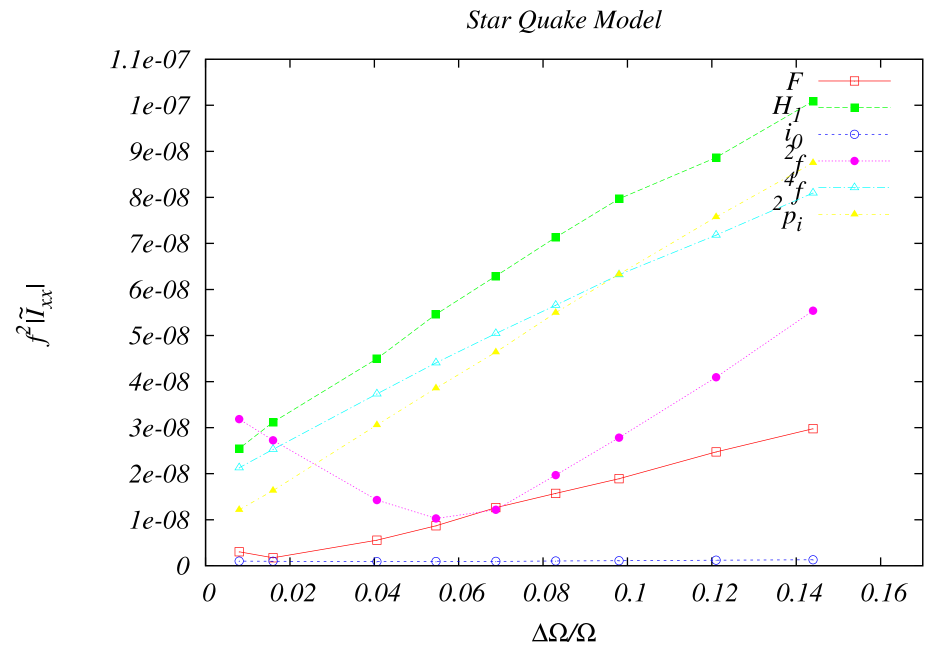

FIG. 6.— This figure shows the effective gravitational wave magnitudes for perturbation one. This data was used to extrapolate back to real glitch magnitudes. The nonphysical behavior exhibited by the ^{2}f mode is most likely caused by numerical errors.

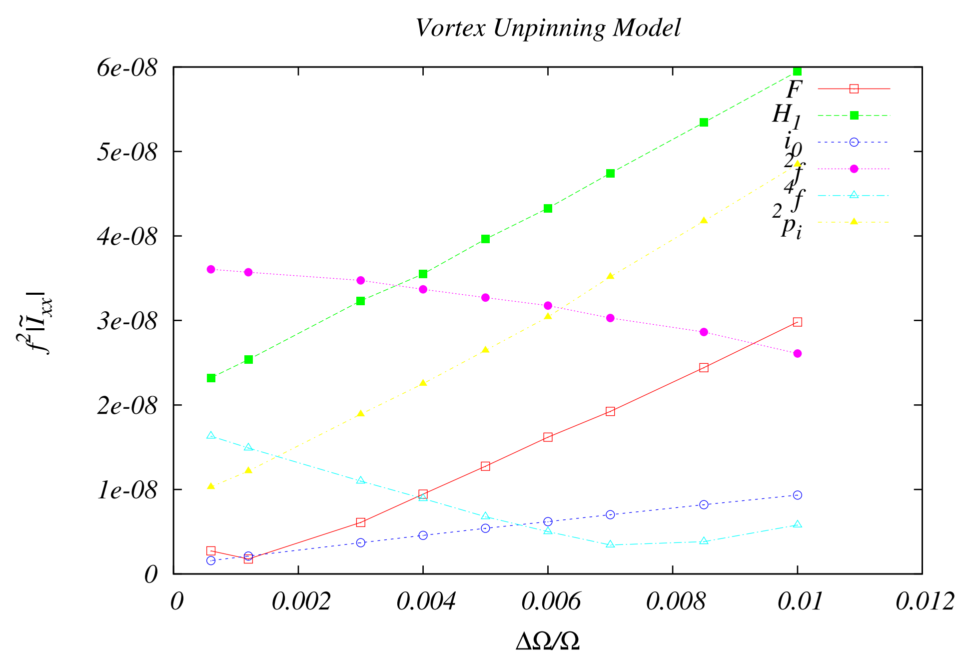

FIG. 7.— This figure shows the effective gravitational wave magnitudes for perturbation two for various glitch magnitudes. We use this data to extrapolate back to realistic glitch magnitudes. The nonphysical behavior of the ^{2}f and ^{4}f mode are currently under investigation.

8. DISCUSSION

	Frequency (kHz)	S.Q.	V.U.
F	1.152	1.11×10^{-27}	1.44×10^{-26}
H_1	3.828	1.10×10^{-24}	1.00×10^{-24}
i_0	0.808	4.20×10^{-26}	6.04×10^{-26}
^2f	1.613	6.58×10^{-25}	1.78×10^{-24}
^4f	2.100	9.12×10^{-25}	7.14×10^{-25}
^2p_i	2.734	3.78×10^{-25}	3.48×10^{-25}

TABLE 1
The above table shows the summary of the final results for the maximum characteristic strain amplitude of each mode excited by the different glitch models. These values assume a glitch amplitude of 10^{-5}, a neutron star mass of $2.0M_\odot$, and a distance of 1 kpc.

Currently, detecting gravitational radiation from a pulsar glitch is very unlikely. The strongest strain amplitude calculated is located in a less sensitive frequency band as well as being an order of magnitude lower than the most sensitive band range of advanced LIGO. Future detectors such as the Einstein Telescope may reach the sensitivity necessary to measure such waves. It is important to reiterate that these strain values are for very good conditions; the highest mass neutron star observed, the highest glitch magnitude observed, and a rather close distance. In addition, we considered the maximum characteristic strain rate which assumes the best orientation of the star to us.

Overall, both perturbations excite the same modes to varying degrees. The vortex unpinning perturbation notably excites the inertial mode of the star which contains the majority of the kinetic energy and is located near the most sensitive frequency range of gravitational wave detectors (aLIGO). The star quake perturbation notably excites the ^{4}f mode more than the vortex unpinning model. Both perturbations exhibit nonphysical results in the ^{4}f and ^{2}f modes. Future work must be done to explain these features if these results are to be considered reputable.

Although the calculated strain amplitudes are below current detector sensitivity, they may be observable in the future depending on their dampening time. For periodic sources, the detection limit is inversely proportional to the observation time. Therefore, if the mode is excited for long enough, the low amplitudes may be overcame by the periodic nature of the signals. Future work could be done to explore dampening and incorporating it into our simulations.

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