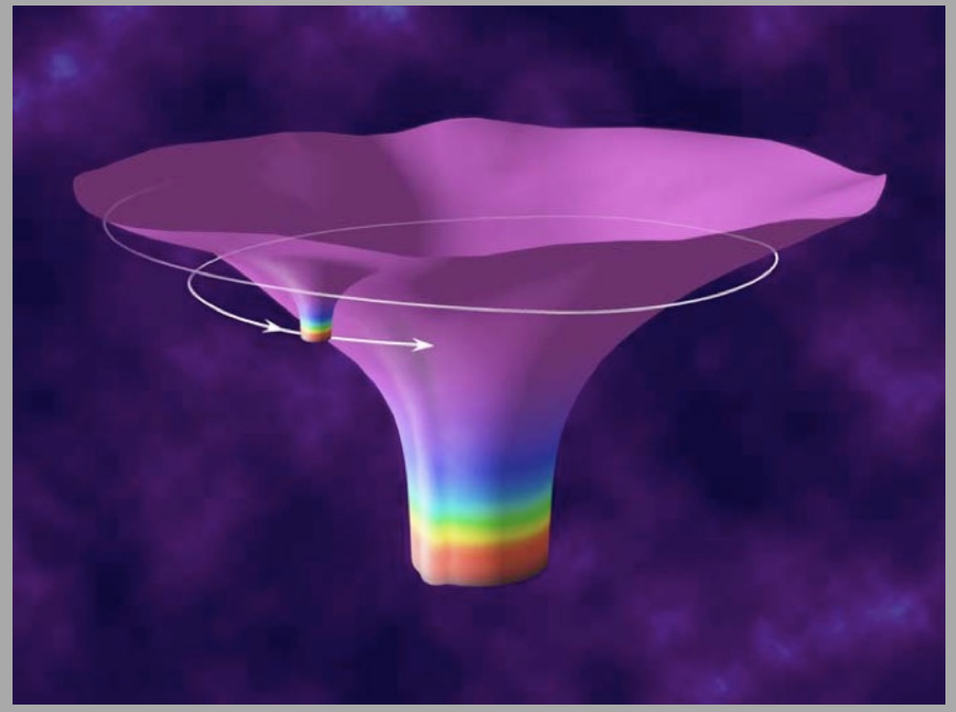
Overcoming the Gauge Problem for the Gravitational Self-Force

Carlos F. Sopuerta

Institute of Space Sciences (CSIC-IEEC)



Work in Collaboration with Priscilla Cañizares (IoA, Cambridge)

INDEX

- Introduction to EMRI modelling
- EMRIs and the Self-Force
- Self-Force in the Regge-Wheeler gauge
- The Particle without Particle Formulation
- Present and Future prospects

Introduction to EMRI modelling

- Extreme-Mass-Ratio inspirals consist of a Stellar-mass Compact Object (SCO)

$$m \sim 1 - 30\, M_\odot$$

orbiting a Massive Black Hole (MBH) at a galactic center

$$M_\bullet \sim 10^5 - 10^7\, M_\odot$$

in the regime where the dynamics is driven by GW emission.

Introduction to EMRI modelling

- Extreme-Mass-Ratio inspirals consist of a Stellar-mass Compact Object (SCO)

$$m \sim 1 - 30\, M_\odot$$

orbiting a Massive Black Hole (MBH) at a galactic center

$$M_\bullet \sim 10^5 - 10^7\, M_\odot$$

in the regime where the dynamics is driven by GW emission.

- They are one of the main sources of GWs for eLISA. The mass ratios of interest are in the range:

$$\mu = \frac{m}{M_\bullet} \sim 10^{-3} - 10^{-7}$$

Introduction to EMRI modelling

- EMRI observations can potentially lead to new discoveries in Astrophysics (Dynamics around galactic nuclei, distribution of masses and spins of MBHs, etc.), in Cosmology (Constraints on galaxy formation models, measurements of cosmological parameters, etc.), and in Fundamental Physics (tests of the no-hair theorem and theories of gravity).

Introduction to EMRI modelling

- EMRI observations can potentially lead to new discoveries in Astrophysics (Dynamics around galactic nuclei, distribution of masses and spins of MBHs, etc.), in Cosmology (Constraints on galaxy formation models, measurements of cosmological parameters, etc.), and in Fundamental Physics (tests of the no-hair theorem and theories of gravity).

- Given that EMRI signals will be buried in the detector data stream it is very important to have very precise gravitational waveform templates.

Introduction to EMRI modelling

- EMRI observations can potentially lead to new discoveries in Astrophysics (Dynamics around galactic nuclei, distribution of masses and spins of MBHs, etc.), in Cosmology (Constraints on galaxy formation models, measurements of cosmological parameters, etc.), and in Fundamental Physics (tests of the no-hair theorem and theories of gravity).

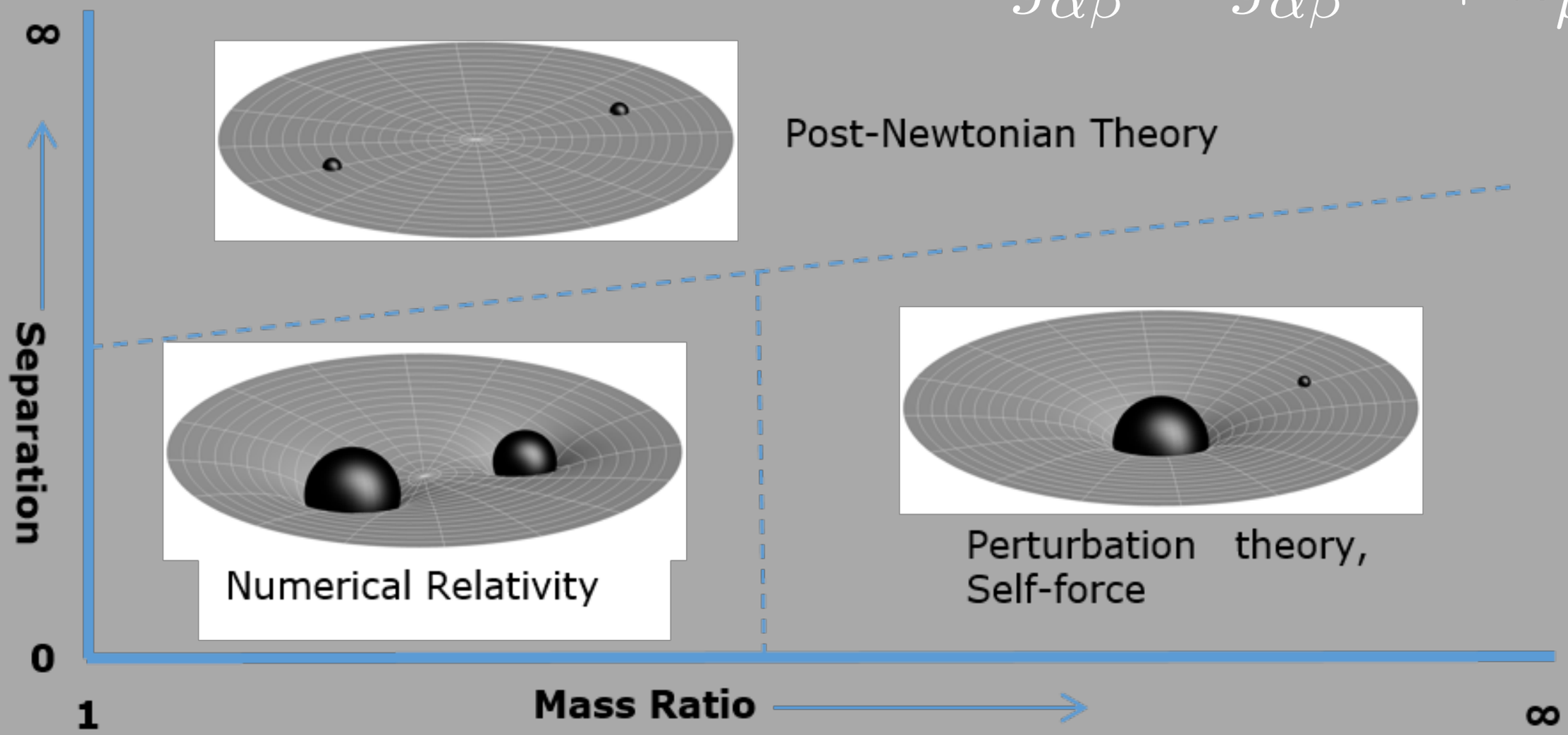
- Given that EMRI signals will be buried in the detector data stream it is very important to have very precise gravitational waveform templates.

- For a space-based detector like eLISA, signals corresponding to the last year before plunge may contain more than 10^5 cycles.

EMRIs and the Self-Force

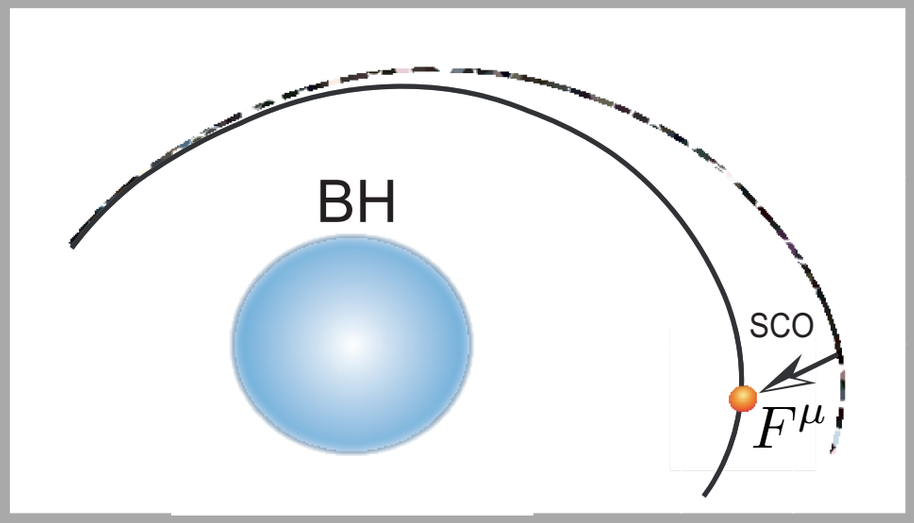
- Due to the extreme mass ratios we can describe the system accurately using BH Perturbation Theory: The spacetime is the MBH spacetime (Kerr metric) with perturbations induced by the SCO.

$$g_{\alpha\beta} = g_{\alpha\beta}^{\mathrm{MBH}} + h_{\mu\nu}$$



EMRIs and the Self-Force

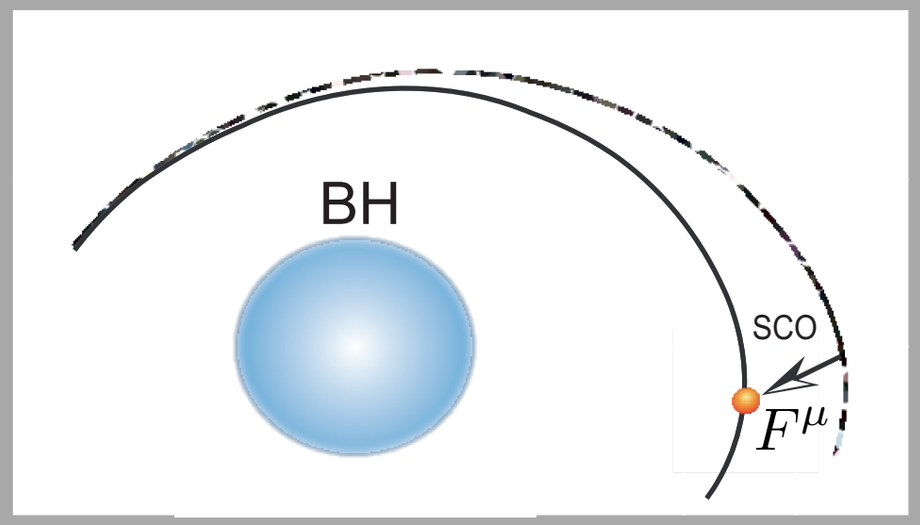
- When we treat the SCO as a point-like object the deviations from geodesic motion can be described by the action of a local force, the self-force. The equation of motion for the SCO is the so-called the MiSaTaQuWa equation [Mino, Sasaki & Tanaka (1997); Quinn & Wald (1997)]:



$$F^\alpha = -\frac{m}{2}\left(g^{\alpha\beta} + \frac{dz^\alpha}{d\tau}\frac{dz^\beta}{d\tau}\right)\left(2\nabla_\rho h_{\beta\sigma} - \nabla_\beta h_{\rho\sigma}\right)\Big|_{z(\tau)}\frac{dz^\rho}{d\tau}\frac{dz^\sigma}{d\tau}$$

EMRIs and the Self-Force

- When we treat the SCO as a point-like object the deviations from geodesic motion can be described by the action of a local force, the self-force. The equation of motion for the SCO is the so-called the MiSaTaQuWa equation [Mino, Sasaki & Tanaka (1997); Quinn & Wald (1997)]:



Needs Regularization

$$F^\alpha = -\frac{m}{2}\left(g^{\alpha\beta} + \frac{dz^\alpha}{d\tau}\frac{dz^\beta}{d\tau}\right)\left(2\nabla_\rho h_{\beta\sigma} - \nabla_\beta h_{\rho\sigma}\right)\Big|_{z(\tau)}\frac{dz^\rho}{d\tau}\frac{dz^\sigma}{d\tau}$$

EMRIs and the Self-Force

- At present, the (first order) self-force has been computed for all cases in the case of a non-rotating MBH (Barack & collaborators) both in the time and frequency domains.

Barack & Sago, PRD **75** 064021 (2007)

Sago, Barack & Detweiler, PRD **78** 124024 (2008)

Barack & Sago, PRD **81** 084021 (2010)

Akcay, Warburton & Barack, PRD **88** 104009 (2013)

- It has been done in the so-called Lorenz gauge:

$$\psi_{\alpha\beta} = h_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} g^{\rho\sigma} h_{\rho\sigma}$$

$$\psi^{\alpha\beta}{}_{;\beta} = 0$$

EMRIs and the Self-Force

- The regularization of the retarded (full) metric perturbations is done from a multipolar expansion:

$$F^\alpha = \sum_{\ell=0}^{\infty} F^\alpha_\ell \qquad \text{(This diverges at the particle location)}$$

EMRIs and the Self-Force

- The regularization of the retarded (full) metric perturbations is done from a multipolar expansion:

$$F^\alpha = \sum_{\ell=0}^{\infty} F^\alpha_\ell \qquad \text{(This diverges at the particle location)}$$

- To obtain the regularized self-force we need to substract a singular piece that can be obtained analytically:

EMRIs and the Self-Force

- The regularization of the retarded (full) metric perturbations is done from a multipolar expansion:

$$F^\alpha = \sum_{\ell=0}^{\infty} F_\ell^\alpha \qquad \text{(This diverges at the particle location)}$$

- To obtain the regularized self-force we need to substract a singular piece that can be obtained analytically:

$$F_\ell^{\mathrm{self},\alpha} = F_\ell^\alpha - F_\ell^{\mathrm{S},\alpha}$$

EMRIs and the Self-Force

- The regularization of the retarded (full) metric perturbations is done from a multipolar expansion:

$$F^\alpha = \sum_{\ell=0}^{\infty} F^\alpha_\ell \qquad \text{(This diverges at the particle location)}$$

- To obtain the regularized self-force we need to substract a singular piece that can be obtained analytically:

$$F^{\mathrm{self},\alpha}_\ell = F^\alpha_\ell - F^{\mathrm{S},\alpha}_\ell$$

Computed analytically in the Lorenz gauge

EMRIs and the Self-Force

- The regularization of the retarded (full) metric perturbations is done from a multipolar expansion:

$$F^\alpha = \sum_{\ell=0}^{\infty} F^\alpha_\ell \qquad \text{(This diverges at the particle location)}$$

- To obtain the regularized self-force we need to substract a singular piece that can be obtained analytically:

$$F^{\mathrm{self},\alpha}_\ell = F^\alpha_\ell - F^{\mathrm{S},\alpha}_\ell$$

Computed numerically in the Lorenz gauge

Computed analytically in the Lorenz gauge

EMRIs and the Self-Force

- Structure of the Singular "force":

$$F_\alpha^{\mathrm{S},\ell} = A_\alpha \left(\ell + \frac{1}{2}\right) + B_\alpha + \frac{C_\alpha}{\ell + \frac{1}{2}} + \frac{D_\alpha}{\left(\ell - \frac{1}{2}\right)\left(\ell + \frac{3}{2}\right)} + \ldots$$

EMRIs and the Self-Force

- Structure of the Singular "force":

$$F_\alpha^{\mathrm{S},\ell} = A_\alpha\left(\ell+\frac{1}{2}\right) + B_\alpha + \frac{C_\alpha}{\ell+\frac{1}{2}} + \frac{D_\alpha}{\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right)} + \cdots$$

Quadratic Divergences

EMRIs and the Self-Force

- Structure of the Singular "force":

$$F_\alpha^{\mathrm{S},\ell} = A_\alpha\left(\ell+\frac{1}{2}\right) + B_\alpha + \frac{C_\alpha}{\ell+\frac{1}{2}} + \frac{D_\alpha}{\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right)} + \cdots$$

Quadratic Divergences

Linear Divergences

EMRIs and the Self-Force

- Structure of the Singular "force":

$$F_\alpha^{\mathrm{S},\ell} = A_\alpha\left(\ell+\frac{1}{2}\right) + B_\alpha + \frac{C_\alpha}{\ell+\frac{1}{2}} + \frac{D_\alpha}{\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right)} + \cdots$$

$A_\alpha\left(\ell+\frac{1}{2}\right)$ → Quadratic Divergences

B_α → Linear Divergences

$\frac{C_\alpha}{\ell+\frac{1}{2}}$ → Logarithmic Divergences

EMRIs and the Self-Force

- Structure of the Singular "force":

$$F_\alpha^{\mathrm{S},\ell} = A_\alpha\left(\ell+\frac{1}{2}\right) + B_\alpha + \frac{C_\alpha}{\ell+\frac{1}{2}} + \frac{D_\alpha}{\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right)} + \cdots$$

- $A_\alpha\left(\ell+\frac{1}{2}\right)$ → Quadratic Divergences
- B_α → Linear Divergences
- $\frac{C_\alpha}{\ell+\frac{1}{2}}$ → Logarithmic Divergences
- $\frac{D_\alpha}{\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right)} + \cdots$ → Convergent Terms that sum up to zero individually

EMRIs and the Self-Force

- Structure of the Singular "force":

$$F_\alpha^{\mathrm{S},\ell} = A_\alpha\left(\ell+\frac{1}{2}\right) + B_\alpha + \frac{C_\alpha}{\ell+\frac{1}{2}} + \frac{D_\alpha}{\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right)} + \cdots$$

- $A_\alpha\left(\ell+\frac{1}{2}\right)$ → Quadratic Divergences
- B_α → Linear Divergences
- $\frac{C_\alpha}{\ell+\frac{1}{2}}$ → Logarithmic Divergences
- $\frac{D_\alpha}{\left(\ell-\frac{1}{2}\right)\left(\ell+\frac{3}{2}\right)} + \cdots$ → Convergent Terms that sum up to zero individually

- So, all what remains is to compute the different harmonics of the retarded field in an efficient and precise way.

Self-Force in the Regge-Wheeler gauge

- The equations for each harmonic of the metric perturbations in the Lorenz gauge are coupled, in contrast with the Regge-Wheeler gauge, where they decouple: We have master wave-type equations (Regge-Wheeler & Zerilli) and all the metric perturbations can be reconstructed from the solution of the master equations.

$$h^{\ell m}_{\alpha\beta} = \begin{pmatrix} p^{\ell m}_{ab}\, Y^{\ell m} & q^{\ell m}_{a}\, Y^{\ell m}_{A} + h^{\ell m}_{a}\, S^{\ell m}_{A} \\ * & r^2 \left(K^{\ell m}\, Y^{\ell m}_{AB} + G^{\ell m}\, Z^{\ell m}_{AB} \right) + h^{\ell m}_{2}\, S^{\ell m}_{AB} \end{pmatrix}$$

Self-Force in the Regge-Wheeler gauge

- The equations for each harmonic of the metric perturbations in the Lorenz gauge are coupled, in contrast with the Regge-Wheeler gauge, where they decouple: We have master wave-type equations (Regge-Wheeler & Zerilli) and all the metric perturbations can be reconstructed from the solution of the master equations.

$$h^{\ell m}_{\alpha\beta} = \begin{pmatrix} p^{\ell m}_{ab}\, Y^{\ell m} & \cancel{a^{\ell m}_{a}}\, Y^{\ell m}_{A} + h^{\ell m}_{a}\, S^{\ell m}_{A} \\ * & r^2 \left(K^{\ell m}\, Y^{\ell m}_{AB} + \cancel{G^{\ell m}}\, Z^{\ell m}_{AB} \right) + \cancel{h^{\ell m}_{2}}\, S^{\ell m}_{AB} \end{pmatrix}$$

Self-Force in the Regge-Wheeler gauge

- **QUESTION:** Can we compute the self-force in the Regge-Wheeler gauge?

Self-Force in the Regge-Wheeler gauge

- **QUESTION:** Can we compute the self-force in the Regge-Wheeler gauge?

- **CANONICAL ANSWER:** No, because the Regge-Wheeler gauge is a <u>singular</u> gauge.

Self-Force in the Regge-Wheeler gauge

PHYSICAL REVIEW D, VOLUME 64, 124003

Gravitational self-force and gauge transformations

Leor Barack
Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, Am Mühlenberg 1, D-14476 Golm, Germany

Amos Ori
Department of Physics, Technion—Israel Institute of Technology, Haifa 32000, Israel
(Received 16 July 2001; published 31 October 2001)

- Main Conclusions:

$$F_{\rm self}^{\rm Gauge\ X,\alpha} = F_{\rm full}^{\rm Gauge\ X,\alpha} - F_{\rm S}^{\rm Lorenz\ Gauge,\alpha}$$

Self-Force in the Regge-Wheeler gauge

PHYSICAL REVIEW D, VOLUME 64, 124003

Gravitational self-force and gauge transformations

Leor Barack
Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, Am Mühlenberg 1, D-14476 Golm, Germany

Amos Ori
Department of Physics, Technion—Israel Institute of Technology, Haifa 32000, Israel
(Received 16 July 2001; published 31 October 2001)

- Main Conclusions:

$$F_{\rm self}^{\rm Gauge\ X,\alpha} = F_{\rm full}^{\rm Gauge\ X,\alpha} - F_{\rm S}^{\rm Lorenz\ Gauge,\alpha}$$

- This is true provided the self-force admits a definite finite value in the Gauge X.

Self-Force in the Regge-Wheeler gauge

PHYSICAL REVIEW D, VOLUME 64, 124003

Gravitational self-force and gauge transformations

Leor Barack
Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, Am Mühlenberg 1, D-14476 Golm, Germany

Amos Ori
Department of Physics, Technion—Israel Institute of Technology, Haifa 32000, Israel
(Received 16 July 2001; published 31 October 2001)

- Main Conclusions:

$$F_{\rm self}^{\rm Gauge\ X,\alpha} = F_{\rm full}^{\rm Gauge\ X,\alpha} - F_{\rm S}^{\rm Lorenz\ Gauge,\alpha}$$

- This is true provided the self-force admits a definite finite value in the Gauge X.
- Since this happens when X=Lorenz, the condition is that the transformation from the Lorenz gauge to the Gauge X would yield a regular finite value for the self-force difference.

Self-Force in the Regge-Wheeler gauge

- Gauge transformation for metric perturbations:

$$h_{\alpha\beta}^{\text{Gauge X}} = h_{\alpha\beta}^{\text{Lorenz Gauge}} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

Self-Force in the Regge-Wheeler gauge

- Gauge transformation for metric perturbations:

$$h_{\alpha\beta}^{\text{Gauge X}} = h_{\alpha\beta}^{\text{Lorenz Gauge}} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

- A basic analysis shows that:

Self-Force in the Regge-Wheeler gauge

- Gauge transformation for metric perturbations:

$$h_{\alpha\beta}^{\text{Gauge X}} = h_{\alpha\beta}^{\text{Lorenz Gauge}} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

- A basic analysis shows that:

- Only for Radial trajectories in the Regge Wheeler gauge can we obtain a finite value for the self-force.

Self-Force in the Regge-Wheeler gauge

- Gauge transformation for metric perturbations:

$$h_{\alpha\beta}^{\text{Gauge X}} = h_{\alpha\beta}^{\text{Lorenz Gauge}} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

- A basic analysis shows that:
- Only for Radial trajectories in the Regge Wheeler gauge can we obtain a finite value for the self-force.
- We cannot get a finite value for the self-force for any type of trajectory in the Radiation gauge.

Self-Force in the Regge-Wheeler gauge

- Gauge transformation for metric perturbations:

$$h_{\alpha\beta}^{\text{Gauge X}} = h_{\alpha\beta}^{\text{Lorenz Gauge}} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

- A basic analysis shows that:
- Only for Radial trajectories in the Regge Wheeler gauge can we obtain a finite value for the self-force.
- We cannot get a finite value for the self-force for any type of trajectory in the Radiation gauge.
- **Can we get around this?**

Self-Force in the Regge-Wheeler gauge

- Gauge transformation for metric perturbations:

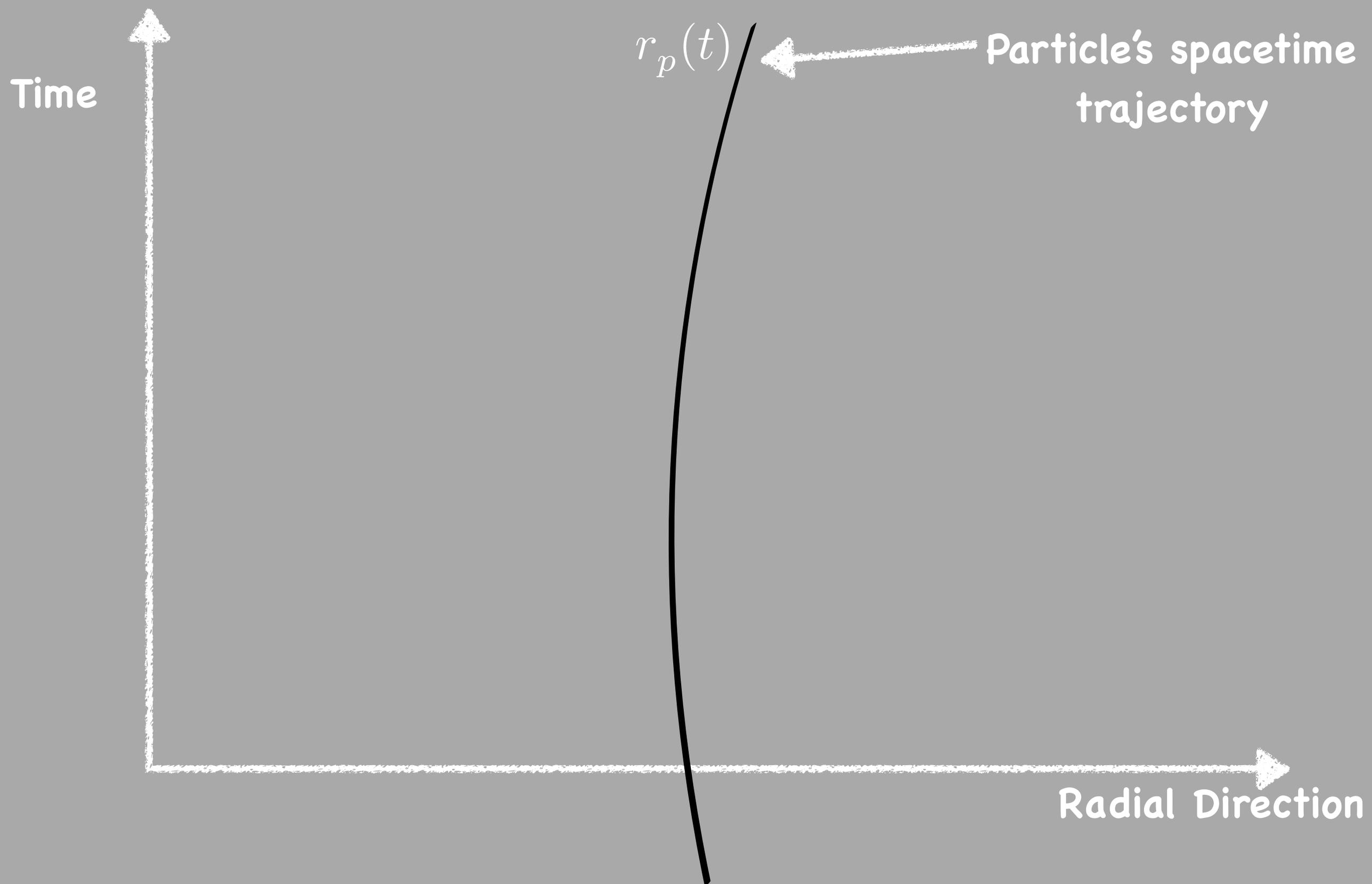
$$h_{\alpha\beta}^{\text{Gauge X}} = h_{\alpha\beta}^{\text{Lorenz Gauge}} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

Key Quantity

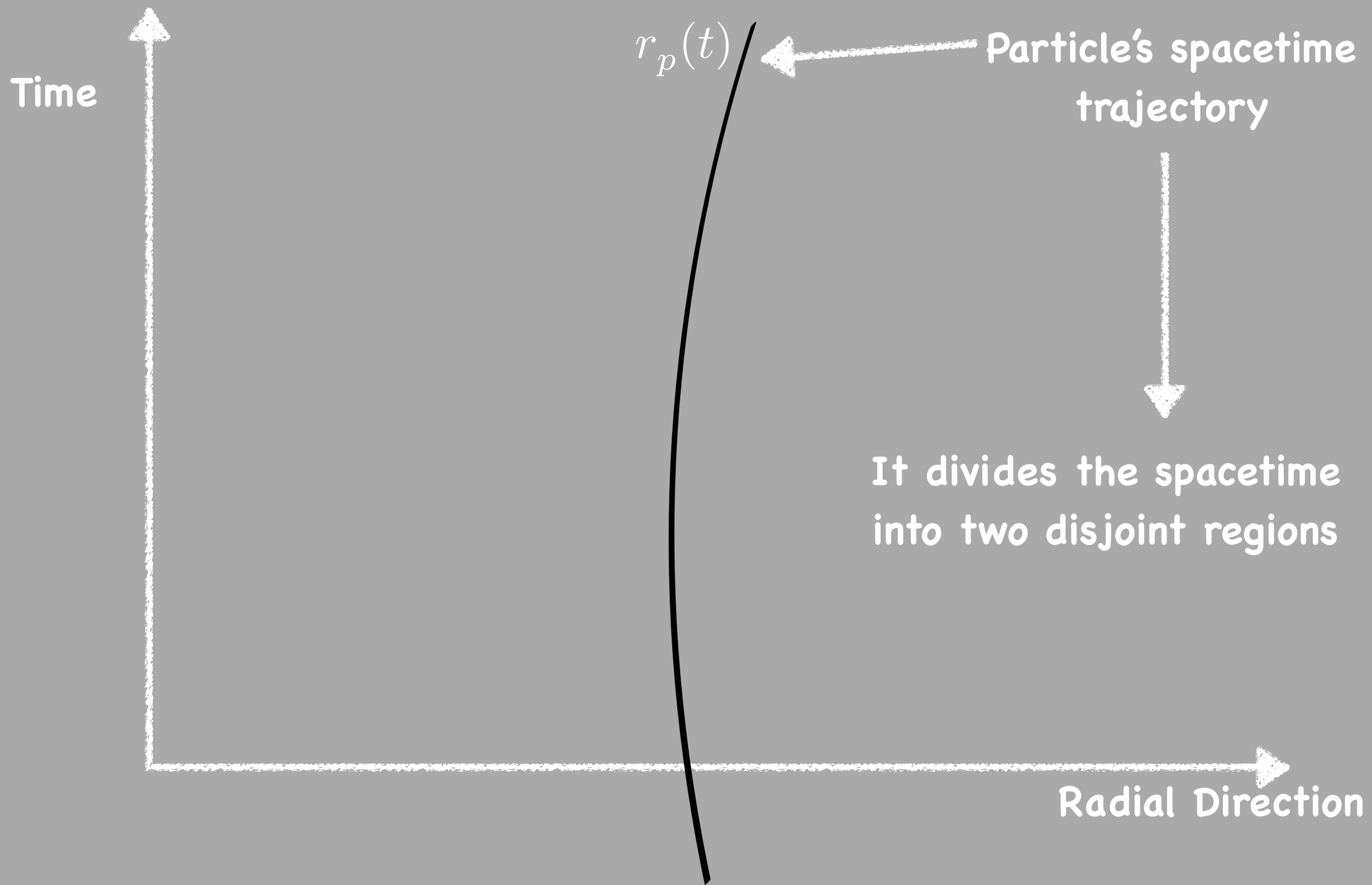
- A basic analysis shows that:
- Only for Radial trajectories in the Regge Wheeler gauge can we obtain a finite value for the self-force.
- We cannot get a finite value for the self-force for any type of trajectory in the Radiation gauge.
- **Can we get around this?**

The Particle without Particle scheme

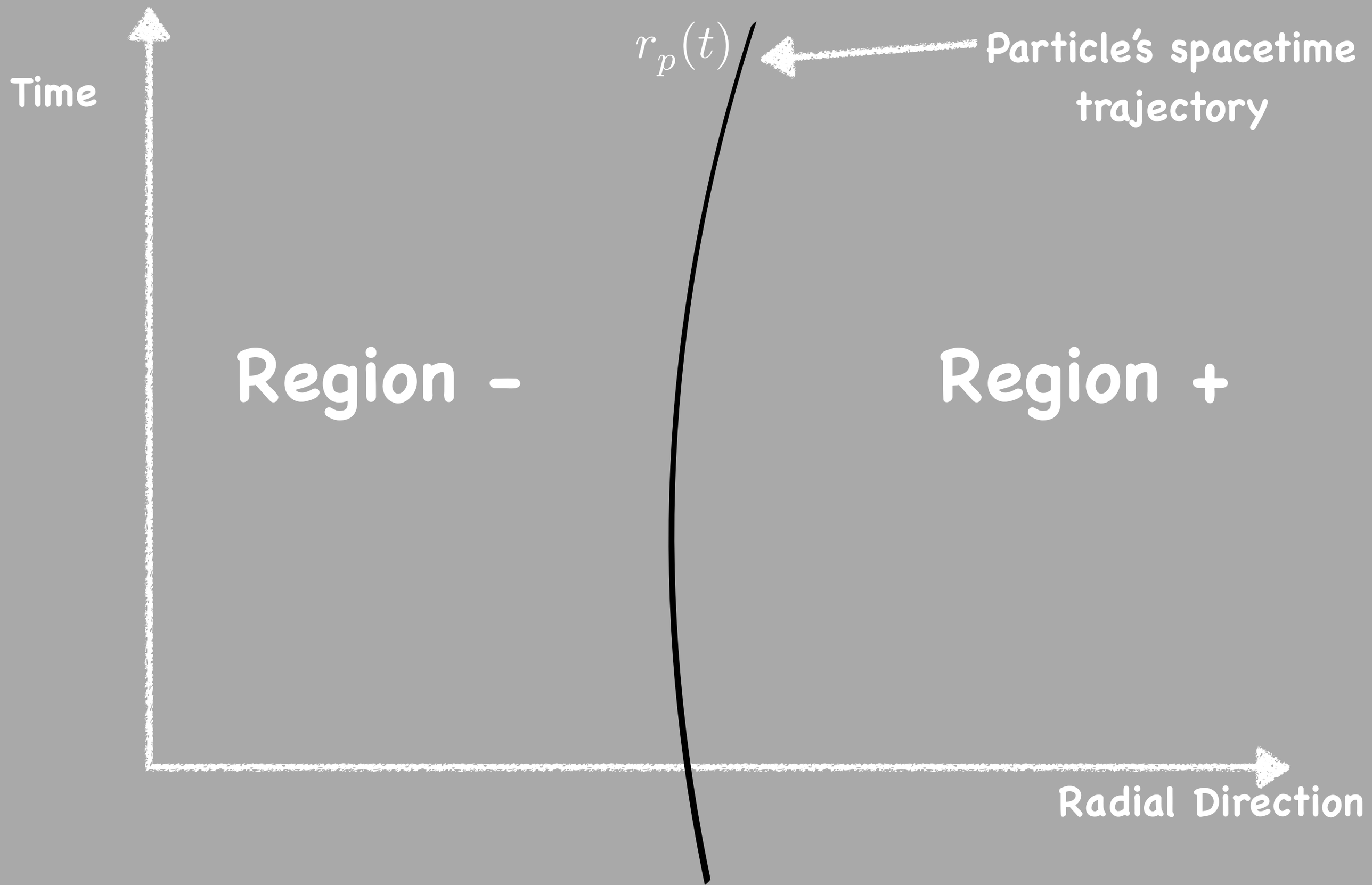
The Particle without Particle scheme



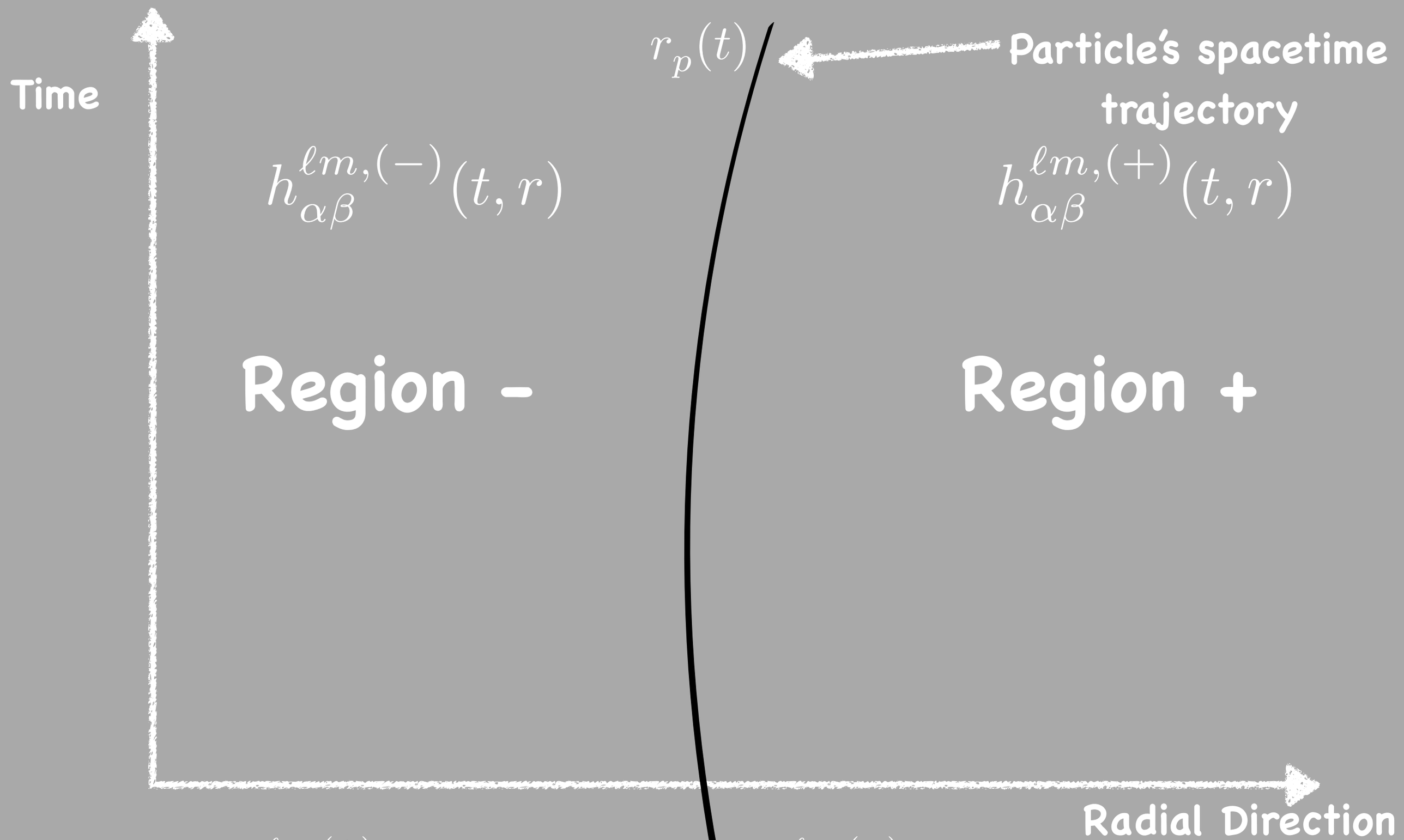
The Particle without Particle scheme



The Particle without Particle scheme

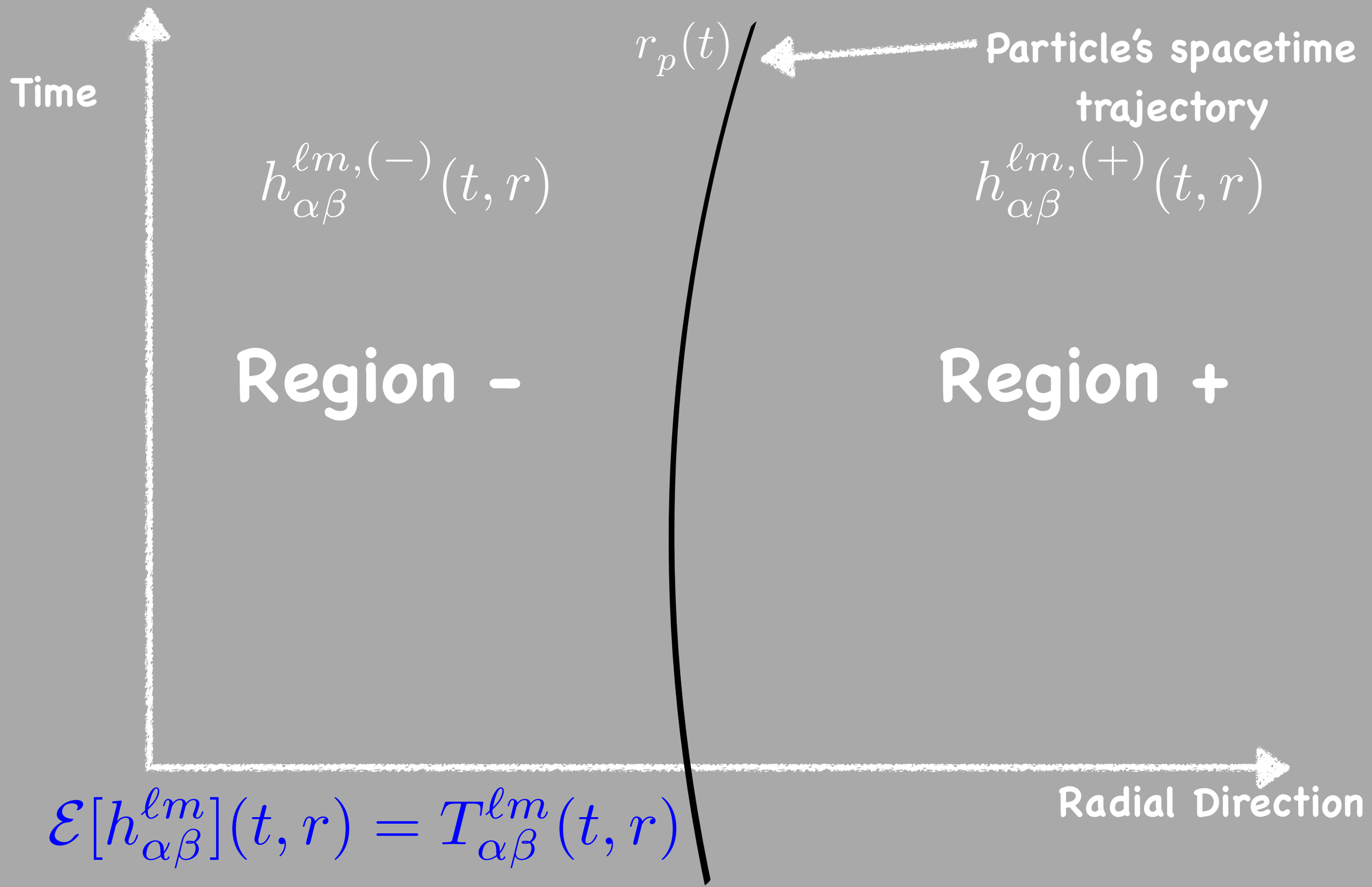


The Particle without Particle scheme



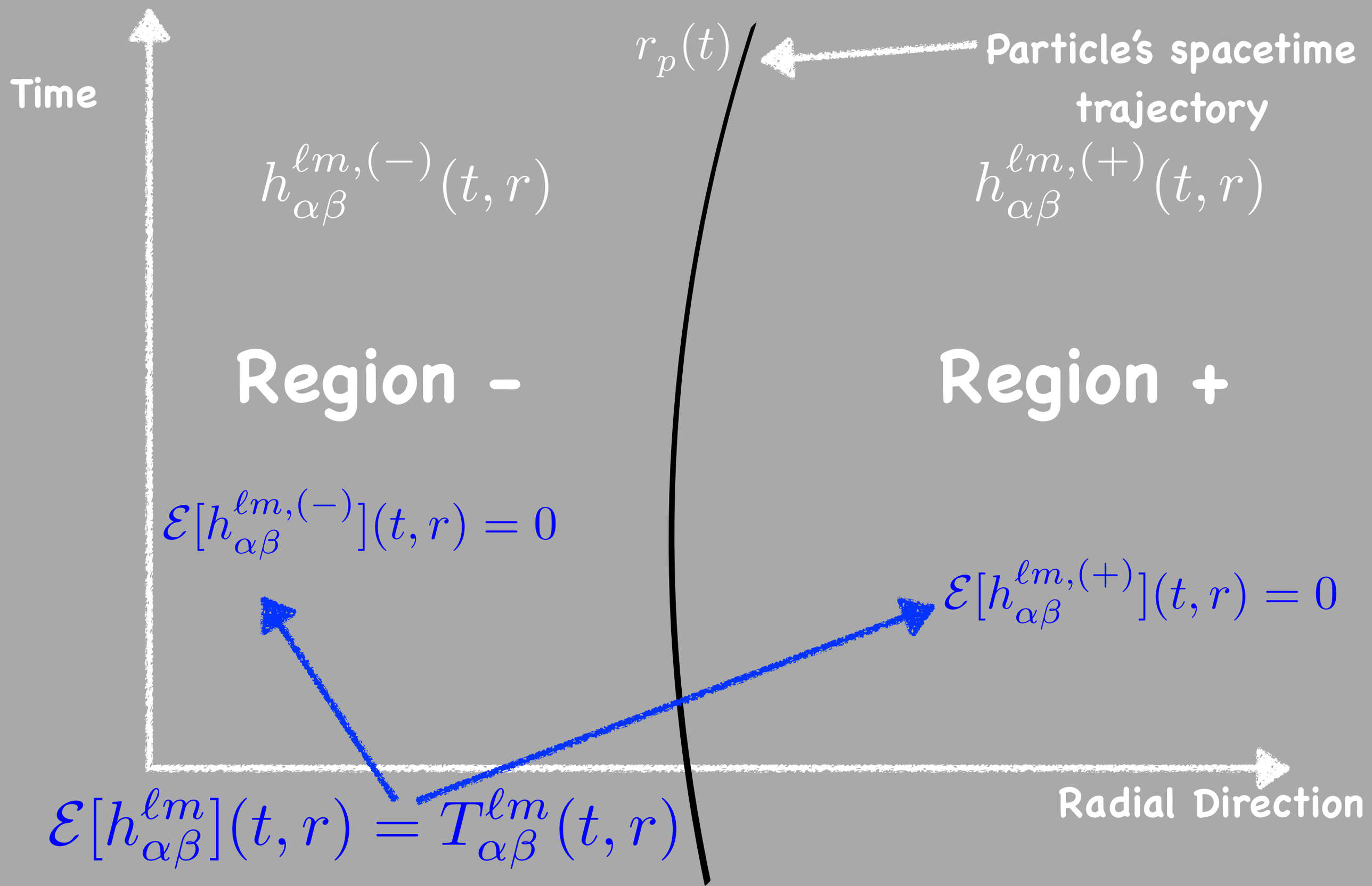
$$h^{\ell m}_{\alpha\beta}(t,r) = h^{\ell m,(+)}_{\alpha\beta}(t,r)\,\Theta\left(r - r_p(t)\right) + h^{\ell m,(-)}_{\alpha\beta}(t,r)\,\Theta\left(r_p(t) - r\right)$$

The Particle without Particle scheme

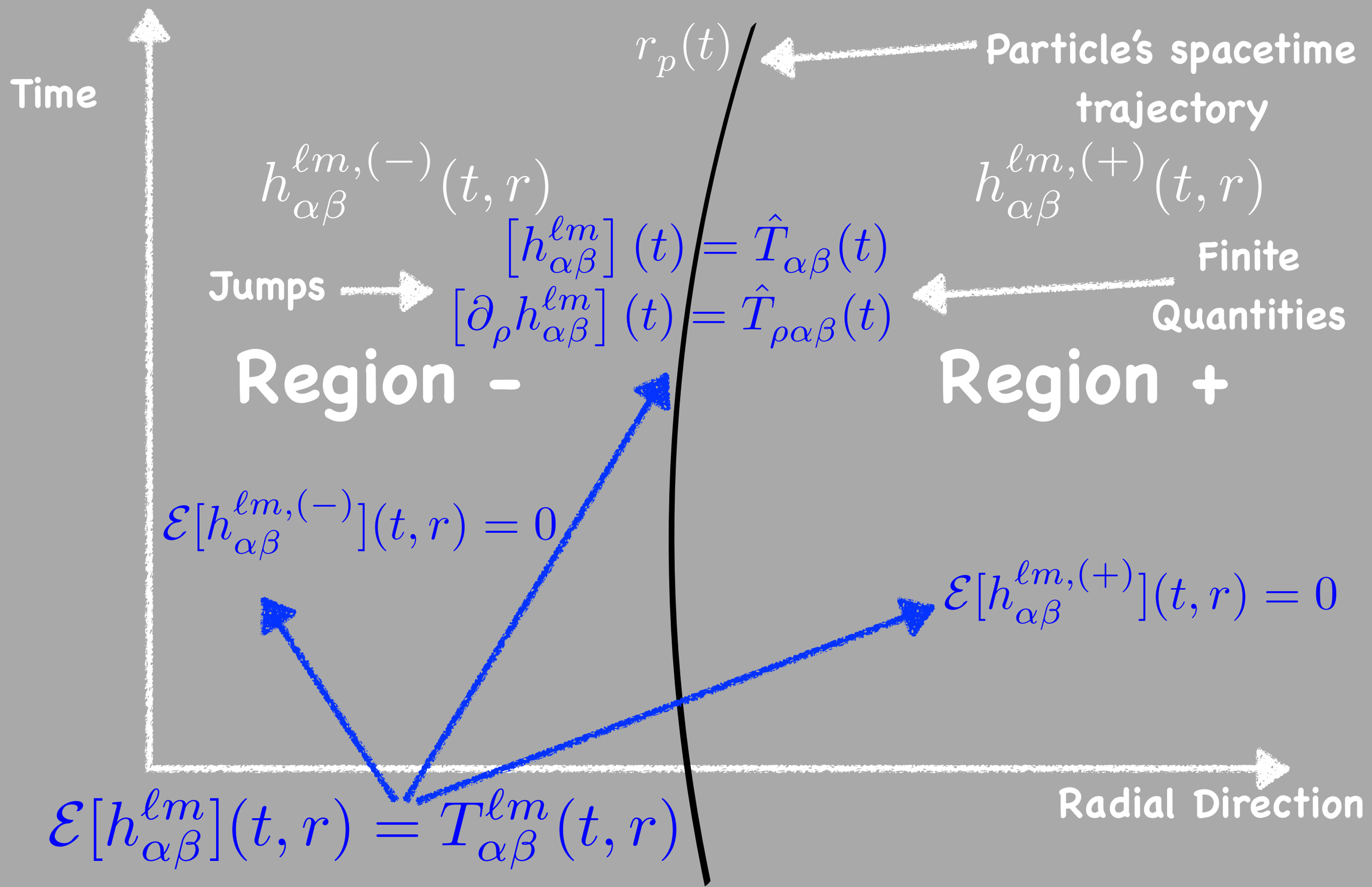


$$\mathcal{E}[h^{\ell m}_{\alpha\beta}](t,r) = T^{\ell m}_{\alpha\beta}(t,r)$$

The Particle without Particle scheme



The Particle without Particle scheme



The Particle without Particle scheme

- It avoids the presence of singularities in our computational domain and also the need for introducing an artificial (spatial) scale in the problem.

Cañizares & CFS, CQG **28** 134011 (2011)

Jaramillo, CFS & Cañizares, PRD **83** 061503 (2011)

Cañizares, CFS & Jaramillo, PRD **82** 044023 (2010)

Cañizares & CFS, PRD **79** 084020 (2009)

The Particle without Particle scheme

- It avoids the presence of singularities in our computational domain and also the need for introducing an artificial (spatial) scale in the problem.

- As a consequence we are left with homogeneous wave-type equations (i.e. without distributional source terms) at the interiors of the two regions. Then, we obtain smooth solutions in both regions.

Cañizares & CFS, CQG **28** 134011 (2011)

Jaramillo, CFS & Cañizares, PRD **83** 061503 (2011)

Cañizares, CFS & Jaramillo, PRD **82** 044023 (2010)

Cañizares & CFS, PRD **79** 084020 (2009)

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Linearized Einstein equations in the Lorenz Gauge + Lorenz Gauge:

$$\left[h^{\mathrm{L}}_{\alpha\beta}\right](t) = 0\,, \qquad \left[\partial_\rho h^{\mathrm{L}}_{\alpha\beta}\right](t) \neq 0$$

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Linearized Einstein equations in the Lorenz Gauge + Lorenz Gauge:

$$\left[h^{\mathrm{L}}_{\alpha\beta}\right](t) = 0\,, \qquad \left[\partial_\rho h^{\mathrm{L}}_{\alpha\beta}\right](t) \neq 0$$

Linearized Einstein equations in the Regge-Wheeler Gauge:

$$\hat{T}\,\delta(r - r_p(t)) = g^{ab} h^{RW,(+)}_{ab}\Theta\left(r - r_p(t)\right) + g^{ab} h^{RW,(-)}_{ab}\Theta\left(r_p(t) - r\right)$$

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Linearized Einstein equations in the Lorenz Gauge + Lorenz Gauge:

$$\left[h^{\mathrm{L}}_{\alpha\beta}\right](t) = 0\,, \qquad \left[\partial_\rho h^{\mathrm{L}}_{\alpha\beta}\right](t) \neq 0$$

Linearized Einstein equations in the Regge-Wheeler Gauge:

$$\hat{T}\,\delta(r - r_p(t)) = g^{ab} h^{RW,(+)}_{ab}\Theta\left(r - r_p(t)\right) + g^{ab} h^{RW,(-)}_{ab}\Theta\left(r_p(t) - r\right)$$

Inconsistency!

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Linearized Einstein equations in the Lorenz Gauge + Lorenz Gauge:

$$\left[h^{\mathrm{L}}_{\alpha\beta}\right](t) = 0\,, \qquad \left[\partial_\rho h^{\mathrm{L}}_{\alpha\beta}\right](t) \neq 0$$

Linearized Einstein equations in the Regge-Wheeler Gauge:

$$\hat{T}\,\delta(r - r_p(t)) = g^{ab} h^{RW,(+)}_{ab}\Theta\left(r - r_p(t)\right) + g^{ab} h^{RW,(-)}_{ab}\Theta\left(r_p(t) - r\right)$$

Zero only for radial trajectories

Inconsistency!

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Linearized Einstein equations in the Lorenz Gauge + Lorenz Gauge:

$$\left[h^{\mathrm{L}}_{\alpha\beta}\right](t) = 0\,, \qquad \left[\partial_\rho h^{\mathrm{L}}_{\alpha\beta}\right](t) \neq 0$$

Linearized Einstein equations in the Regge-Wheeler Gauge:

$$\hat{T}\,\delta(r - r_p(t)) = g^{ab} h^{RW,(+)}_{ab}\Theta\left(r - r_p(t)\right) + g^{ab} h^{RW,(-)}_{ab}\Theta\left(r_p(t) - r\right)$$

Zero only for radial trajectories

Inconsistency!

Why?

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Because we assumed that the Regge-Wheeler is a regular gauge that admits the regular decomposition:

$$h^{RW}_{\alpha\beta} = h^{RW,(+)}_{\alpha\beta}\Theta\left(r - r_p(t)\right) + h^{RW,(-)}_{\alpha\beta}\Theta\left(r_p(t) - r\right)$$

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Because we assumed that the Regge-Wheeler is a regular gauge that admits the regular decomposition:

$$h^{RW}_{\alpha\beta} = h^{RW,(+)}_{\alpha\beta}\Theta\left(r - r_p(t)\right) + h^{RW,(-)}_{\alpha\beta}\Theta\left(r_p(t) - r\right)$$

But the gauge transformation equations dictate a different structure for the metric perturbations in the Regge-Wheeler Gauge:

$$h^{RW}_{\alpha\beta} = h^{L}_{\alpha\beta} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

$$\xi_\alpha = \xi^{(+)}_\alpha\,\Theta\left(r - r_p(t)\right) + \xi^{(-)}_\alpha\,\Theta\left(r_p(t) - r\right)$$

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

Because we assumed that the Regge-Wheeler is a regular gauge that admits the regular decomposition:

$$h^{RW}_{\alpha\beta} = h^{RW,(+)}_{\alpha\beta}\Theta\left(r - r_p(t)\right) + h^{RW,(-)}_{\alpha\beta}\Theta\left(r_p(t) - r\right)$$

But the gauge transformation equations dictate a different structure for the metric perturbations in the Regge-Wheeler Gauge:

$$h^{RW}_{\alpha\beta} = h^{L}_{\alpha\beta} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$$

$$\xi_\alpha = \xi^{(+)}_\alpha\,\Theta\left(r - r_p(t)\right) + \xi^{(-)}_\alpha\,\Theta\left(r_p(t) - r\right)$$

And this induces the Dirac delta terms that make the previous equation consistent. In this way we have controlled the singularities of the problem so that we finally can obtain a finite value for the self-force.

The Particle without Particle scheme

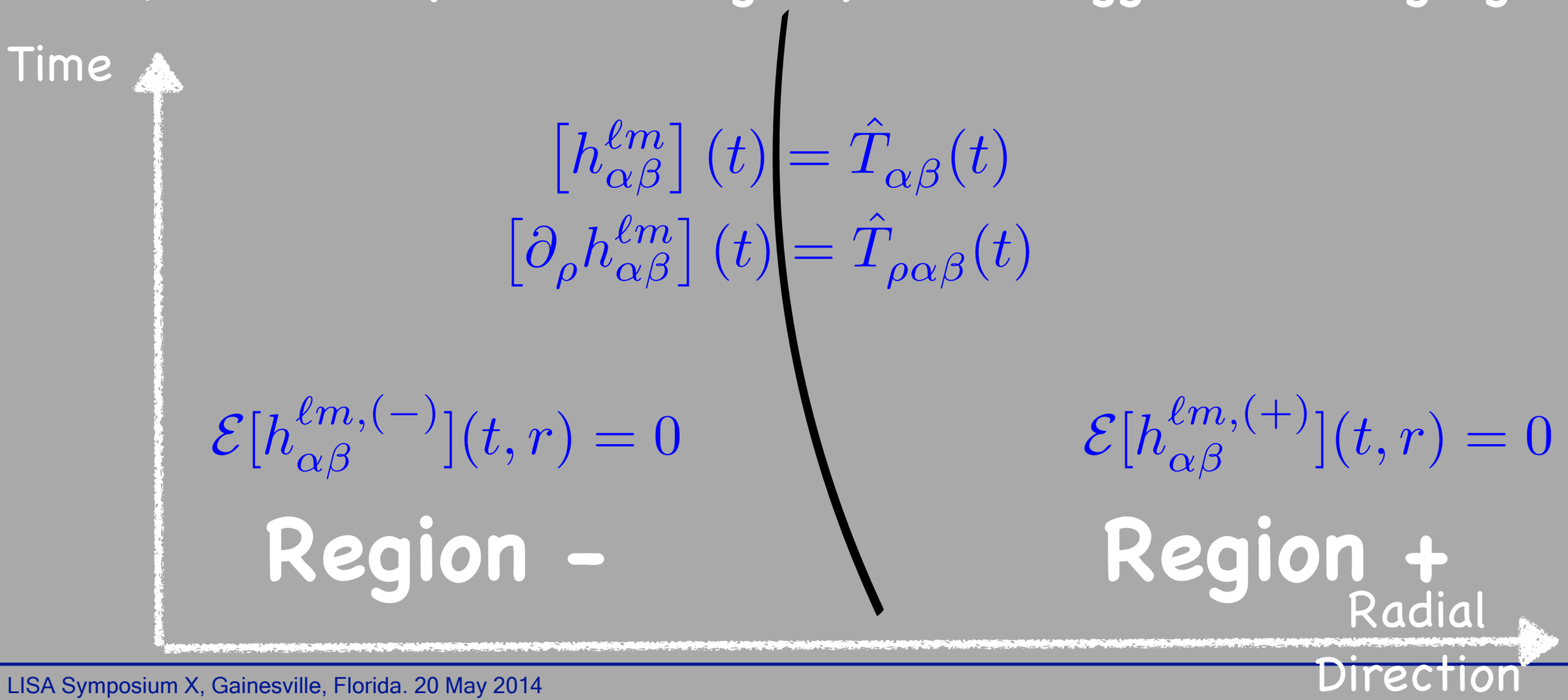
- From the Lorenz Gauge to the Regge-Wheeler Gauge:

In summary, the Particle-without-Particle formulation provides a clean method to control the singularities that appear in the gauge transformation and, as a consequence, provides a well-defined, finite, self-force (from both regions) in the Regge-Wheeler gauge.

The Particle without Particle scheme

- From the Lorenz Gauge to the Regge-Wheeler Gauge:

In summary, the Particle-without-Particle formulation provides a clean method to control the singularities that appear in the gauge transformation and, as a consequence, provides a well-defined, finite, self-force (from both regions) in the Regge-Wheeler gauge.



Present and Future prospects

- Numerical Implementation: The Particle-without-Particle formalism has been implemented successfully for the scalar case with pseudospectral collocation methods (with spectral convergence). We are extending it for the gravitational case in the Regge-Wheeler gauge.

Present and Future prospects

- Numerical Implementation: The Particle-without-Particle formalism has been implemented successfully for the scalar case with pseudospectral collocation methods (with spectral convergence). We are extending it for the gravitational case in the Regge-Wheeler gauge.

- Analysis of other gauges: Radiation gauge, etc.

Present and Future prospects

- Numerical Implementation: The Particle-without-Particle formalism has been implemented successfully for the scalar case with pseudospectral collocation methods (with spectral convergence). We are extending it for the gravitational case in the Regge-Wheeler gauge.

- Analysis of other gauges: Radiation gauge, etc.

- Extension to Kerr: uncertain in the time domain (logarithmic singularities). Good Prospects in the frequency domain (adapting the method of extended solutions).