

# Coherence of Power Lines

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## Abstract

One of the major obstacles in detecting a stochastic gravitational wave background, is the manifestation of correlated noise between detectors. A possible significant source of this correlated noise is the electric field emanating from 60 Hz power sources. In this report, a method of characterizing the coherence of these sources is presented. This method is then applied to 60 Hz power line data from the E7 run. The analysis produces results pertaining to the effects of integration time on the coherence between the LIGO sites.

## 1. Introduction

One of the major anticipated sources of a gravitational wave signal is the stochastic gravity wave background. The method of detection will be primarily based on calculating correlation between the different sites. A major opponent of this form of detection is correlated noise. Currently, the cross correlation of the noise between sites is expected to be proportional to the square root of the integration time. This model would allow a stochastic gravity wave background, which has a cross correlation proportional to the integration time, to be detected with a long enough integration time. A possible source of correlated noise is the electric field emanating from the power lines. The focus of this report will be on a characterization of this noise, which will include coherence testing. The coherence test will help determine if using longer stretches of data will help the stochastic background outweigh the power lines contribution to the cross correlation.

In order to ascertain the values mentioned previously, it was necessary to extract the properties of the 60 Hz noise from the data. This task was completed using the QMLR Line Monitor. The parameters, amplitude ( $a(t)$ ) and phase ( $\Psi(t)$ ), of the LIGO power monitor (PM) signals were measured. The 60 Hz signal, not noise for now, was represented as  $a(t)\cos(\Psi(t))$ , where  $\Psi(t)$  is defined as  $2\pi ft + \phi(t)$ . The measured values of these parameters will be addressed in Section 2. Upon a thorough description of these attributes, Section 3 will cover the power coherence between Livingston and Hanford power monitors. The channels that were used in this analysis were L0:PEM-LVEA\_V1 and H0:PEM-LVEA2\_V1 with sampling rates of 2048 Hz. Approximately 340 hours of Livingston data and 410 hours of Hanford data were analyzed using the offline version of the LineMonitor. Another goal was to analyze the cross correlation of the 60 Hz signals in the AS\_Q channels from the two km interferometer in Hanford, and the four km interferometer in Livingston. Results from this analysis will be presented also in Section 3. The interferometers only produced useful data in the AS\_Q channels when they were in lock, and for this report data was only taken if the interferometer was in lock for more than 4 seconds<sup>1</sup>. This was the only condition used to determine the viability of the data, and resulted in approximately 270 hours of data for Livingston and 178 hours of data for Hanford. The analyzed data was stored in sixteen-second frame files on servers local to each site. The starting GPS time for Livingston was 693588640, and for Hanford was 693591696. The majority of the data in the report will be from the E7 run, although there will be some comparisons of ten hours of data taken from the E4 run. The data analyzed was not continuous, which did not create a problem due to the nature of the analysis. The basic parameters of the LineMonitor had the seed frequency set to 60 Hz, processed five succeeding harmonics, and analyzed the line spectra in four second intervals.

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<sup>1</sup> The lock condition of the interferometers was based on a reading of 59 or 61 from the LSC-LA\_State\_Bits\_Read channels from respective sites.

## 2. Parameters of the 60 Hz Signals

The parameters of the 60 Hz signals were measured using data segments of four seconds long. For each data segment  $d_i$ , the average amplitude  $a_i$  and the phase  $\Psi_i$  were measured. The power frequency was estimated as a derivative of the phase  $\Psi(t)$ .

### 2.1 Amplitude

Figures 1 and 2 show the amplitude of the power monitor signals at each site. An important aspect of these graphs is the stability of the power amplitude. Both data sets do not deviate more than 1% from the mean. Another interesting aspect of the amplitude data is the obvious oscillations in the Livingston data. They have a period of approximately 12 hours. The minimums of these oscillations correspond to times between 5:30-11:30 AM and PM. These are the times when the most current is being drawn from the power grid resulting in a reduced output. Figures 3 and 4 show the Lomb periodogram<sup>2</sup> of the amplitude for both power monitors.

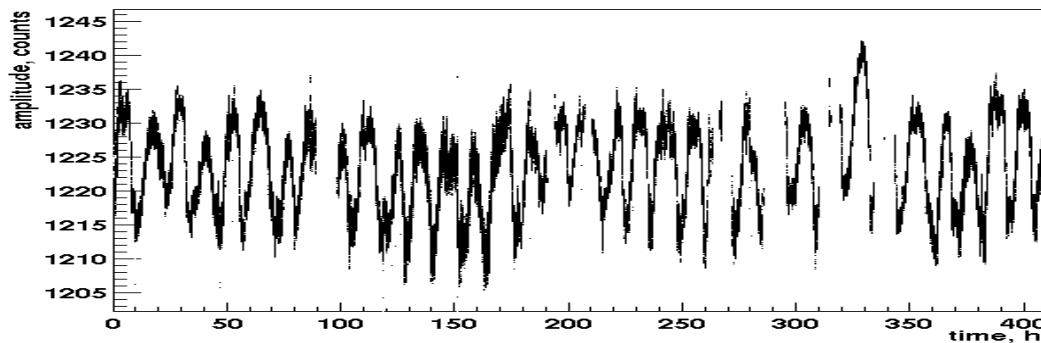


Figure 1. Amplitude: L0:PEM-LVEA\_V1

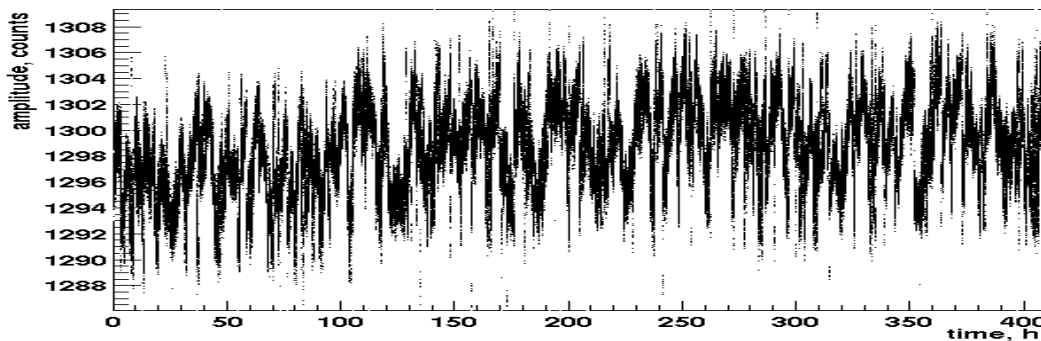


Figure 2. Amplitude: H0:PEM-LVEA2\_V1

<sup>2</sup> The methods of calculation for the Lomb periodogram are discussed in depth in Numerical Recipes(ch13.8).

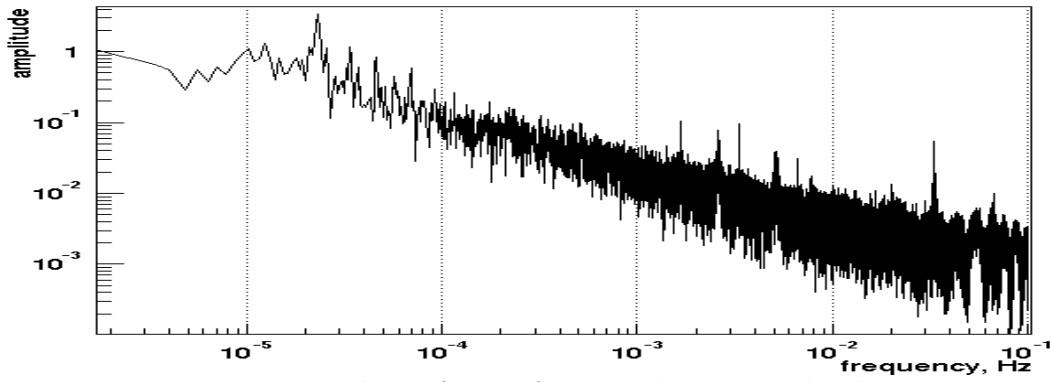


Figure 3. Lomb Periodogram: L0:PEM-LVEA\_V1

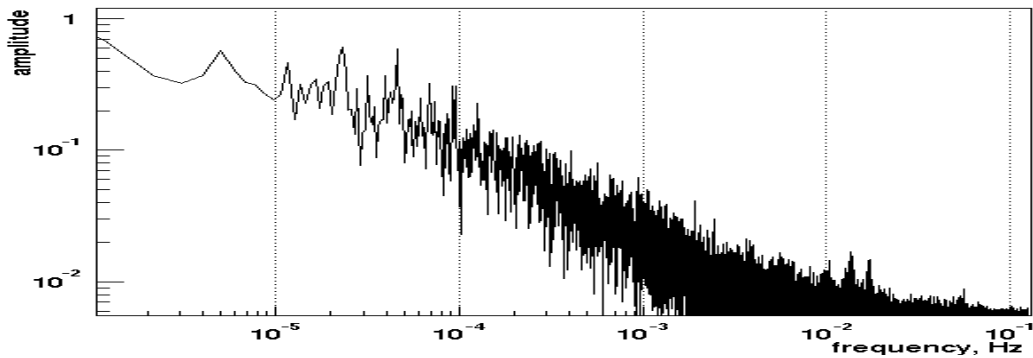


Figure 4. Lomb Periodogram: H0:PEM-LVEA2\_V1

The 60 Hz signals that coupled into the dark ports of the interferometers had amplitude as follows in figs. 5 and 6.

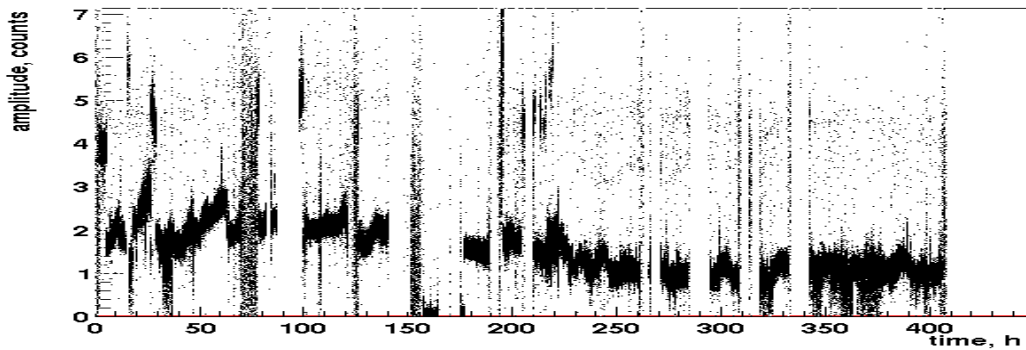


Figure 5. Amplitude: L1:LSC-AS\_Q

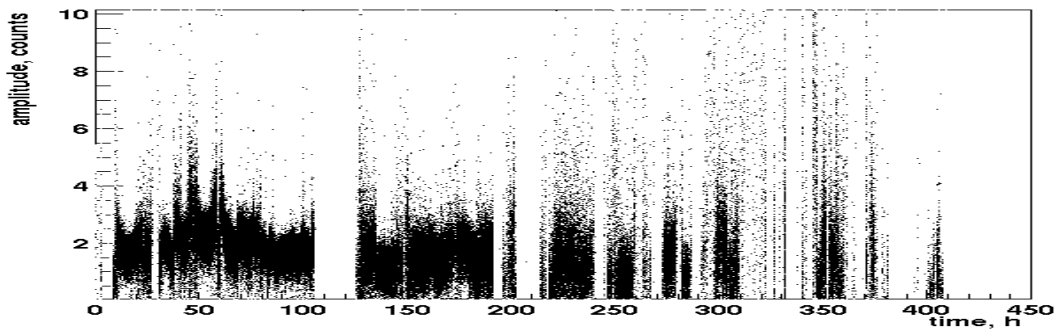


Figure 6. Amplitude: H2:LSC-AS\_Q

## 2.2 Phase

Figure 7 graphs the phase as a function of time for the power monitor in Livingston.

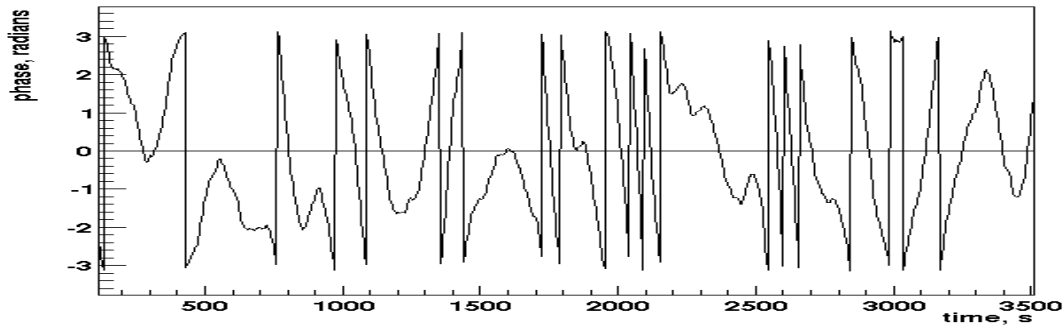


Figure 7. Livingston  $\phi(t)$  Relative to GPS Start Time 693588640

Another method of phase characterization was the reconstruction of the phase as a linear function of time. The idea behind the construction was that given the average frequency for an interval, the beginning phase, and the time, it would be possible to predict the linear evolution of the phase using the formula below.

$$\Psi_n = \omega_{n-1} \Delta t + \Psi_{n-1}$$

The reconstructed phase will not be strictly linear, and in order to further analyze the phase this deviation from a straight line was calculated. This was simply done by fitting the data with a line, and subtracting the mean. The linear phase from Livingston had a slope of approximately 376.988 radians, while the linear phase's slope from Hanford was 376.991 radians. The mean of the Livingston phase was approximately -282.9 radians, while the mean from Hanford was -150.1 radians. Figures 8 and 9 show these results. An expected result in the phase variation was a harmonic component with a period of one day. This would be a result of the power companies maintaining the grid frequency and keeping the average number of 60 Hz cycles constant per diem. Figures 10 and 11 show the phase spectrum for both sites.

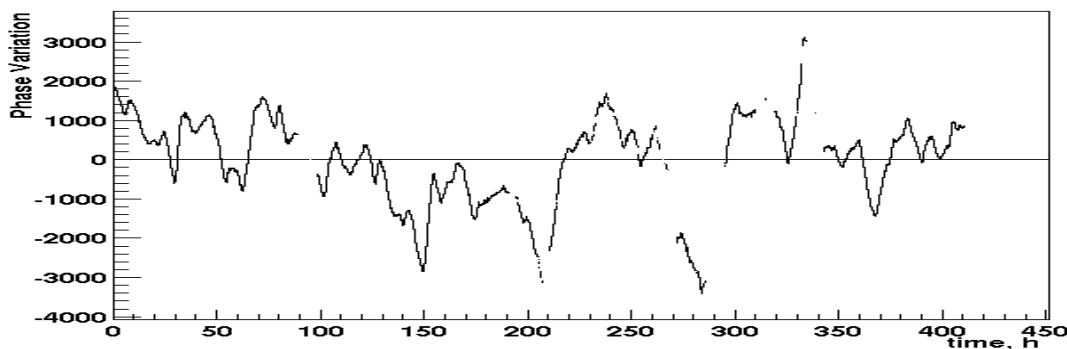


Figure 8. Phase Variation : L0:PEM-LVEA\_V1

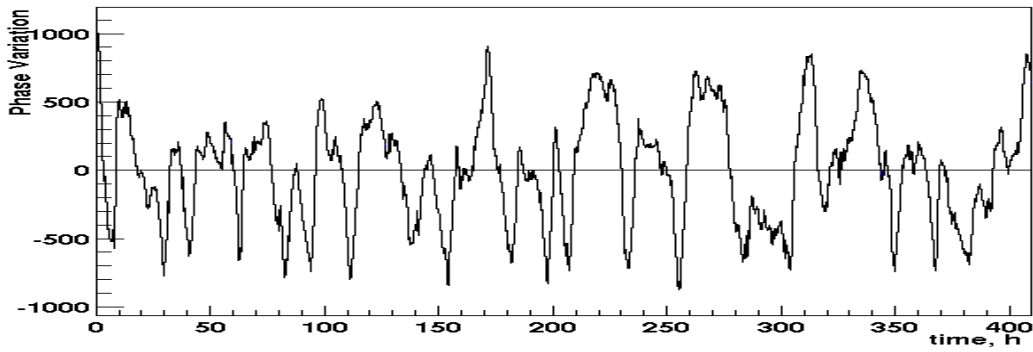


Figure 9. Phase Variation: H0:PEM-LVEA2\_V1

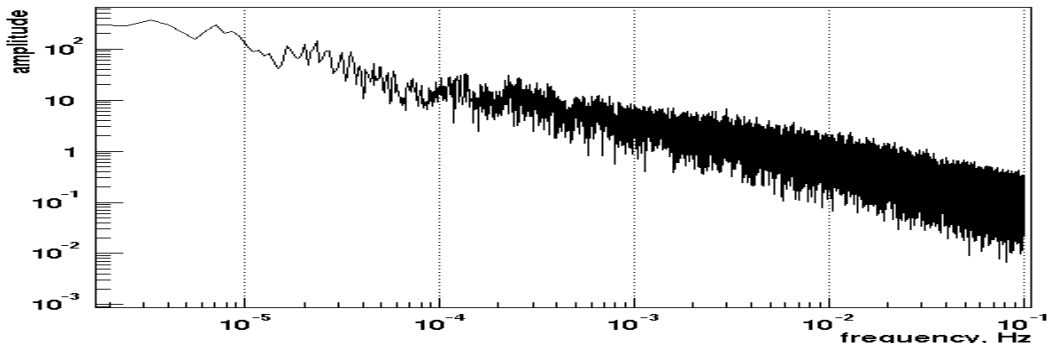


Figure 10. Phase Variation Lomb Periodogram: L0:PEM-LVEA\_V1

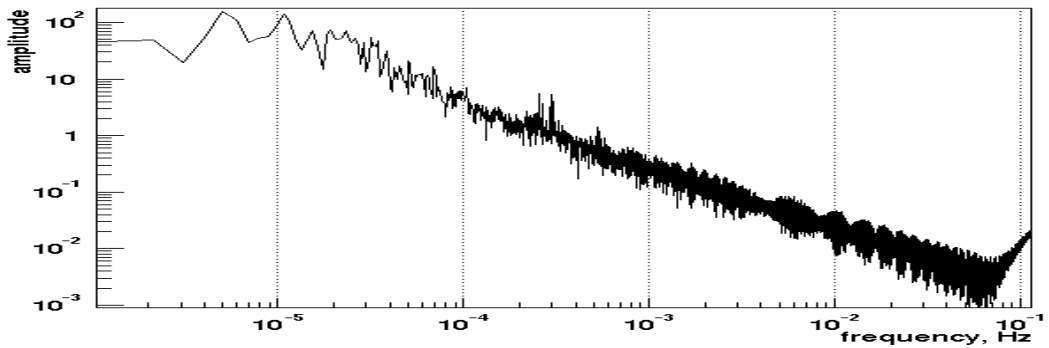


Figure 11. Phase Variation Lomb Periodogram: H0:PEM-LVEA2\_V1

### 2.3 Frequency

During a four second time interval the power frequency doesn't change much. The Line Monitor measures average frequency, which is used as an estimate of instantaneous power frequency. Figures 12 and 13 are histograms of the measured line frequency. It is noticed that both sites have slightly asymmetric distributions with more samples tending towards a lower frequency. The coherence time can be roughly estimated as  $\frac{1}{2.355\sigma}$ , which is the inverse of the width of the distribution at half the maximum number of entries. This results in  $\tau = 32.14$  seconds for Livingston and  $\tau = 37.38$  seconds for Hanford.

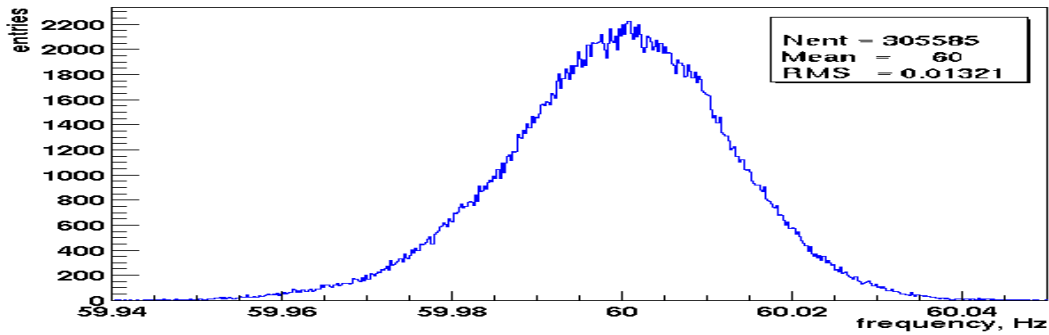


Figure 12. Frequency Distribution: L0:PEM-LVEA\_V1

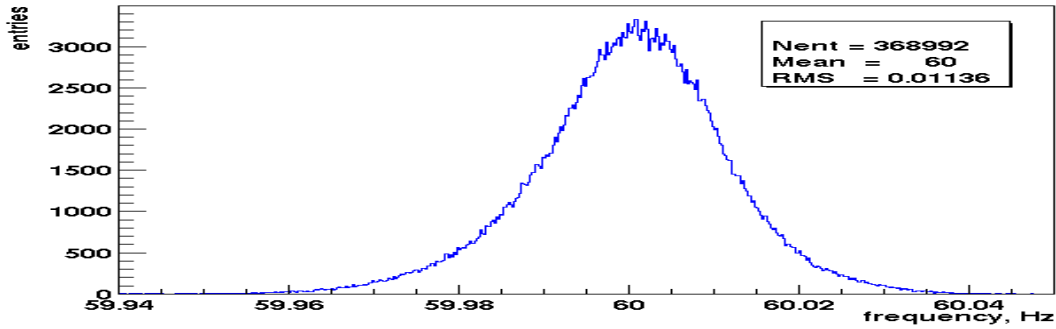


Figure 13. Frequency Distribution: H0:PEM-LVEA2\_V1

### 3. Coherence of 60 Hz Lines

A sum of two harmonic oscillations  $s_L(t)$  and  $s_H(t)$  with the same frequency is also a harmonic oscillation

$$s(t) = s_L(t) + s_H(t) = A \cdot \sin(\omega t + \theta),$$

where the amplitude  $A$  is given by the following equation

$$A^2 = a_L^2 + a_H^2 + 2a_L a_H \cos(\phi_L - \phi_H),$$

and the average (over the time interval  $T$ ) square amplitude is<sup>3</sup>

$$\overline{A^2} = a_L^2 + a_H^2 + 2a_L a_H \frac{1}{T} \int \cos(\phi_L - \phi_H) dt = a_L^2 + a_H^2 + 2a_L a_H \overline{\cos(\Delta\phi)}.$$

If the phase difference remains constant during time  $T$ , the signals  $s_L(t)$  and  $s_H(t)$  are coherent. In case, if the phase difference  $\Delta\phi$  changes randomly in time and the observation time  $T_{\text{tot}}$  is long enough, the  $s_L(t)$  and  $s_H(t)$  are not coherent and the interference term in the last equation is zero.

For coherence study approximately 410 hours of the E7 run data starting at the GPS time 693588640 was analyzed. Figure 14 shows the distribution of  $\Delta\phi$  ( $T=4\text{sec}$ ) for the L0:PEM-LVEA\_V1 and H0:PEM-LVEA2\_V1 channels at corresponding times. In comparison to earlier trials, the longer runs produce a  $\Delta\phi$  distribution significantly more uniform.

<sup>3</sup> The amplitudes  $a_H$  and  $a_L$  are assumed to be constant.

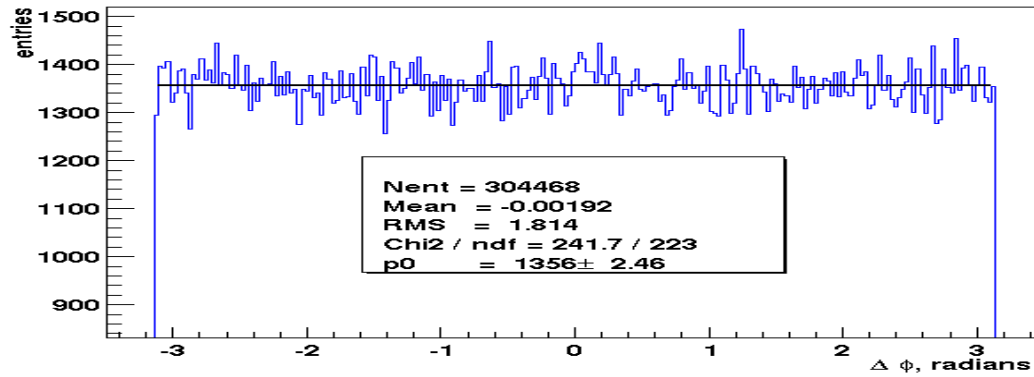


Figure 14.  $\Delta\phi$  Distribution at  $\tau = 0s$

### 3.1 Coherence Coefficient

To characterize the  $\Delta\phi$  uniformity the *coherence coefficient*  $\gamma$  is used

$$\gamma = \frac{1}{N} \left| \sum_{k=1}^N \exp(i\Delta\phi_k) \right|, \quad N = T_{tot}/T.$$

In our case  $\gamma$  equals 0.008, which is consistent with the uniform phase distribution. In a similar way the coherence of signals  $s_L(t)$  and  $s_H(t+\tau)$  can be calculated, where  $\tau$  is a time delay between two signals<sup>4</sup>. The coherence coefficient  $\gamma$  as a function of  $\tau$  is shown on Figure 15. Figure 16 is a histogram of the coherence coefficient, as a function of  $\tau$ , squared. This is an exponential distribution, which will allow the calculation of the significance level. Using the slope calculated from the histogram fit, the significance level, which is a probability to yield measured coherence coefficient assuming the uniform distribution of  $\Delta\phi$  is plotted in figure 17. The significance level was calculated as  $\exp(-60950\gamma^2)$ .

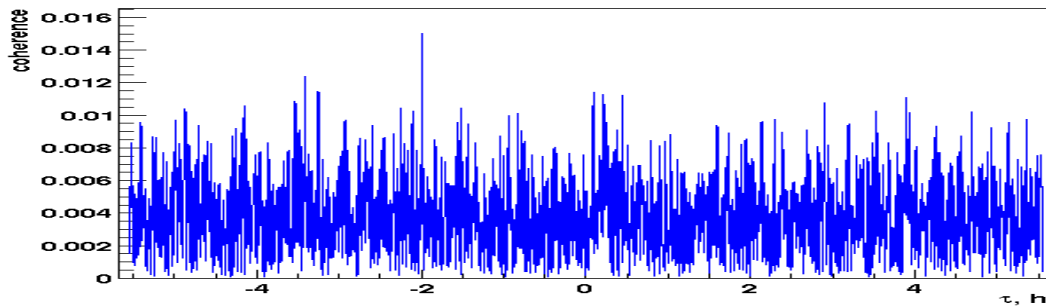


Figure 15. Coherence as a function of  $\tau$

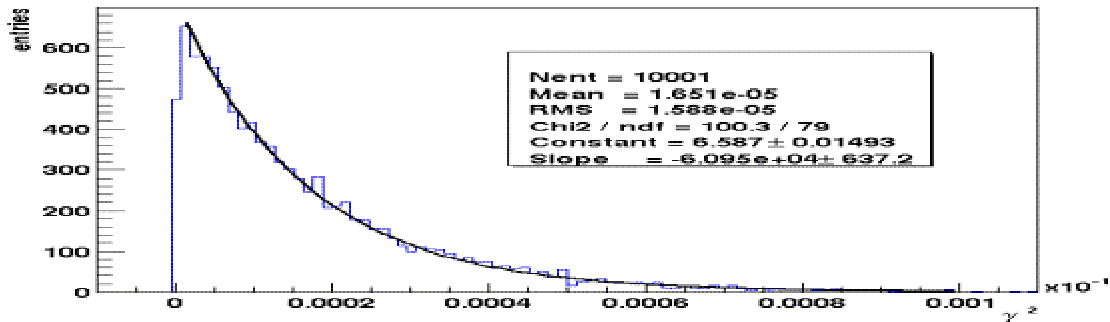


Figure 16.  $\gamma^2$  Distribution

<sup>4</sup> In this paper, the time delay shifts Livingston data.

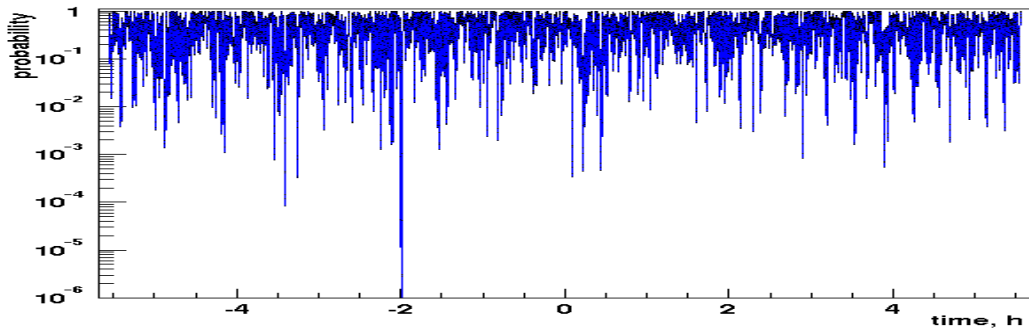


Figure 17. Significance level as a function of  $\tau$

Note that when  $\tau$  equals -1.994h the coherence coefficient,  $\gamma$ , is around 0.015 and the significance level is much less than 0.01%. It is an indication of the presence of a small degree of coherence between the LLO and LHO power mains. This result could possibly be attributed to the two hour difference in local times between sites. Figure 18 shows the phase distribution at this value of  $\tau$ .

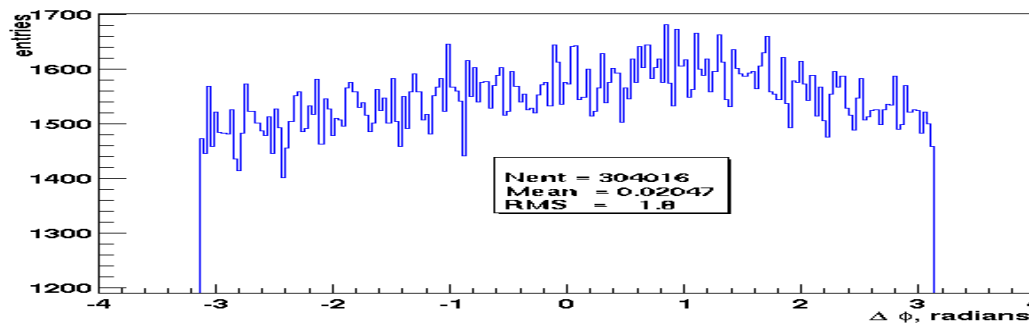


Figure 18.  $\Delta\phi$  Distribution at  $\tau = -1.99h$

### 3.2 Integration Time and Coherence

Previously mentioned, was the expected relation between the coherence coefficient and the integration time, the total amount of time each coefficient was calculated from. As anticipated,  $\gamma \propto \frac{1}{\sqrt{N}}$ . Figure 19 shows the observed data, which possessed minor correlation, and the corresponding fit. Figure 20 introduces  $\chi(t)$  as a function of  $\gamma^{-2}(t)$  and plots its rate of change with respect to integration time.

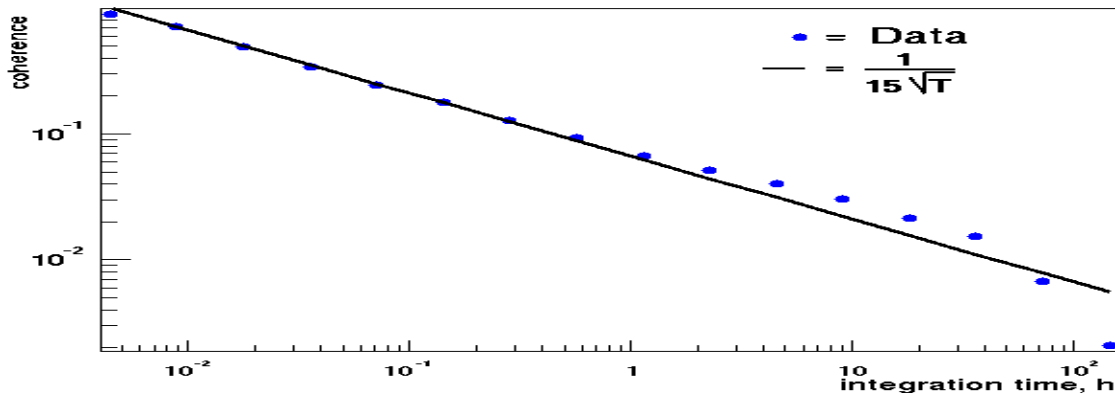


Figure 19. Coherence vs. Integration Time for Experimental Data

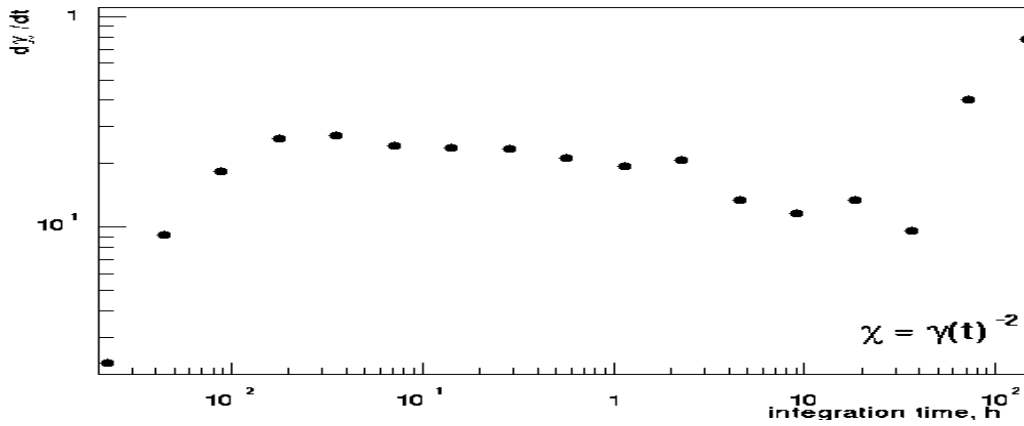


Figure 20. Rate of Change of  $\chi(t)$  vs. Integration Time

### 4. Interpretation

One possible explanation of the observed coherence between the LLO and LHO power monitors is the following. In ideal case the phase of each monitor is a linear function of time:  $\phi = \omega_0 t + const$ . Then the monitors are perfectly coherent. In real life there is an additional (hopefully random) phase  $\varphi(t)$ , so  $\phi = \omega_0 t + \varphi(t) + const$ . Lets assume the phase  $\varphi$  has a harmonic term  $r \cos(\nu t + \theta)$  and the frequency  $\nu$  is the same for both sites

$$\varphi(t) = r \cos(\nu t + \theta) + \eta(t),$$

where  $\eta$  is a random phase. Then the phase difference between sites would be

$$\Delta\phi = r_L \cos(\nu t + \theta_L) - r_H \cos(\nu t + \theta_H) + \eta_L(t) - \eta_H(t).$$

If the  $\eta$  term is small, the coherence is determined by the phase difference  $\theta_L - \theta_H$ . In this case we would expect to see equidistant peaks in the  $\gamma(\tau)$  curve. This picture would be more complicated if there are several modulation frequencies  $\nu$ . This model agrees with what we see in the Fourier spectra of the phase difference (Figure 22), which shows that  $\Delta\phi, t$  equals 0s, has a modulation at frequencies around 0.5, 1., 2. mHz for the E4 run. Figures 20 and 21 show that if there is modulation at these frequencies it is buried in the noise.

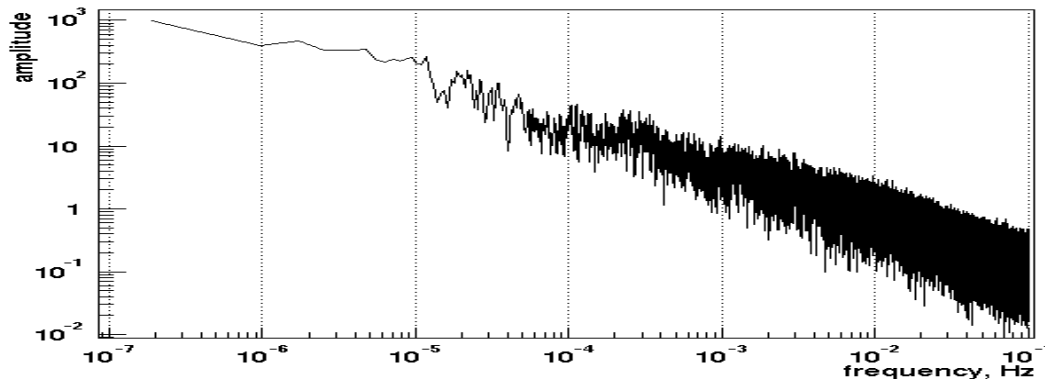


Figure 20.  $\Delta\Psi$  Lomb Periodogram  $\tau=0s$

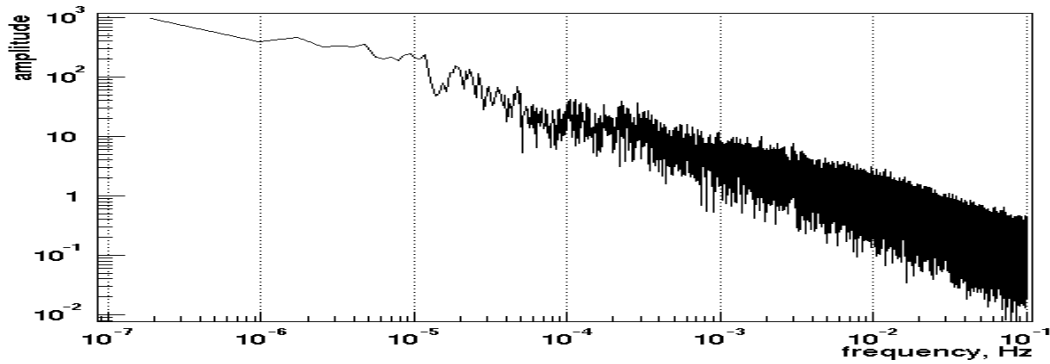


Figure 21.  $\Delta\Psi$  Lomb Periodogram  $\tau=-1.99h$

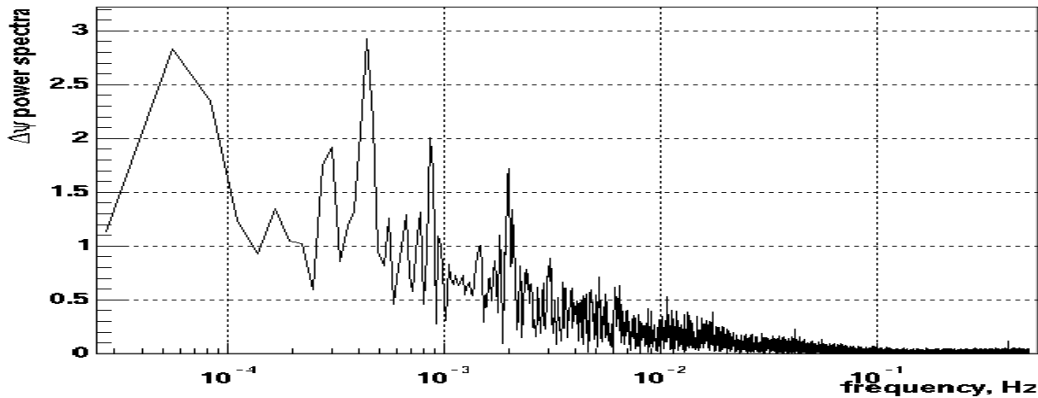


Figure 22.  $\Delta\Psi$  Fourier Spectra from E4 Run(10 hours)

## 5 Conclusion

The effect of integration time on coherence is a positive result. The analysis performed on E7 data clarifies the relation between integration time and coherence. This relation leads to an optimistic outlook on the future of line removal. The implication would be that for any existing harmonic components that contribute to the cross correlation in this manner, will be averaged out over lengthy periods of time. Ideally, the ASQ channels will also need further coherence testing when there is a larger amount of lock data available, ie. the science run, which will provide further substance for this report.