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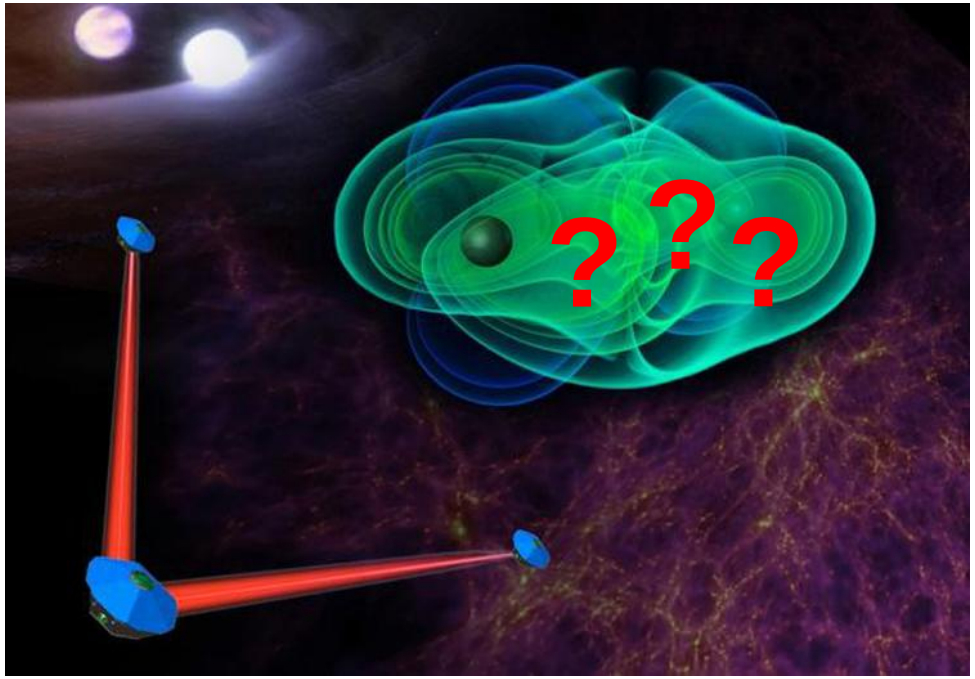

Testing GR with Time Domain Waveforms

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Outline / Motivation

- ➔ Gravitational waves from compact binary inspirals provide strong tests of GR (black hole binaries: v/c ~ 0.3)
- ➔ Want to check GR: if wrong, then all our parameter estimation is wrong
- ➔ Among possible ways to test GR: test PN coefficients
- ➔ A lot of work in this has already been done in context of aLIGO and classic LISA by Nico, Frans, Neil, .. : parametrized post-Einsteinian (ppE) scheme - frequency domain waveform with generically perturbed PN coefficients
- ➔ Wanted to use a modified time domain waveform to this end

PN Expansion $\xrightarrow{\text{Luc}}$ TD waveform $\xrightarrow{\text{FFT}}$ FD waveform $\xrightarrow{(..|..)}$ Data Analysis

- ➔ Aim: investigate effect of possible L3 mission configurations on our ability to test GR (focus on arm length, number of links)

Gravitational-wave tests of GR

➔ **Gravitational wave tests of GR with compact binary inspiral signals**

$$S = \int d^4x \sqrt{-g} \left(\kappa_g R + \frac{\alpha}{4} \vartheta R_{\nu\mu\rho\sigma}{}^* R^{\mu\nu\rho\sigma} - \frac{\beta}{2} \nabla_\mu \vartheta \nabla^\mu \vartheta + \mathcal{L}_{\mathrm{mat}} \right)$$

modified theory

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modified theory

Check waveform **only** for certain theory ← direct test

modified gravitational wave evolution

$$\tilde{h}\sim\tilde{h}^{GR}\left[1-4i\Psi_{CS}\frac{F_+^2+F_\times^2\zeta\left(1+\zeta^2\right)}{F_+^2\left(1+\zeta^2\right)^2+4F_\times^2\zeta^2}\right]$$

Gravitational-wave tests of GR

➔ **Gravitational wave tests of GR with compact binary inspiral signals**

$$\{a, \alpha, b, \beta\}$$

modified theory

generic test

Check data for **any** possible deviation from GR, manifesting in GWs

No one-to-one mapping between modified waveform and particular theories

modified gravitational wave evolution

e.g. ppE scheme by Nico Yunes+

$$\tilde{h}_{\rm ppE}(f) = \tilde{h}_{\rm GR}(f) \left(1 + \sum_a \alpha_a \, u^a\right) e^{i \sum_b \beta_b \, u^b}$$

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➔ Works in frequency domain, motivated with the SPA of the Fourier integral

➡ Time domain is closer to theory, what about a time domain GR testing framework?

Waveform models

Time Domain $x = (GM\omega/c^3)^{2/3}$ $\Theta(t) = \frac{\eta c^3}{5GM}(t_c - t)$

$$\omega(\Theta) = \frac{c^3}{8GM}\left[\Theta^{-3/8} + \left(\frac{743}{2688} + \frac{11}{32}\eta\right)\Theta^{-5/8} - \frac{3\pi}{10}\Theta^{-6/8} + \left(\frac{1855099}{14450688} + \frac{56975}{258048}\eta + \frac{371}{2048}\eta^2\right)\Theta^{-7/8}\right]$$

$$\Phi(\Theta) = \Phi_C - \frac{1}{\eta}\left[\Theta^{5/8} + \left(\frac{3715}{8064} + \frac{55}{96}\eta\right)\Theta^{3/8} - \frac{3\pi}{4}\Theta^{2/8} + \left(\frac{9275495}{14450688} + \frac{284875}{258048}\eta + \frac{1855}{2048}\eta^2\right)\Theta^{1/8}\right]$$

$$h_{+,\times} = \frac{2GM\eta}{c^2 D_L}x\left[H^{(0)}_{+,\times} + x^{1/2}H^{(1/2)}_{+,\times} + xH^{(1)}_{+,\times} + x^{3/2}H^{(3/2)}_{+,\times} + x^2 H^{(2)}_{+,\times}\right]$$

(quasi-circular, non-spinning compact binary inspiral)

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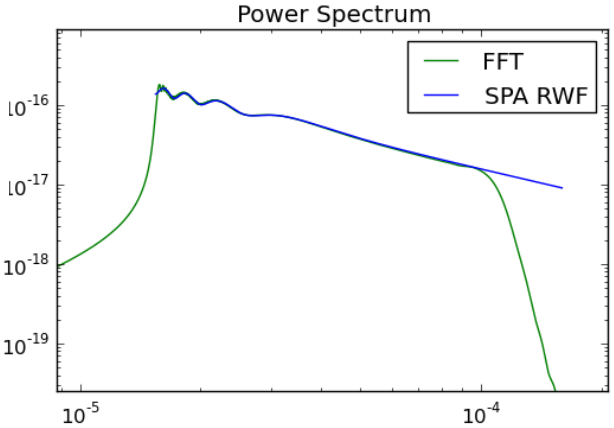
Stationary Phase Approximation (at dominant harmonic)

$$\tilde{h}(f) = \int \mathrm{d}t\, e^{2\pi i f t}\, h(t) \approx \frac{\mathcal{A}[t(f)]}{2\sqrt{\dot{F}(t)|_{t=t(f)}}}\, e^{i\Psi(f)}$$

$$\Psi(f) = 2\pi f t(f) - 2\Phi[t(f)] - \frac{\pi}{4}$$



Modified waveforms

Frequency domain (ppE scheme)

$$\tilde{h}_{\rm ppE}(f) = \tilde{h}_{\rm GR}(f) \left(1 + \sum_a \alpha_a \, u^a\right) e^{i \sum_b \beta_b \, u^b} \qquad u = \frac{G\mathcal{M}}{c^3}\pi f$$

Sampson, Yunes, Cornish 2013: Leading order correction enough to detect departures from GR:

$$\Psi_{\rm ppE}(u) = \Psi_{\rm GR}(u) + \beta_b u^b \qquad A_{\rm ppE}(u) = (1 + \alpha_a u^a)\, A_{\rm GR}(u)$$

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Time Domain (proposal)

$$\Phi_{\rm NGR}(\Theta) = \Phi_{\rm GR}(\Theta) - \frac{1}{\eta}\sum_i \alpha_i \Theta^{\frac{5-2i}{8}}$$

(no amplitude correction)

$$\omega_{\rm NGR}(\Theta) = \omega_{\rm GR}(\Theta) + \frac{c^3}{8GM}\sum_i \frac{5-2i}{5}\alpha_i \Theta^{\frac{-3-2i}{8}}$$

(i = 0, 1/2, 1, 3/2, 2)

up to 2PN order

Where to go with this?

- ➔ Explore the accuracy with which a LISA-like detector can measure the theory parameters α_i with MCMC (in preparation)

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- A lot of theory work in context of ppE scheme has been done by Nico, Neil, ..
  Is there a way to relate modified time domain waveforms to the ppE framework?

could also be sum of multiple corrections

$$\Psi_{\rm ppE}(u) = \Psi_{\rm GR}(u) + \overbrace{\beta_b u^b}$$

$$\Phi_{\rm NGR}(\Theta) = \Phi_{\rm GR}(\Theta) - \frac{1}{\eta}\sum_i \alpha_i \Theta^{\frac{5-2i}{8}}$$

$\alpha_i(b, \beta) = ?$

Relating modified waveforms

$$\Psi(u) = 2\left[\frac{c^3}{G\mathcal{M}} t(u)\, u - \Phi(u)\right] - \frac{\pi}{4} \qquad \text{(SPA)}$$

$$\Rightarrow \frac{d\Psi}{du} = 2\frac{c^3}{G\mathcal{M}}\, t(u) \qquad \left(\frac{d\Phi}{du}(u) = \frac{dt}{du}\frac{c^3}{G\mathcal{M}} u\right)$$

$$u = \frac{G\mathcal{M}}{c^3}\pi f$$

Relating modified waveforms

$$t(u) = \frac{1}{2}\frac{G\mathcal{M}}{c^3}\frac{d\Psi}{du}$$

$$u(t) = \frac{G\mathcal{M}}{c^3}\frac{d\Phi}{dt}$$

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$$u = \frac{G\mathcal{M}}{c^3}\pi f$$

$$u[\Theta(u)]_{2\mathrm{PN}} = u\left(1 + \sum_{k=0}^{4} u^{k/3}\underbrace{\mathcal{A}_k}_{=0}\right) = u$$

Truncate at 2PN,
work in linear order of α_i and β

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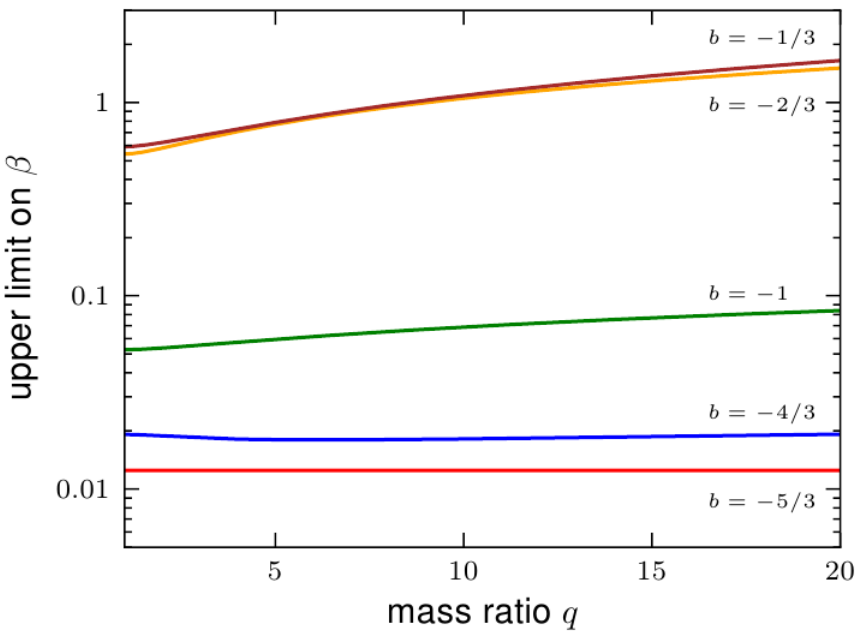
	0PN	0.5PN	1PN	1.5PN	2PN
b	-5/3	-4/3	-1	-2/3	-1/3
α_0	16β	0	0	0	0
$\alpha_{1/2}$	0	$8\beta\eta^{1/5}$	0	0	0
α_1	$-16\beta\Phi_1$	0	$4\beta\eta^{2/5}$	0	0
$\alpha_{3/2}$	$-\frac{32}{3}\beta\Phi_{3/2}$	$-\frac{32}{5}\beta\Phi_1\eta^{1/5}$	0	$2\beta\eta^{3/5}$	0
α_2	$16\beta\left(\frac{4}{5}\Phi_1^2 - \frac{1}{3}\Phi_2\right)$	$-\frac{64}{15}\beta\Phi_{3/2}\eta^{1/5}$	$-\frac{12}{5}\beta\Phi_1\eta^{2/5}$	0	$\beta\eta^{4/5}$

Regime of validity I

- ➔ Used linear approximation to derive $\alpha_i(b, \beta)$
- ➔ In order for the framework to stay a perturbation of GR, require $\frac{\alpha_i}{\Phi_i} < 0.2$
- ➔ For $\alpha_i(b, \beta)$ this gives different 'upper limits' on β, depending on b



Is this a problem for the linear approximation?

Regime of validity II

Is this a problem for the linear approximation?

➔ Evaluate the residual

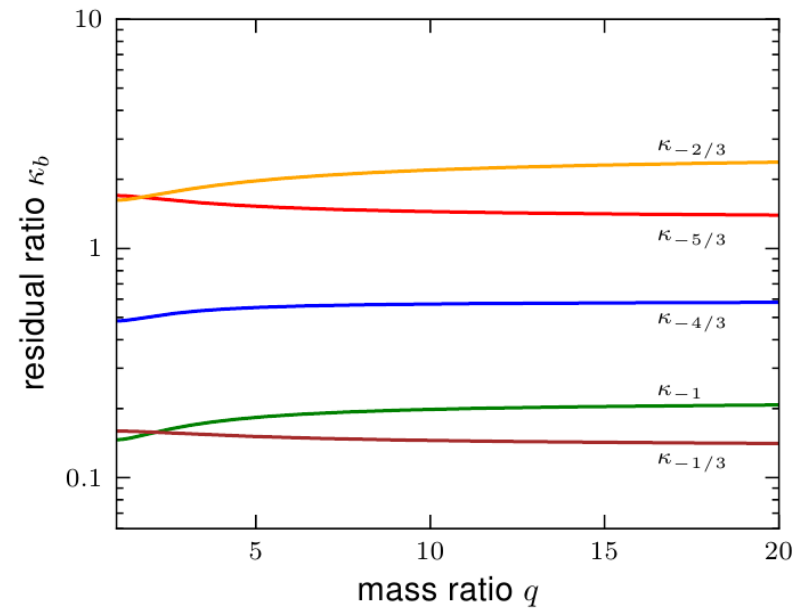
$$u[\Theta(u)] = u + \delta u(u)$$

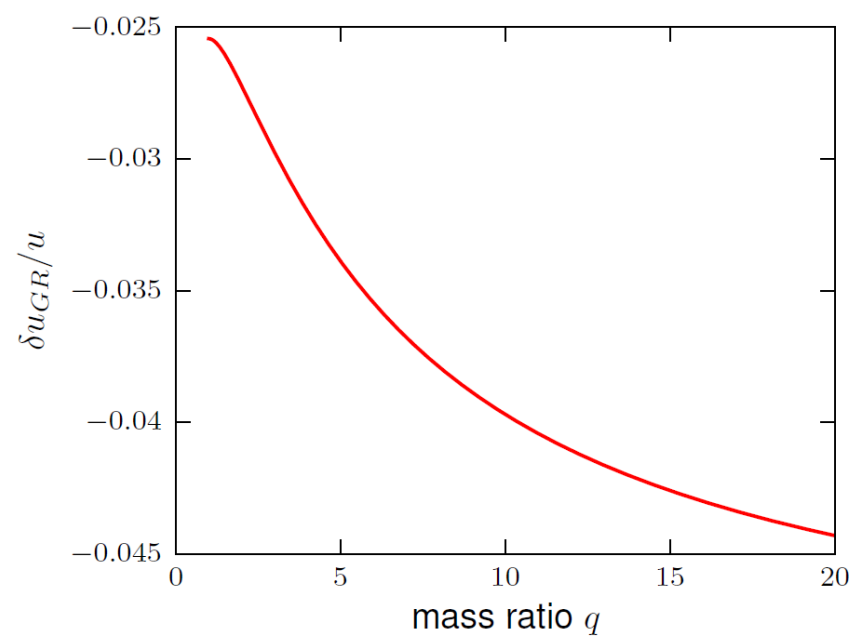
$$\delta u = \delta u_{\rm GR} + \delta u_{\rm NGR}$$

➔ Compute the ratio

$$\kappa_b = \left.\frac{\delta u_{\rm NGR}(u_{\rm max})}{\delta u_{\rm GR}(u_{\rm max})}\right|_{\beta=\beta_{\rm max}(b)}$$

➔ Looks alright!





Summary

- ➔ Modified time domain waveforms are more natural to use than modified 'frequency domain' waveforms (SPA).

- ➔ Modified time domain waveforms can be related with modified frequency domain waveforms (ppE) by employing the SPA in good approximation.

- ➔ Will add 5 theory parameters: 9+5 = 14 dimensional parameter space for non-spinning binaries

- ➔ Stay tuned for parameter estimation simulations!