Memory effect from spinning unbound binaries

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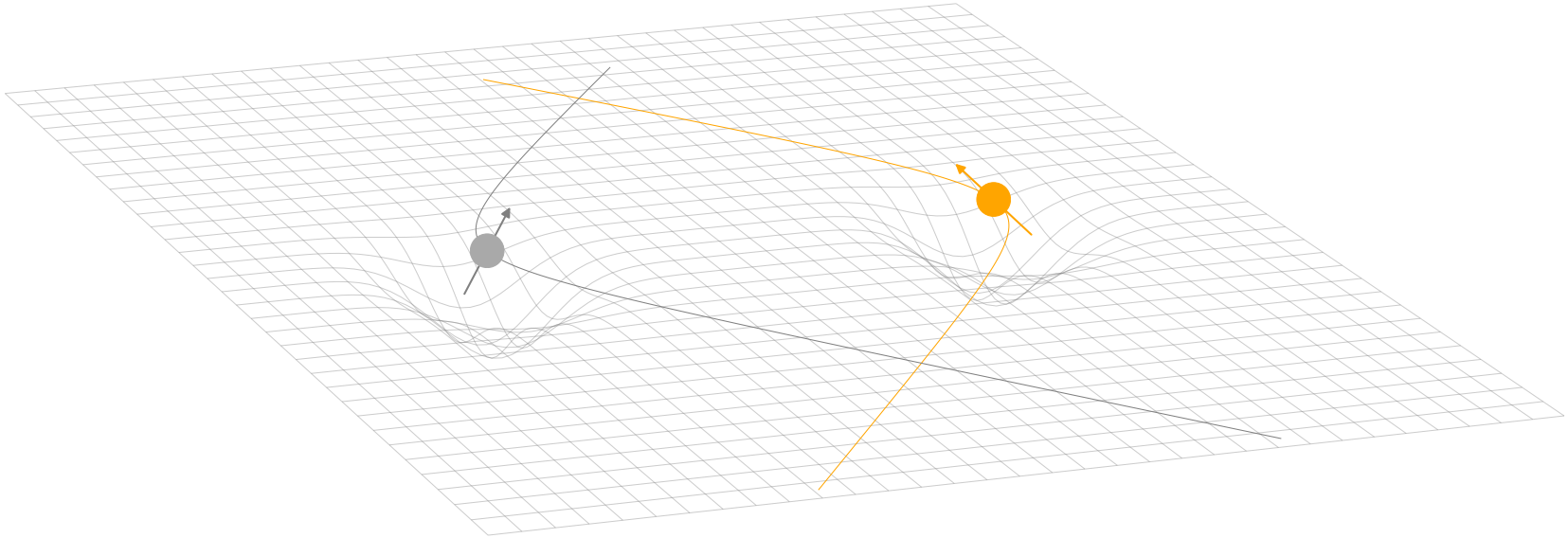

in collaboration with A. Gopakumar, A. Gupta, P. Jetzer

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Outline

A method to generate **waveform templates** for **spinning** binaries on **hyperbolic orbits**

Spinning binaries

- How to obtain PN accurate $h_{+,\times}$
- How to obtain PN accurate dynamics
- Waveforms: Memory effect <u>in $h_\times$ and h_+</u>

Non-spinning binaries

- How to obtain PN accurate $h_{+,\times}$
- How to obtain PN accurate dynamics
- Waveforms: Memory effect <u>only in $h_\times$</u>

How to obtain $h_{+,\times}$ for spinning binaries

Polarization states expressions for general orbits are well-known up to high PN orders, including spin contributions

$$h_{+,\times} = -\frac{G\mu}{c^4 R}\left(h^N_{+,\times} + \frac{1}{c}h^{0.5}_{+,\times} + \frac{1}{c^2}h^1_{+,\times} + \frac{1}{c^2}h^S_{+,\times}\right)$$

The leading order contribution - the **quadrupolar order** - reads

$$h_+|_Q = \frac{2G\mu}{c^4 R}\left\{(\boldsymbol{p}\cdot\boldsymbol{v})^2 - (\boldsymbol{q}\cdot\boldsymbol{v})^2 - \frac{Gm}{r}\left[(\boldsymbol{p}\cdot\boldsymbol{n})^2 - (\boldsymbol{q}\cdot\boldsymbol{n})^2\right]\right\}$$

$$h_\times|_Q = \frac{4G\mu}{c^4 R}\left\{(\boldsymbol{p}\cdot\boldsymbol{v})(\boldsymbol{q}\cdot\boldsymbol{v}) - \frac{Gm}{r}(\boldsymbol{p}\cdot\boldsymbol{n})(\boldsymbol{q}\cdot\boldsymbol{n})\right\}$$

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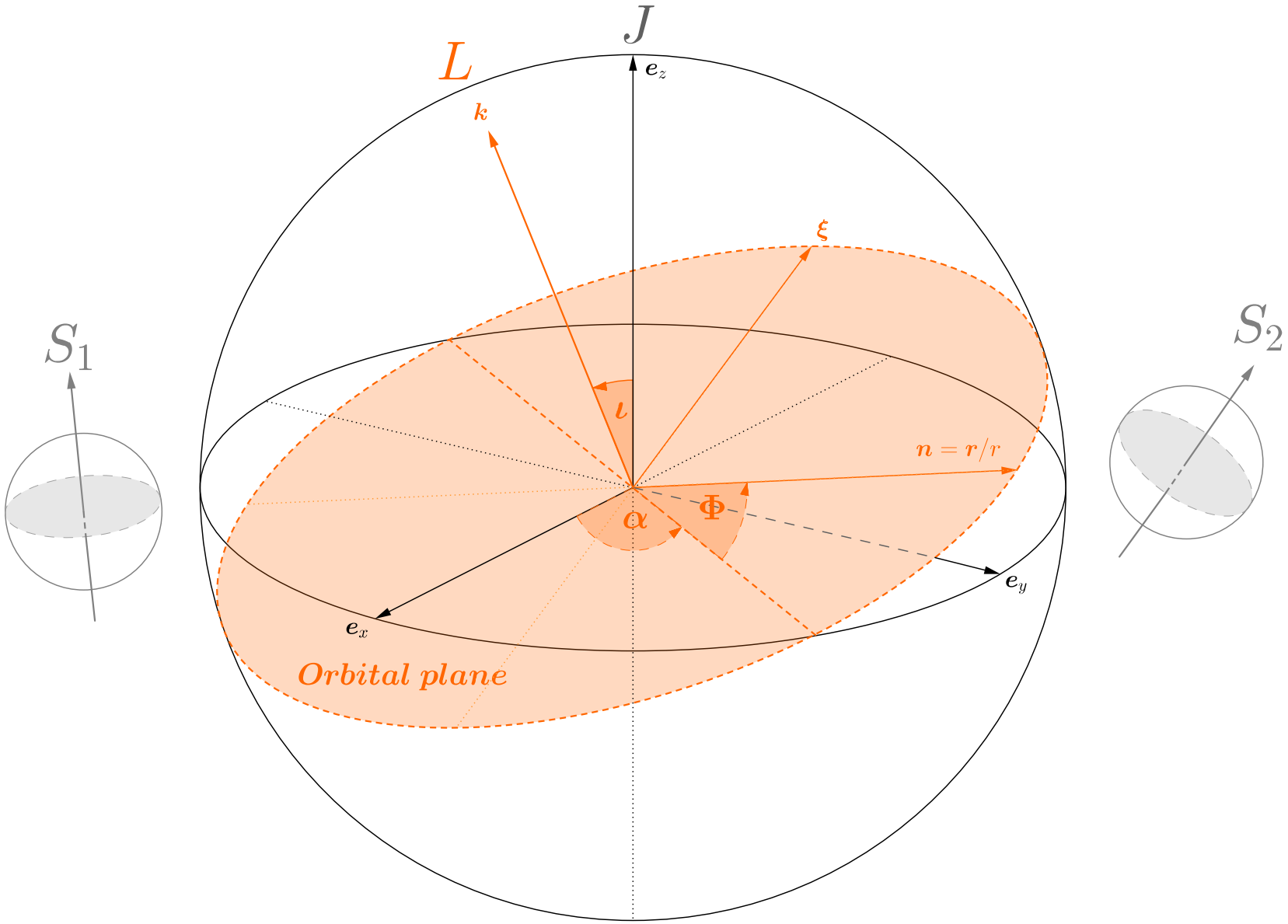
$\longrightarrow$ need to get these dot products
i.e. describe the dynamics

The Precessing Dance



Explicit $h_{+,\times}$ for spinning binaries

- Express dynamics in terms of the 3 Eulerian angles Φ, α, ι
- Write dot products in $h_{+,\times}$ as functions of $r, \dot{r}, \Phi, \dot{\Phi}, \alpha, \iota$
- Get (very long) explicit exressions for $h_{+,\times}$ for **spinning** compact binaries moving on general non-circular orbits

$$\boxed{h_{+,\times}(t) = h_{+,\times}\left(r(t), \dot{r}(t) \ , \Phi(t), \dot{\Phi}(t), \alpha(t), \iota(t) \right)}$$

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$$h_{+,\times}(t) = h_{+,\times}\left(\underbrace{r(t), \dot{r}(t)}_{\text{radial part}}, \underbrace{\Phi(t), \dot{\Phi}(t), \alpha(t), \iota(t)}_{\text{angular part}}\right)$$

Radial part of the dynamics

Radial part **has** Keplerian-type parametric solution

Hyperbolic Kepler equation $l := \bar{n}(t - t_0) = e_{\rm t} \sinh v - v$
can be solved numerically for $v(l)$ through **Mikkola's method**.

The **1.5PN-accurate solution** for r and $\dot{r}$ turns out to be

$$r(l) = \frac{Gm}{c^2} \frac{1}{\bar{\xi}^{2/3}} \left\{ e_{\rm t} \cosh v - 1 + \bar{\xi}^{2/3}(\cdots) + \bar{\xi}\,(\cdots) \right\}$$

$$\dot{r}(l) = \bar{\xi}^{1/3} \frac{c\, e_{\rm t} \sinh v}{e_{\rm t} \cosh v - 1} \left\{ 1 - \bar{\xi}^{2/3}(\cdots) \right\}$$

Angular part of the dynamics

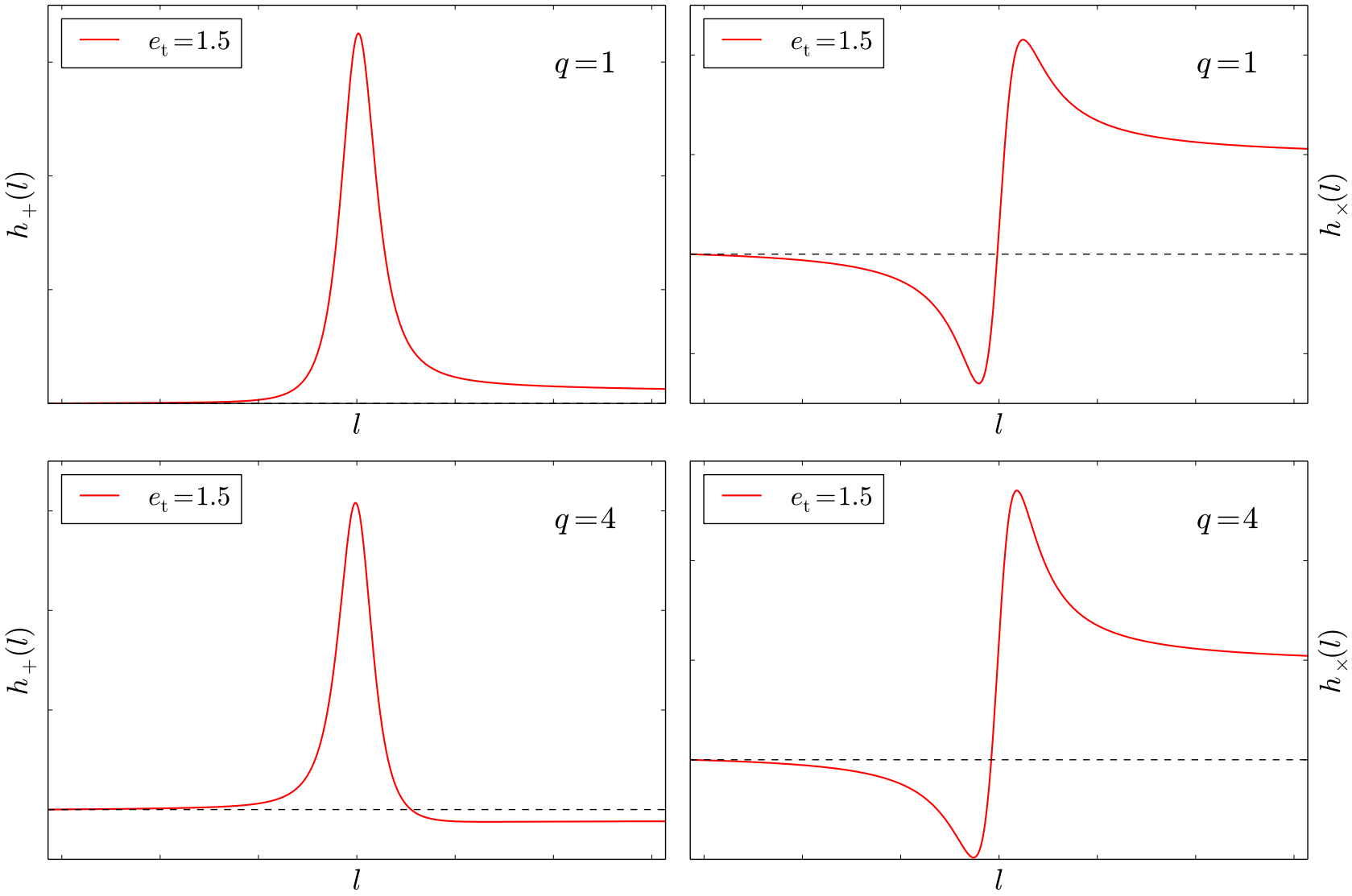
Angular part **has no** parametric solution

Express angular evolution through set of coupled differential eqs.

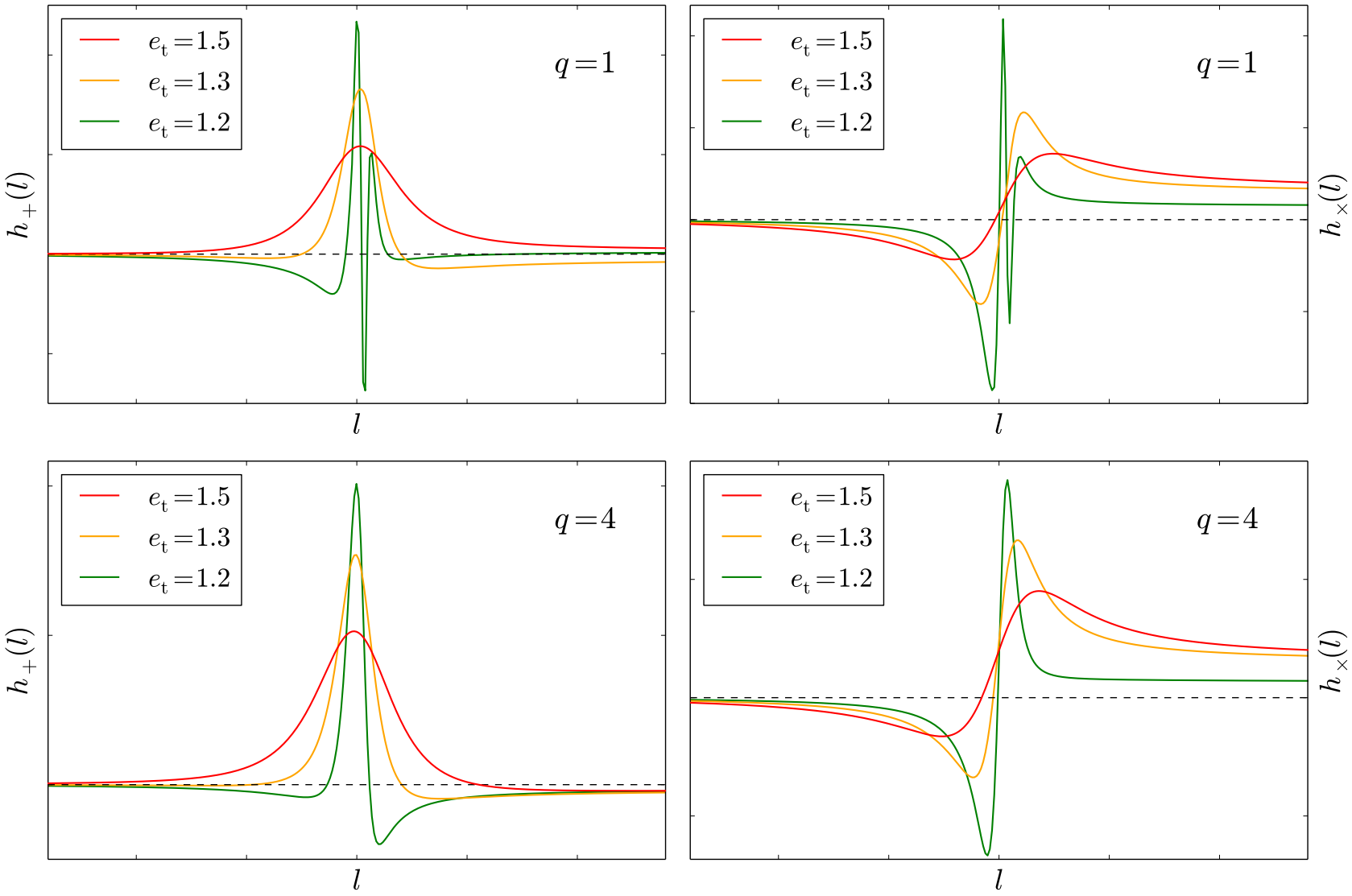
- we need **9** precessional equations for $\dot{\boldsymbol{S}}_1, \dot{\boldsymbol{S}}_2, \dot{\boldsymbol{L}}$
- we need the **1** evolution equation for $\dot{\Phi}(t)$
- can also include **2** radiation reaction equations for $\dot{e}_{\mathrm{t}}$ and $\dot{\bar{n}}$

$\rightarrow$ **Numerically solve this set of 12 differential equations** and get $\dot{\Phi}(t), \Phi(t), \alpha(t), \iota(t)$ at any time during the interaction

Waveform for spinning binaries



Waveform for spinning binaries



Memory effect

The **Memory effect** we see in the plots is

$$\boxed{\Delta h^{\text{mem}}_{+,\times} = \lim_{t\to+\infty} h_{+,\times}(t) - \lim_{t\to-\infty} h_{+,\times}(t)}$$

For **spinning binaries** **<u>both</u>** polarizations show memory effect

$$\Delta h^{\text{mem}}_{+} \neq 0 \qquad \text{and} \qquad \Delta h^{\text{mem}}_{\times} \neq 0$$

How to get $h_{+,\times}$ for non-spinning binaries

What about **NON**-spinning binaries?

How to get $h_{+,\times}$ for non-spinning binaries

In non-spinning binaries, **both radial and angluar** parts of the dynamics have parametric solutions:

$$
\begin{aligned}
r &= \frac{Gm}{c^2}\frac{1}{\bar{\xi}^{2/3}}\left\{e_{\mathrm{t}}\cosh v - 1 + \bar{\xi}^{2/3}(\cdots)\right\} \\
\dot{r} &= \bar{\xi}^{1/3}\,\frac{c\,e_{\mathrm{t}}\,\sinh v}{e_{\mathrm{t}}\,\cosh v - 1}\left\{1 + \bar{\xi}^{2/3}(\cdots)\right\} \\
\phi &= 2\arctan\left[\sqrt{\tfrac{e_\phi+1}{e_\phi-1}}\tanh\frac{v}{2}\right]\left\{1 + \bar{\xi}^{2/3}(\cdots)\right\} \\
\dot{\phi} &= \frac{\bar{n}\sqrt{e_{\mathrm{t}}^2 - 1}}{(\,e_{\mathrm{t}}\,\cosh v - 1\,)^2}\left\{1 + \bar{\xi}^{2/3}(\cdots)\right\}
\end{aligned}
$$

How to get $h_{+,\times}$ for non-spinning binaries

Polarization states $h_{+,\times} = -\frac{G\mu}{c^4 R}\left(h^N_{+,\times} + \frac{1}{c}h^{0.5}_{+,\times} + \frac{1}{c^2}h^1_{+,\times}\right)$

$$h^N_+ = 2r\dot{r}\dot{\phi}(1+C_\theta^2)\sin 2\phi + (1+C_\theta^2)\left(z + r^2\dot{\phi}^2 - \dot{r}^2\right)\cos 2\phi + S_\theta^2\left(\dot{r}^2 + r^2\dot{\phi}^2 - z\right) ,$$

$$h^N_\times = 2C_\theta\left(z + r^2\dot{\phi}^2 - \dot{r}^2\right)\sin 2\phi - 2C_\theta 2r\dot{r}\dot{\phi}\cos 2\phi ,$$

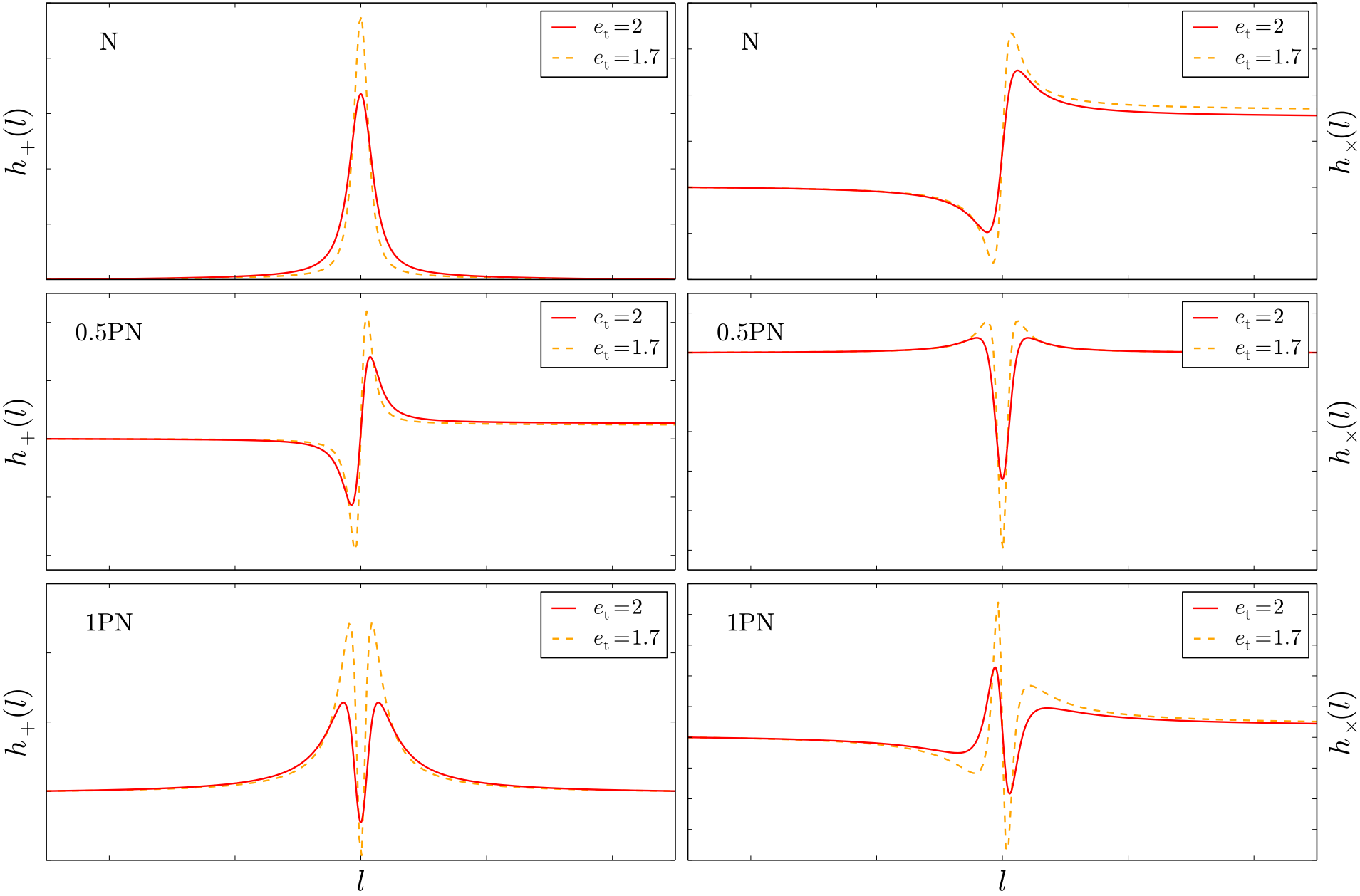
$$\begin{aligned} h^{0.5}_+ = \frac{S_\theta}{2}(X_2 - X_1)\Big[&(3C_\theta^2 - 1)\left(\dot{r}^2 + r^2\dot{\phi}^2 - 2z\right)\dot{r}\cos\phi + (1+C_\theta^2)\left(\dot{r}^2 - 3r^2\dot{\phi}^2 - 2z\right)\dot{r}\cos 3\phi \\ &- \left(2(1+3C_\theta^2)\dot{r}^2 - 2S_\theta^2 r^2\dot{\phi}^2 - (5+7C_\theta^2)z + (1-C_\theta^2)\left(6\dot{r}^2 - 2r^2\dot{\phi}^2 - 7z\right)\cos 2\phi\right) r\dot{\phi}\sin\phi\Big] , \end{aligned}$$

$$\begin{aligned} h^{0.5}_\times = \frac{C_\theta S_\theta}{2}(X_2 - X_1)\Big[&\left(2\dot{r}^2 + 2r^2\dot{\phi}^2 - 5z\right) r\dot{\phi}\cos\phi + \left(6\dot{r}^2 - 2r^2\dot{\phi} - 7z\right) r\dot{\phi}\cos 3\phi \\ &+ 4\dot{r}\left(\dot{r}^2 - r^2\dot{\phi}^2 - 2z + \left(\dot{r}^2 - 3r^2\dot{\phi}^2 - 2z\right)\cos 2\phi\right)\sin\phi\Big] , \end{aligned}$$

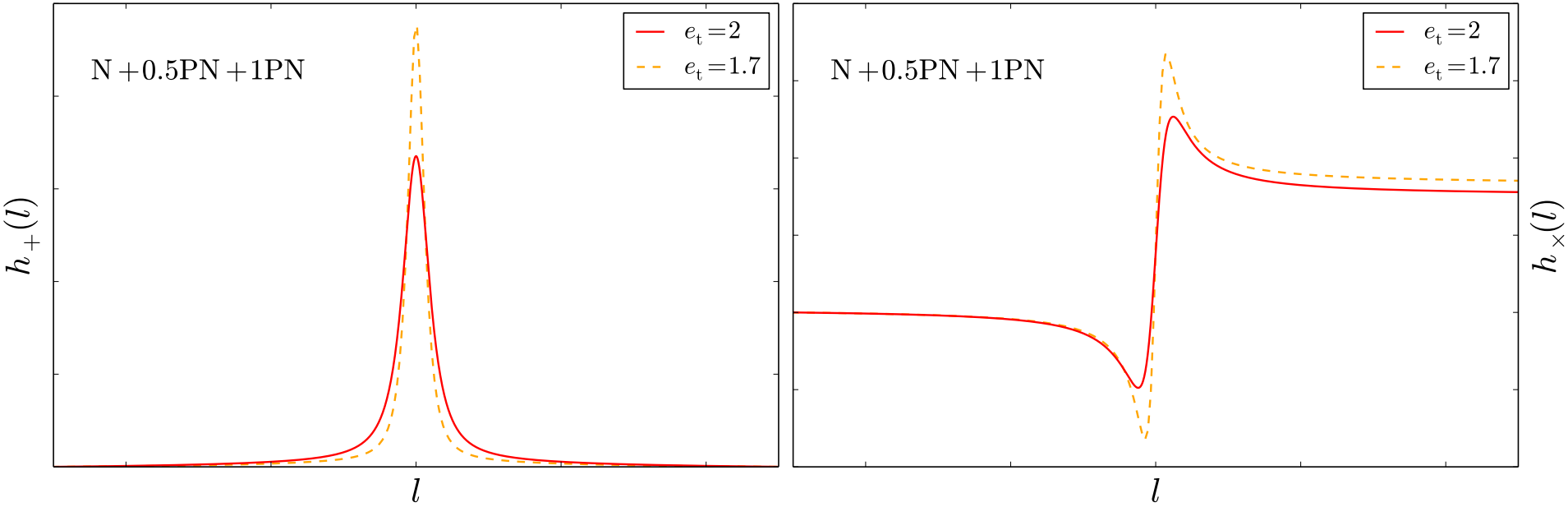
$$\begin{aligned} h^1_+ = \frac{1}{24}\Big[&-18S_\theta^4(\dot{r}^4 + r^4\dot{\phi}^4)(3\eta - 1) + r^2\dot{\phi}^2 z S_\theta^2(51\eta - 69 + C_\theta^2 39(1-3\eta)) + z^2 \times \\ &\times S_\theta^2(116 + 7(1-3\eta)(1-3C_\theta^2)) - 26\dot{r}^2 S_\theta^2 r^2\dot{\phi}^2(3\eta - 1) + 6\dot{r}^2 z S_\theta^2(9C_\theta^2(1-3\eta) + 3 + 13\eta)\Big] \\ &+ \cos 2\phi\,\frac{1}{2}\Big[(\dot{r}^4 - r^4\dot{\phi}^4)(1 - C_\theta^2(1 + 2S_\theta^2))(3\eta - 1) + r^2\dot{\phi}z\Big((2+3\eta)(1+C_\theta^2) + S_\theta^2(3\eta - 1)(4 + C_\theta^2)\Big) \\ &- \dot{r}^2 z\Big((3+2\eta)(1+C_\theta^2) + 6C_\theta^2 S_\theta^2(3\eta - 1)\Big) + z^2\frac{1}{3}\Big(7C_\theta^2 S_\theta^2(3\eta - 1) - 29(1+C_\theta^2)\Big)\Big] \end{aligned}$$

some usual PN mess...

Waveform for non-spinning binaries



Waveform for non-spinning binaries



Non-spinning binary system with masses $m_1 = 8M_\odot, m_2 = 13M_\odot$, $r_{\text{min}} = 2 \times 10^9 m$, and $R = 21000$ ly. ($\sim$ Hulse-Taylor pulsar)

$\longrightarrow$ **Only the $\times$-polarization shows a memory!**

$\longrightarrow$ **Both signals are symmetric w.r.t. $t = 0$**

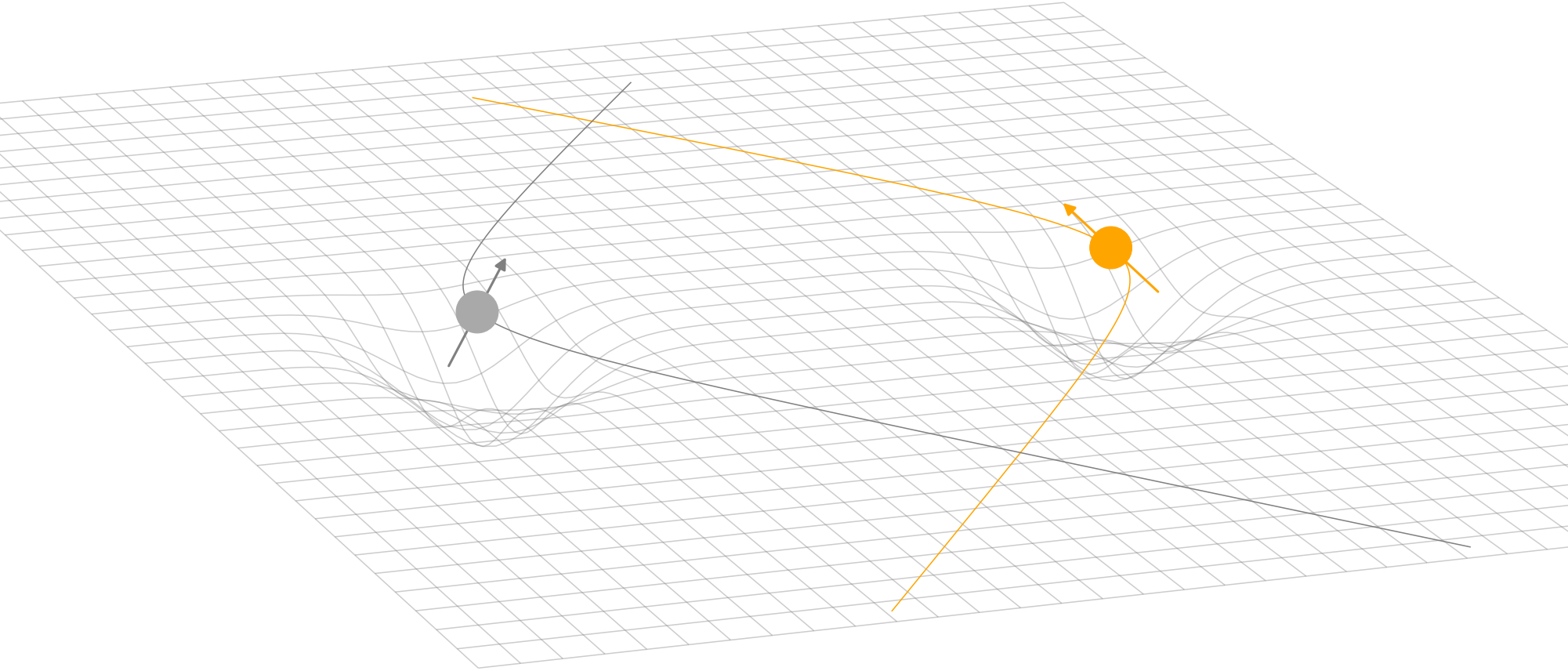
Summary

1. We provide a new prescription to generate ready-to-use templates for **spinning** binaries on **hyperbolic** orbits, up to **1.5PN** order in the dynamics and **1PN** for polarization states.

2. We include **leading order spin-orbit coupling**, as well as **radiation reaction** effects.

3. We show the existence of a **linear memory effect**, which is responsible for a non-vanishing amplitude of **both** polarizations.

4. In the literature there is **disagreement** about the waveforms of signals from hyperbolic encounters. With our new and fast method we suggest a solution to the problem.



Thank you!