The Minimum Amount Of Observations Needed To Determine If the Universe Has A Mass Gap

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Observations of gravitational waves, emitted from coalescing compact binaries allow physical parameters of the binary to be determined. The two purposes of this experiment is to use the mass parameters inferred from observations, such as the chirp mass, total mass, and the symmetric mass ratio (μ, M, η) to theoretically determine how many neutron star - black hole binaries will need to be observed to determine if the universe has a mass gap between low mass black holes and high mass neutron stars or not, as well as, to see if the proposed method will be able to accurately identify differences among universes, such as metallicity and merger rates. Both problems are carried out by generating different universes and models of the universes, then comparing the models to all of the universes. As for the mass gap problem, the difference in universes lies with cutoff for component masses, generating mass gap and no gap distributions. The metallicity and merger rate problem is done by using the component mass distributions for NSBH systems that already take into account metallicity, merger rates and other parameters from S. Stevenson et al. (2015) [5]. Multivariate distributions are generated of chirp mass vs. total mass and chirp mass vs. the symmetric mass ratio which are compared between the mass gap and the no gap universe, keeping the types of distributions consistent. The Kolmogorov-Smirnov Test and the Anderson-Darling Test are utilized to determine how well the observed sample compares to the entire population of the theoretical universe. From these plots and tests it is estimated how many observations are needed to accurately model the universe and to make an accurate prediction of whether a mass gap exists or not.

1. INTRODUCTION

Black holes and neutron stars are formed in supernovae, the collapse of massive stars. Depending on the mass of the collapsed star it will form a black hole, neutron star, or a white dwarf. At the time of collapse, if the star is roughly the mass of the Sun, then it will form a white dwarf. If the star is greater than the Chandrsekar limit, but less than 3 $M_\odot$ it will become a neutron star. If the collapsing star is larger than 3 $M_\odot$ it will, most likely, form a black hole.

Gravitational waves observations allow us to investigate physical properties of black holes (BH's) and neutron stars (NS's). Gravitational waves (GW's) are emitted from interacting, massive objects, in space time. The luminosity of GW's emitted by the interacting objects is proportional to the compactness of the masses to the fifth power ($L \propto \sigma^5$). [4] Black holes and neutron stars are the focus of GW detection's because they are the most compact objects in the universe. These GW observations are made using ground based interferometer's such as LIGO, VIRGO, KAGRA, etc.. Observations with the ground based detectors allow for potential unravel of the mass distributions for black holes and neutron stars. Verification of these distributions allows us to begin to try and answer questions regarding the evolution of black holes and neutron stars. [2]

Currently, there is some controversy about whether a mass gap exists between high mass NS's and low mass BH's where no compact object can be. There is even disagreement as to where the gap exists. M. Hannam et al. (2013) claims the mass gap lies from $\sim$ 2 - 4 $M_\odot$, while T. Littenberg et al. (2015) suggests that the gap is between $\sim$ 3 - 5 $M_\odot$. [2] [3] Both of the referenced experiments investigate how accurately the component masses of compact binaries can be measured, in order to attempt to conclude the existence of a mass gap or not. The issue with using the component masses is that they are measured with extremely poor accuracy, ranging between 100-200%.

Here we argue the extremes of both, assuming the gap is between $\sim$ 2 - 5 $M_\odot$. However, the mass parameters that will be taken into consideration for this experiment are chirp mass, total mass and the symmetric mass ratio, which can all be deduced from GW observations.

Chirp mass (μ) is the most accurately recorded mass parameter because it is the leading order of the phase for GW's.

$$\mu = \eta^{3/5} M = \frac{m_1 m_2^{3/5}}{(m_1 + m_2)^{-1/5}} \tag{1}$$

The symmetric mass ratio (η) of binaries is not measured as accurately as the chirp mass because it is only seen in high order corrections, as well as a degeneracy between the symmetric mass ratio and the angular momentum of the components.[2]

$$\eta = \frac{m_1 m_2}{(m_1 + m_2)^2} \tag{2}$$

This degeneracy also effects the accuracy as to how well the component masses (m_1, m_2) can me measured. Fortunately, for this experiment we assume the components are not spinning. Finally, we consider the total mass (M) of the binary. It will be discussed later in Section 2 that the total mass of the binary can be measured with greater accuracy than the symmetric mass ratio.

Using these mass parameters instead of the component masses, investigate how many observations will be needed to accurately model the universe, which will eventually allow us to conclude if the mass gap exists or not. In the following sections we will discuss in great detail, the steps we took to arrive at our estimated value of observations. Section 2 describes the necessary procedures to reproduce this experiment and gives descriptions and reasons for using various tests and models.The results of testing for a few of the many models that could be used, are presented in Section 3. Section 4 provides an summary of the project as well as possible future endeavors that can expand or improve this project.

2. METHOD

2.1. Mass Gap Vs. No Mass Gap

Our study does not assume the universe contains a mass gap or not, however these are the two options. In order to remain impartial to either scenario, both options are tested. To simulate these two universes, mass cutoffs are applied to the mass distributions of NS's and BH's. For the mass gap scenario we apply hard cutoffs at 1.2 and 2 $M_{\odot}$ for the NS and 5 and 15 $M_{\odot}$ for the BH. There are BH's with masses much greater than 15 solar masses, however for this experiment they are not considered. The universe with out a mass gap ranges for the NS from 1.2 - 3 $M_{\odot}$ and 2 - 15 $M_{\odot}$ for BH's.

Four models of the universe are generated and tested. To better explain the universes; there is one universe with a mass gap and one without. Two distributions are used for NS's and BH's, uniform and normal distributions. However, the normal distributions for BH's are generated based of of the standard deviation and mean values of component mass from the universe models used in M. Dominik et al. (2012).[1]

The goal of the project is to be able to distinguish the mass gap universes from the no gap universes. To do this, an estimate of how reliable the observed mass parameters will be in an actual observational experiment must be made. The parameters of analytic interest are μ , M, and η, which can all be calculated from the original component mass distributions. From K.G. Arun et al. (2008) we took the accuracy of measurement for chirp mass and eta to be .00679 and .01459, respectively, as well as the correlation coefficient ($C_{\mu\eta} = .9294$). However, to find the accuracy of measurement for the total mass the propagation of uncertainty is used (Equation 3).

$$\sigma_M^2 = |\frac{\partial M}{\partial \mu}|^2 \sigma_\mu^2 + |\frac{\partial M}{\partial \eta}|^2 \sigma_\eta^2 + 2\frac{\partial^2 M}{\partial \mu \partial \eta}\sigma_{\mu\eta} \tag{3}$$

Substituting equations 1 and 2 into equation 3 gives the resulting equation:

$$\sigma_M^2 = (\frac{\sigma_\mu}{\mu})^2 + (\frac{3\sigma_\eta}{5\eta})^2 + \frac{6\sigma_{\mu\eta}}{5\mu\eta}. \tag{4}$$

Now to calculate the covariance between chirp mass and the symmetric mass ratio, the following expression is needed:

$$\sigma_{\mu\eta} = C_{\mu\eta}\sigma_\mu\sigma_\eta. \tag{5}$$

With this method the fractional error determined for total mass (σ_M) is 0.00834, thus theoretically proving that the total mass of a compact binary can be determined with a higher accuracy than the symmetric mass ratio.

Once the accuracy of measurement is known for all mass parameters, 1-Dimensional distributions are generated as a test to prove that the measured values of the parameters will produce a plot similar to the theoretical plot. If the measured and theoretical plots are similar then the corresponding parameter can be used throughout the experiment, producing good results. To determine how the observed and theoretical 1-Dimensional distributions compare, the Kolmogorov-Smirnov Test (KS Test) is used.

The KS Test is a nonparametric test making it very general, where it does not assume that the distributions being tested are of any particular type. It works by creating a Cummulative Distribution Function (CDF) and measuring the greatest distance (D-Statistic) between the two plots, then computing the probability (P-Value) that a distance that large or larger will be measured. If the P-Value is greater than the significance level, then it is concluded that there is not enough evidence to overrule the null hypothesis, meaning the distributions originate from the same population. The null hypothesis states that the plots being tests come from identical distributions. One disadvantage of the KS Test is that it weighs the center of the distributions much more heavily than the tails of the distribution. Calculating the D-statistic is straight forward as seen in Equation6.

$$D_{KS} = \sqrt{\frac{mn}{m+n}}|F_m(x) - F_n(x)| \tag{6}$$

To find the P-Value, the following equation is used:

$$P = \frac{N_D}{n} \tag{7}$$

where 'n' is the number of permutations and N_D is the number of D-statistics that are greater than the observed D_{KS}.

To restrict the tests further, multivariate distributions of μ vs. η and μ vs. M are generated, however these are not completely random 2-Dimensional plots. There is some correlation, as mentioned earlier, between all of the parameters because they can all be derived from m_1 and m_2. Figure 1 depicts the correlation between parameters of the 2-Dimensional plot. Furthermore, the additional dimension to the plots makes the situation of the model not matching the universe much more probable. Meaning, when a test statistic that is nonzero is much more significant then that of a 1-Dimensional plot.

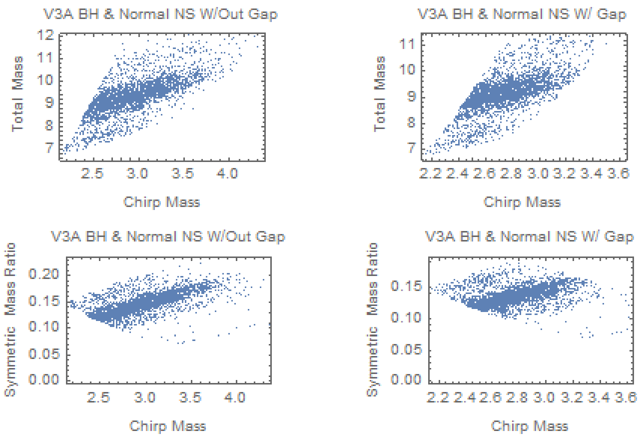

Figure 1: These plots clearly depict the correlation between the mass parameters utilized throughout this project. There is a positive correlation between total mass and chirp mass and a negative correlation between the symmetric mass ratio and chirp mass. The sign of the correlation is determined by the sign of the slope of the best fit line of the plots. The correlation restricts the 2-D plot making the test statistics more significant.

Using the multivariate distributions the minimum number of actual binary observations can be estimated by comparing the distributions of the same BH and NS models to the two different universes. This is tested by trial and error essentially. We start by testing 100 observations to the entire population of the universe model (comparing 100 observations of the measured μ and M plot to the entire theoretical μ and M plot of the universe with and without a mass gap). We begin with 100 observations because that is the projected number of actual observations that will be recorded by aLIGO/aVIRGO in the next two years. To compare the 2-Dimensional plots we use the KS Test, as well as, the Anderson-Darling Test (AD Test).

The AD Test is very similar to the KS Test, however it is used because it weighs the ends of the distributions more heavily than the KS Test. The bottom of Figure 2 shows that for some of these distributions the majority of the difference is at the tails. The AD Test is more sensitive then the KS Test because it uses the specific distribution to calculate the critical values, however, having to calculate the critical values is also a disadvantage compared to the KS Test. Determining the D-Statistic for the AD Test is a little more involved than the KS Test. Equation 8 is the formula for finding the D_{AD} statistic.

$$D_{AD} = \frac{1}{mn} \sum_{i=1}^{m+n} \frac{(N_i Z_{m+n-ni})^2}{i Z_{n+m-i}} \tag{8}$$

Where Z_{n+m} is the combined and ordered samples X_n and Y_m.

The KS and AD Test statistic is calculated 100 times, then averaged and the standard deviation is determined for every comparison of the multivariate plots to the two different universes (Reference Tables 5, 5, 5, 5). We then repeat the multivariate steps for 200, 400, 800 and 1000 observations, in order to determine how many observations are needed to be able to accurately model the universes population.

2.2. Metallicity and Merger Rate Differences

To test the sensitivity of the proposed method in the previous Section 2.1, the test is used to determine slight differences in universe models given by S. Stevenson et al. (2015) based on the metallicity and merger rate of the binaries, as well as other parameters. [5] The same steps are applied as above, however, this time 100 and 1000 samples are taken from each universe model 100 times and compared each theoretical universe population using the KS and AD statistical tests. These tests produce the average P-Value and standard deviation of the P-Value, which can be seen in Table 5 and Table 5.

3. RESULTS

3.1. Mass Gap Vs. No Mass Gap

Using the Standard A Model and Variation 3A Model, given by S. Stevenson et al. (2015), we test our method described above to see if we can distinguish a universe with a mass gap from a universe without a mass gap and if so, what is the minimum observations needed.[5] Again, our models were generated as a normal and uniform distribution with and without a mass gap applied, based on the mean and standard deviation of the models from S. Stevenson et al. (2015) and M. Dominik et al. (2012). This produces four different universes for each model.

As seen if Tables 5 and 5 (Tables of the KS Test on the models) and Tables 5 and 5 (Tables of the AD Test on the models) the proposed method using the KS and AD Tests, high average P-Values are returned. Having high average P-Values means that the distributions being compared come from the same or similar populations. This allows us to argue that our proposed method will accurately determine if a mass gap exists or not in our universe.

Now, to answer the question of what the minimum number of observations of NSBH binaries are needed to distinguish between the two universes. Initially, this comparison was made using 100 observations, then 200, 400, 800 and 1000. As depicted in Figure 2, which shows the probability density and cumulative distribution functions (PDF and CDF) for 100 and 1000 observations, there is a very insignificant difference between the two amounts of observations. The only noticeable difference is the number of observations in each bin, as shown in Figure 2 (Top). Only needing 100 observations to accurately determine the existance of a mass gap is extremely significant because it is projected that in the next two years 100 observations of NSBH systems should be recorded.

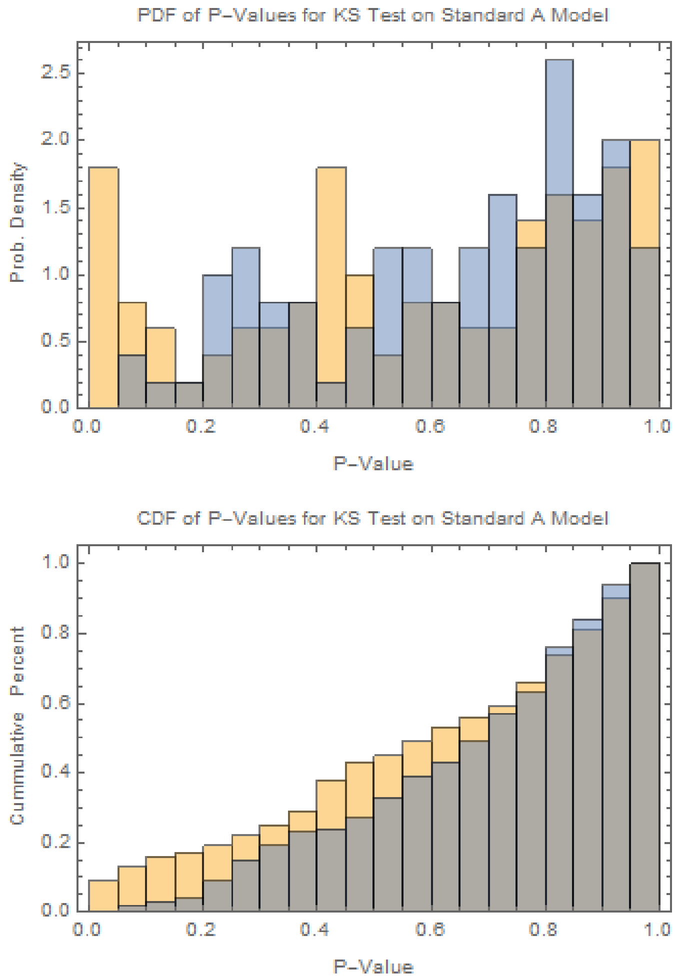

Figure 2: This figure shows the PDF (Top) and CDF (Bottom) of the P-Values when comparing the Standard A Model universe with a mass gap, 100 times to models of the same universe with 100 (Yellow) and 1000 (Blue) observations, using the Kolmogorov-Smirnov Test. As seen, the CDF of 100 and 1000 observations does not have a very significant difference and the average P-Value for the 100 observations is well above the significance level. This concludes that the 100 observations is not as great 1000 observations, but works well enough to complete the experiment.

3.2. Metallicity and Merger Rate Differences

Using the same method as used to compare a mass gap universe to a no gap universe, eleven of the universe models in S. Stephenson et al. (2015) are compared. Again, the difference between these models are the metallicity and merger rates, which is not as significant as a component mass cutoff.

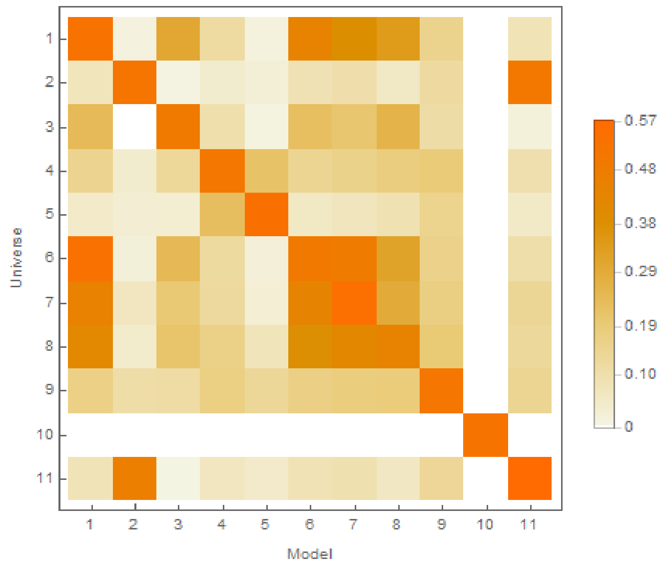

Figure 3: This is a color mapping of Table 5, showing the P-Values using the Anderson-Darling Test. The X-axis are the populations and the Y-axis are the models. The numbers one through eleven correspond to the Standard A Model through Variation 12 A Model, omitting Variation 8 and 10 A Models. It depicts that there is, on average, high P-values for the 100 observed samples compared to corresponding population. Also, this graphic clearly shows the symmetry of the table.

As illustrated in Figures 3 and 4 the different universes can be distinguished from each other for the most part, with a few exceptions. However, these exceptions are symmetric and need to be analyzed further to understand why that is. Tables 5 and 5 show the actual data that Figures 3 and 4 illustrate. In all except the first column the highest average P-Values occur when the universe model is tested to its corresponding population. This was the expected outcome, but again more analysis needs to be made as to why that is not the case in the first column.

Much like in Section 3.1, there is a very insignificant difference between using 100 and 1000 observations to compare the eleven universes with the KS and AD Tests. The comparison between 100 and 1000 observations when comparing Variation 12 A Model to the corresponding population is shown in Figure 5. The tables and figures that are shown only use 100 observations.

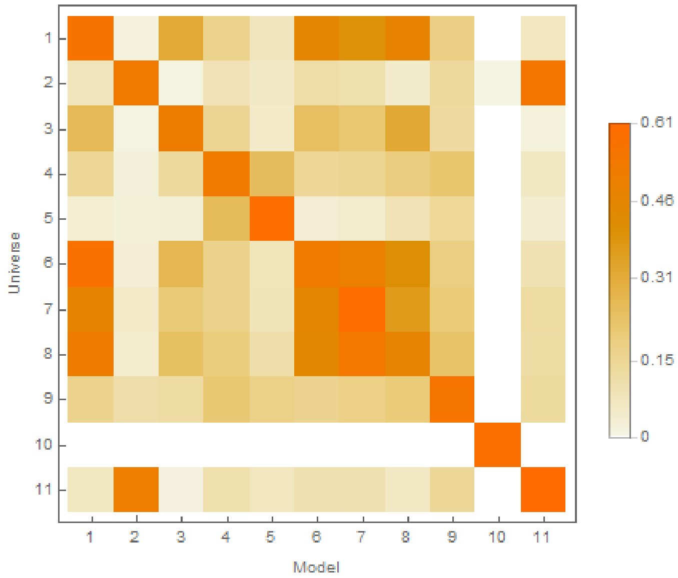

Figure 4: This is a color mapping of Table 5, showing the P-Values using the Kolmogorov-Smirnov Test. The X-axis are the populations and the Y-axis are the models. The numbers one through eleven correspond to the Standard A Model through Variation 12 A Model, omitting Variation 8 and 10 A Models. It depicts that there is, on average, high P-values for the 100 observed samples compared to corresponding population. Also, this graphic clearly shows the symmetry of the table. The

4. CONCLUSION

Section 3.1 allows us to conclude that the proposed method is powerful enough to distinguish between a universe with and without a mass gap using only 100 observations of NSBH binaries. The evidence supporting our claim is shown in Figures 5, 5, 5, 5. The only flaw is that the P-Values are averages, meaning that some of the individual P-Values are very small, below the significance level of 0.05. If an actual experiment was performed with only 100 observations, there it is probable that a P-Value below the significance level is produced, not allowing for a conclusion of either gap or no gap to be drawn.

However, this is a very simplistic model where the only parameters considered are the mass parameters. Our proposed method should be tested to distinguish between a universe with and without gap that also accounts for metallicity, merger rates, spin, and the spin mass ratio degeneracy of the binaries, to see if the method's strength is any different considering more than just mass parameters. Furthermore, our model is only of NSBH binaries, which are the binaries that must be observed to determine a mass gap, but this experiment should be carried out in a realistic universe with BNS, BBH, and NSBH binaries just to see if anything changes.

Not a lot can drawn from our results discussed in Section 3.2 because the issue of symmetric high average P-Values between various models and universes has yet to be explored. As of now, our conclusion is that our method is not powerful enough to distinguish between universes that vary by metallicity and merger rates and not mass parameters. However, this conclusion may not be the case because we lack an understanding as to why the high P-Values are symmetric with various models and populations.

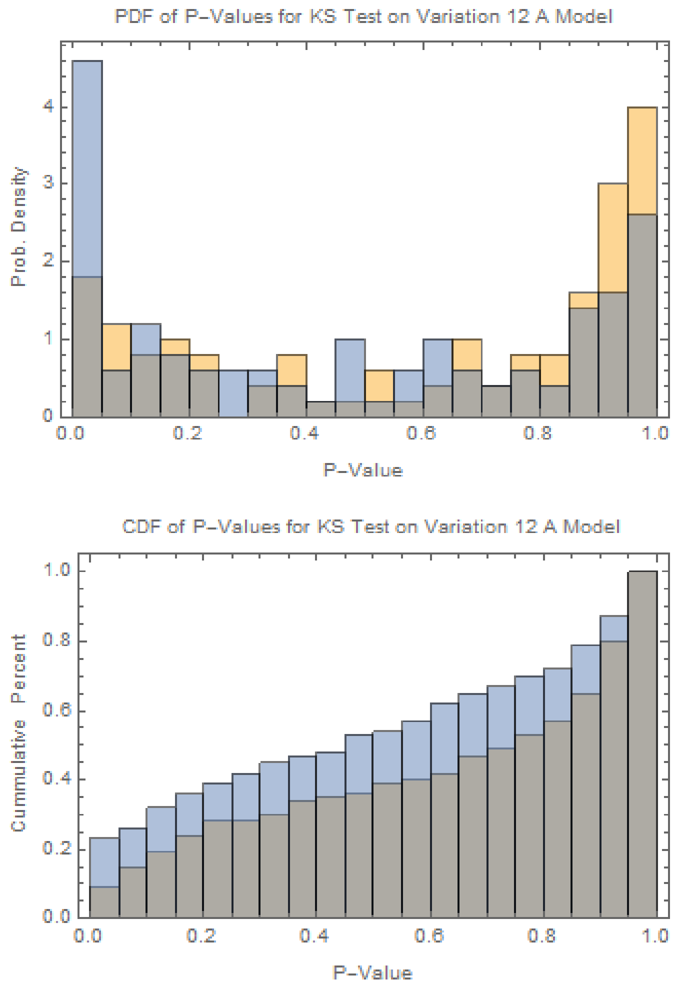

Figure 5: This figure shows the PDF (Top) and CDF (Bottom) of the P-Values when comparing the Variation 12 A Model universe, 100 times to models of the same universe with 100 (Yellow) and 1000 (Blue) observations, using the Kolmogorov-Smirnov Test. As seen, the CDF of 100 and 1000 observations does not have a very significant difference and the average P-Value for the 100 observations is well above the significance level. This concludes that the 100 observations is not as great 1000 observations, but works well enough to complete the experiment.

5. ACKNOWLEDGMENTS

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[1] Michal Dominik, Krzysztof Belczynski, Christopher Fryer, Daniel E Holz, Emanuele Berti, Tomasz Bulik, Ilya Mandel, and Richard O'Shaughnessy. Double compact objects. i. the significance of the common envelope on merger rates. *The Astrophysical Journal*, 759(1):52, 2012.

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[4] Bangalore Suryanarayana Sathyaprakash. Gravitational waves and astrophysical sources. *Comptes Rendus Physique*, 14(4):272–287, 2013.

[5] Simon Stevenson, Frank Ohme, and Stephen Fairhurst. Distinguishing compact binary population synthesis models using gravitational-wave observations of coalescing binary black holes. *arXiv preprint arXiv:1504.07802*, 2015.

Table I: This is a table of the average P-Values and the standard deviation of the P-Values resulting from 100 multivariate Anderson-Darling Test comparing eleven of the universes from S. Stevenson et al (2015) to models of those universes consisting of 100 observations. The average P-Value is the left value in the parentheses and the standard deviation is on the right. The diagonal of the table represents the model compared to its corresponding population. The diagonal contains the highest average P-value for that particular column, except for the first column. Notice that with a high P-Value, there is a relatively high standard deviation. This can be a concern, but is discussed in Section 4

	M_{SA}	M_{V1A}	M_{V2A}	M_{V3A}	M_{V4A}	M_{V5A}	M_{V6A}	M_{V7A}	M_{V9A}	M_{V11A}	M_{V12A}
U_{SA}	(0.519,0.352)	(0.0,0.0)	(0.169,0.282)	(0.0,0.0)	(0.0,0.0)	(0.400,0.364)	(0.310,0.341)	(0.236,0.258)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)
U_{V1A}	(0.0,0.0)	(0.508,0.347)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.488,0.352)
U_{V2A}	(0.068,0.154)	(0.0,0.0)	(0.474,0.341)	(0.0,0.0)	(0.0,0.0)	(0.048,0.128)	(0.021,0.083)	(0.101,0.195)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)
U_{V3A}	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.489,0.332)	(0.036,0.096)	(0.0,0.0)	(0.0,0.0)	(0.001,0.006)	(0.007,0.012)	(0.0,0.0)	(0.0,0.0)
U_{V4A}	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.049,0.093)	(0.528,0.347)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)
U_{V5A}	(0.522,0.325)	(0.0,0.0)	(0.069,0.158)	(0.0,0.0)	(0.0,0.0)	(0.476,0.338)	(0.460,0.338)	(0.193,0.245)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)
U_{V6A}	(0.414,0.328)	(0.0,0.0)	(0.012,0.043)	(0.0,0.0)	(0.0,0.0)	(0.400,0.317)	(0.528,0.357)	(0.153,0.236)	(0.001,0.004)	(0.0,0.0)	(0.0,0.0)
U_{V7A}	(0.361,0.289)	(0.0,0.0)	(0.028,0.088)	(0.0,0.001)	(0.0,0.0)	(0.308,0.294)	(0.374,0.304)	(0.401,0.351)	(0.011,0.024)	(0.0,0.0)	(0.0,0.0)
U_{V9A}	(0.0,0.001)	(0.0,0.0)	(0.0,0.0)	(0.0,0.003)	(0.0,0.0)	(0.0,0.001)	(0.002,0.008)	(0.003,0.009)	(0.499,0.324)	(0.0,0.0)	(0.0,0.0)
U_{V11A}	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.512,0.359)	(0.0,0.0)
U_{V12A}	(0.0,0.0)	(0.435,0.344)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.572,0.367)

Table II: This is a table of the average P-Values and the standard deviation of the P-Values resulting from 100 multivariate Kolmogorov-Smirnov Test comparing eleven of the universes from S. Stevenson et al (2015) to models of those universes consisting of 100 observations. The average P-Value is the left value in the parentheses and the standard deviation is on the right. The diagonal of the table represents the model compared to its corresponding population. The diagonal contains the highest average P-value for that particular column, except for the first column. Notice that with a high P-Value, there is a relatively high standard deviation. This can be a concern, but is discussed in Section 4

	M_{SA}	M_{V1A}	M_{V2A}	M_{V3A}	M_{V4A}	M_{V5A}	M_{V6A}	M_{V7A}	M_{V9A}	M_{V11A}	M_{V12A}
U_{SA}	(0.545,0.362)	(0.0,0.0)	(0.178,0.272)	(0.001,0.004)	(0.0,0.0)	(0.426,0.361)	(0.315,0.325)	(0.437,0.326)	(0.003,0.011)	(0.0,0.0)	(0.0,0.0)
U_{V1A}	(0.0,0.0)	(0.509,0.341)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.527,0.336)
U_{V2A}	(0.077,0.160)	(0.0,0.0)	(0.483,0.318)	(0.0,0.002)	(0.0,0.0)	(0.058,0.146)	(0.028,0.093)	(0.179,0.256)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)
U_{V3A}	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.500,0.329)	(0.066,0.119)	(0.0,0.0)	(0.0,0.0)	(0.007,0.022)	(0.029,0.050)	(0.0,0.0)	(0.0,0.0)
U_{V4A}	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.072,0.125)	(0.589,0.342)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)
U_{V5A}	(0.560,0.333)	(0.0,0.0)	(0.081,0.175)	(0.001,0.005)	(0.0,0.0)	(0.498,0.340)	(0.472,0.329)	(0.329,0.311)	(0.007,0.020)	(0.0,0.0)	(0.0,0.0)
U_{V6A}	(0.426,0.333)	(0.0,0.0)	(0.018,0.060)	(0.001,0.006)	(0.0,0.0)	(0.415,0.319)	(0.589,0.348)	(0.263,0.299)	(0.015,0.040)	(0.0,0.0)	(0.0,0.0)
U_{V7A}	(0.484,0.318)	(0.0,0.0)	(0.057,0.148)	(0.010,0.033)	(0.0,0.0)	(0.410,0.338)	(0.517,0.329)	(0.429,0.355)	(0.044,0.075)	(0.0,0.0)	(0.0,0.0)
U_{V9A}	(0.0,0.001)	(0.0,0.0)	(0.0,0.0)	(0.022,0.038)	(0.001,0.008)	(0.0,0.001)	(0.003,0.009)	(0.012,0.053)	(0.527,0.322)	(0.0,0.0)	(0.0,0.0)
U_{V11A}	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.567,0.340)	(0.0,0.0)
U_{V12A}	(0.0,0.0)	(0.482,0.354)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.0,0.001)	(0.0,0.0)	(0.610,0.363)

Table III: This table shows the average P-Values calculated from a Kolmogorov-Smirnov Test for four universes using the mean and standard deviation from Variation 3 A Model given by S. Stevenson et al. (2015). The four universes consist are a normal distribution (UN & MN) with and without a mass gap, as well as an uniform distribution (UU & MU) with and without a mass gap. The universe population models consist of 10,000 NSBH binaries which are compared to models consisting of 100 observations. The diagonal of the table has the highest P-Value for that particular column and row, which means that our proposed method can distinguish between universes with and without a mass gap.

	MN_{NoGap}	$MN_{W/Gap}$	MU_{NoGap}	$MU_{W/Gap}$
UN_{NoGap}	(0.521,0.328)	(0.004,0.011)	(0.0,0.0)	(0.0,0.0)
$UN_{W?Gap}$	(0.012,0.032)	(0.561,0.288)	(0.0,0.0)	(0.0,0.0)
UU_{NoGap}	(0.0,0.0)	(0.0,0.0)	(0.578,0.296)	(0.0,0.0)
$UU_{W/Gap}$	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.518,0.351)

Table IV: This table shows the average P-Values calculated from a Anderson-Darling Test for four universes using the mean and standard deviation from Variation 3 A Model given by S. Stevenson et al. (2015). The four universes consist are a normal distribution (UN & MN) with and without a mass gap, as well as an uniform distribution (UU & MU) with and without a mass gap. The universe population models consist of 10,000 NSBH binaries which are compared to models consisting of 100 observations. The diagonal of the table has the highest P-Value for that particular column and row, which means that our proposed method can distinguish between universes with and without a mass gap.

	MN_{NoGap}	$MN_{W/Gap}$	MU_{NoGap}	$MU_{W/Gap}$
UN_{NoGap}	(0.513,0.344)	(0.001,0.002)	(0.0,0.0)	(0.0,0.0)
$UN_{W?Gap}$	(0.0,0.0)	(0.508,0.301)	(0.0,0.0)	(0.0,0.0)
UU_{NoGap}	(0.0,0.0)	(0.0,0.0)	(0.547,0.313)	(0.0,0.0)
$UU_{W/Gap}$	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.490,0.361)

Table V: This table shows the average P-Values calculated from a Kolmogorov-Smirnov Test for four universes using the mean and standard deviation from the Standard A Model given by S. Stevenson et al. (2015). The four universes consist are a normal distribution (UN & MN) with and without a mass gap, as well as an uniform distribution (UU & MU) with and without a mass gap. The universe population models consist of 10,000 NSBH binaries which are compared to models consisting of 100 observations. The diagonal of the table has the highest P-Value for that particular column and row, which means that our proposed method can distinguish between universes with and without a mass gap.

	MN_{NoGap}	$MN_{W/Gap}$	MU_{NoGap}	$MU_{W/Gap}$
UN_{NoGap}	(0.491,0.322)	(0.077,0.126)	(0.0,0.0)	(0.0,0.0)
$UN_{W?Gap}$	(0.123,0.149)	(0.496,0.301)	(0.0,0.0)	(0.0,0.0)
UU_{NoGap}	(0.0,0.0)	(0.0,0.0)	(0.494,0.356)	(0.0,0.0)
$UU_{W/Gap}$	(0.0,0.0)	(0.0,0.0)	(0.001,0.005)	(0.453,0.316)

Table VI: This table shows the average P-Values calculated from a Anderson-Darling Test for four universes using the mean and standard deviation from the Standard A Model given by S. Stevenson et al. (2015). The four universes consist are a normal distribution (UN & MN) with and without a mass gap, as well as an uniform distribution (UU & MU) with and without a mass gap. The universe population models consist of 10,000 NSBH binaries which are compared to models consisting of 100 observations. The diagonal of the table has the highest P-Value for that particular column and row, which means that our proposed method can distinguish between universes with and without a mass gap.

	MN_{NoGap}	$MN_{W/Gap}$	MU_{NoGap}	$MU_{W/Gap}$
UN_{NoGap}	(0.467,0.320)	(0.062,0.108)	(0.0,0.0)	(0.0,0.0)
$UN_{W/Gap}$	(0.096,0.128)	(0.475,0.306)	(0.0,0.0)	(0.0,0.0)
UU_{NoGap}	(0.0,0.0)	(0.0,0.0)	(0.481,0.355)	(0.0,0.0)
$UU_{W/Gap}$	(0.0,0.0)	(0.0,0.0)	(0.0,0.0)	(0.430,0.322)