Albert Einstein Institute
Max Planck Institute for Gravitational Physics and
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Low noise laser interferometer design using IfoCAD

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Gerhard Heinzel and Karsten Danzmann

http://www.geo600.uni-hannover.de/ifocad/

European Space Agency
ESA

Centre for Quantum Engineering and Space-Time Research

Leibniz Universität Hannover
Germany

German Aerospace Center

Max Planck Society
Germany

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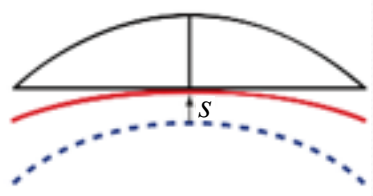
Low noise laser interferometer design using IfoCAD

- C/C++ Library **for laser interferometer design and optimization**
- **public C version (→ google), C++ version on request**
- developed at AEI since ~2010
- features
  - **propagation of various beam types** (Gaussian beams incl. higher order; general astigmatism; FFT-optics)
  - **photodiode readout signals**
  - **optimization routines**

Interferometer Signals

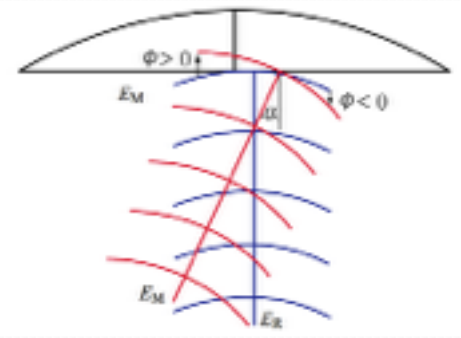
Phase:
readout of longitudinal shifts



$$\phi = k \cdot s := \frac{2\pi}{\lambda} s$$

DWS:
(differential wavefront sensing)

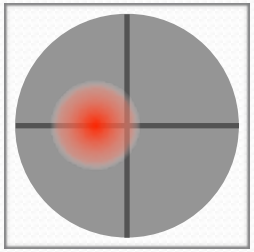
measure of beam tilt



$$\mathrm{DWS_h} = \phi_{\mathrm{left}} - \phi_{\mathrm{right}}$$

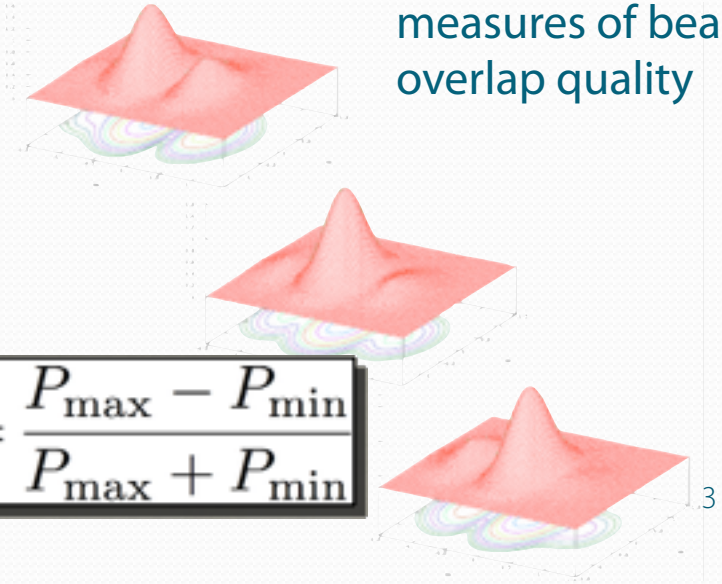
DPS:
(differential power sensing)

measure of beam displacement



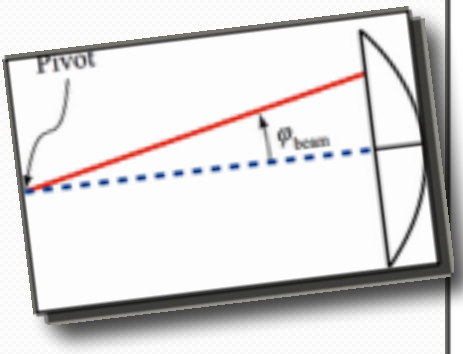
$$\mathrm{DPS_h} = \frac{\bar{P}_{\mathrm{left}} - \bar{P}_{\mathrm{right}}}{\bar{P}_{\mathrm{left}} + \bar{P}_{\mathrm{right}}}$$

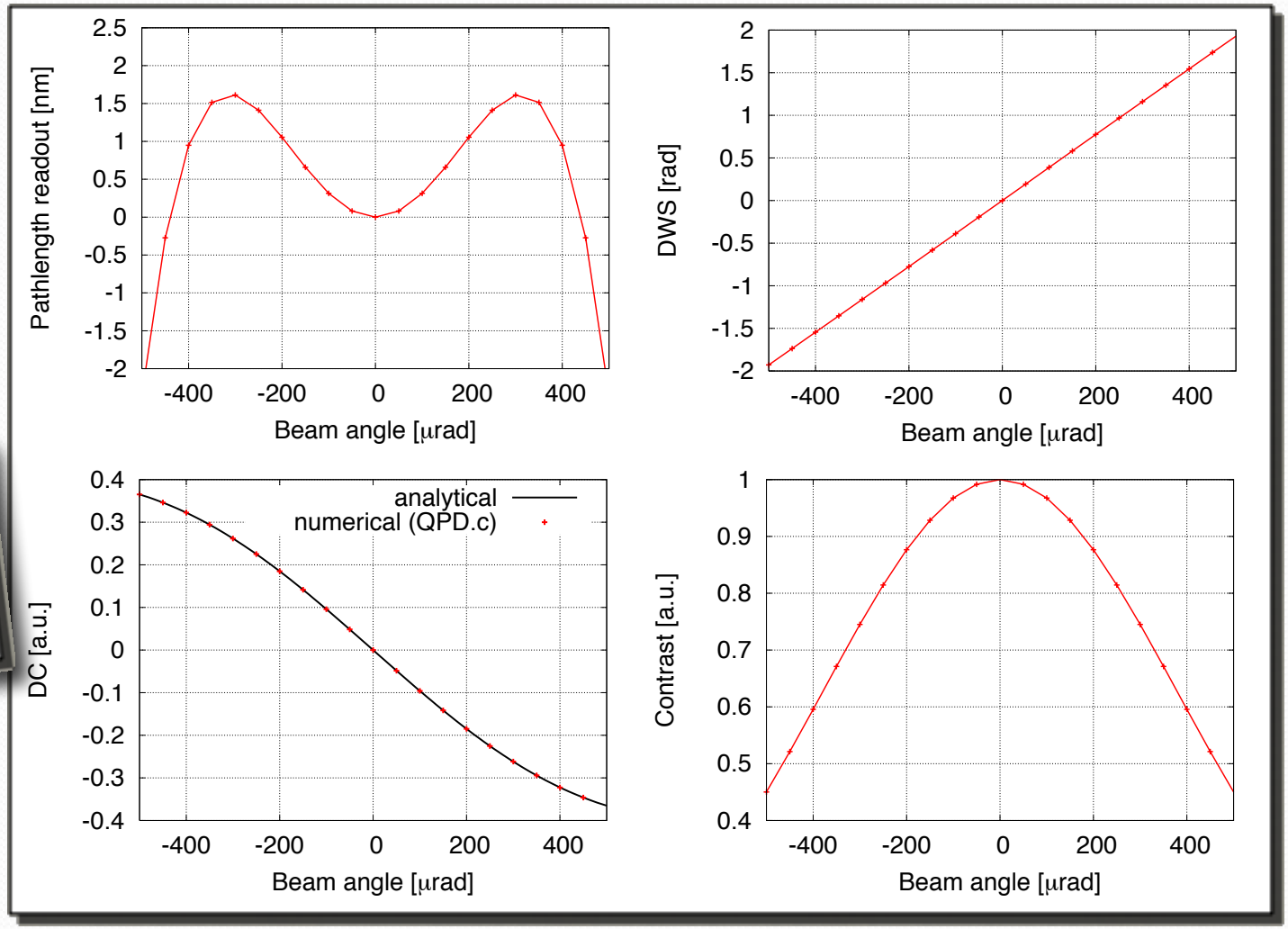
Contrast:
measures of beam overlap quality



$$c := \frac{P_{\max} - P_{\min}}{P_{\max} + P_{\min}}$$

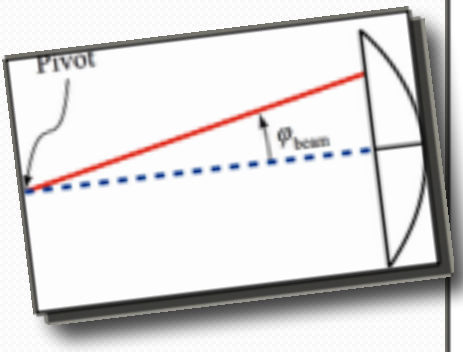
Phase on a quadrant diode?

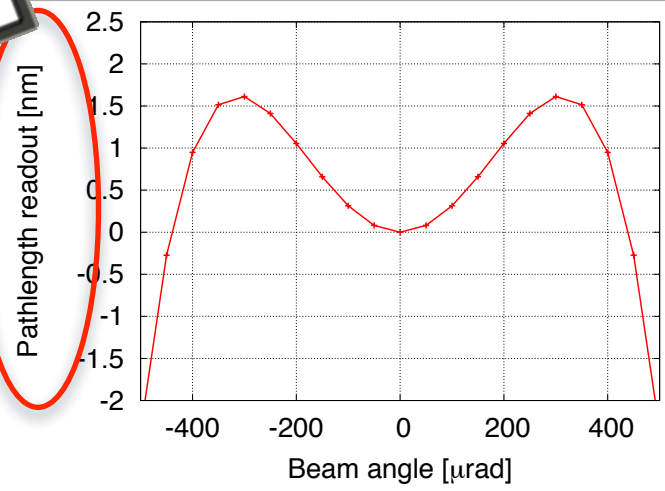


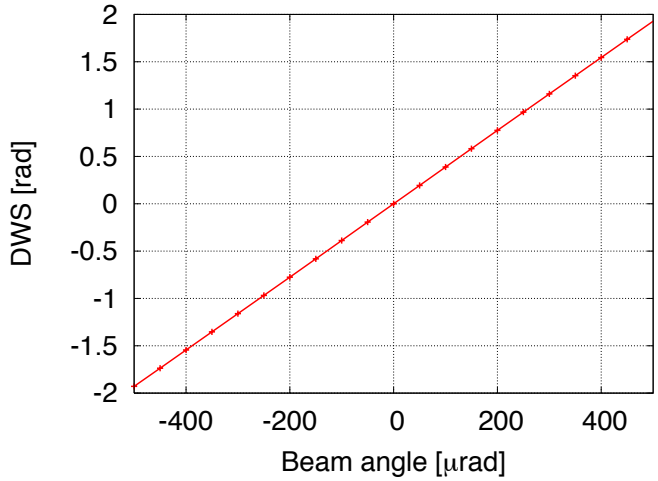


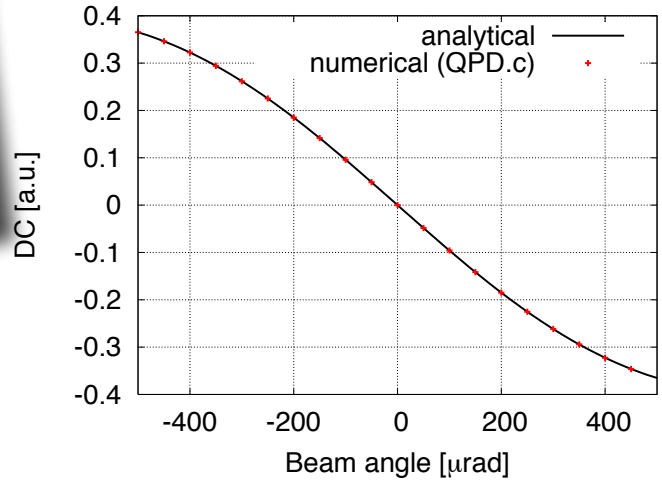
Phase on a quadrant diode?

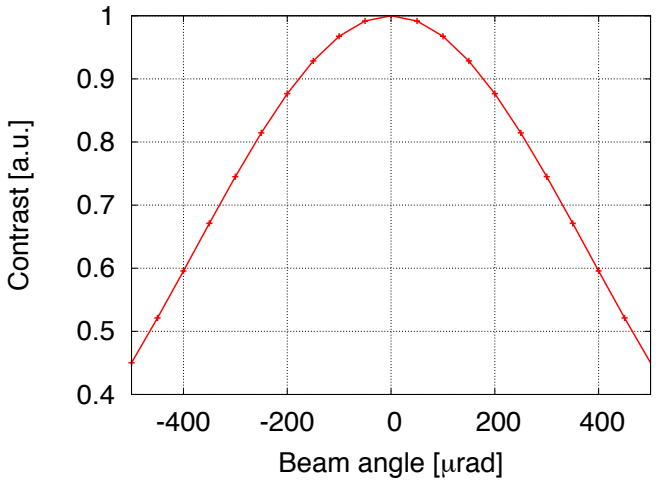
what is this **exactly**?











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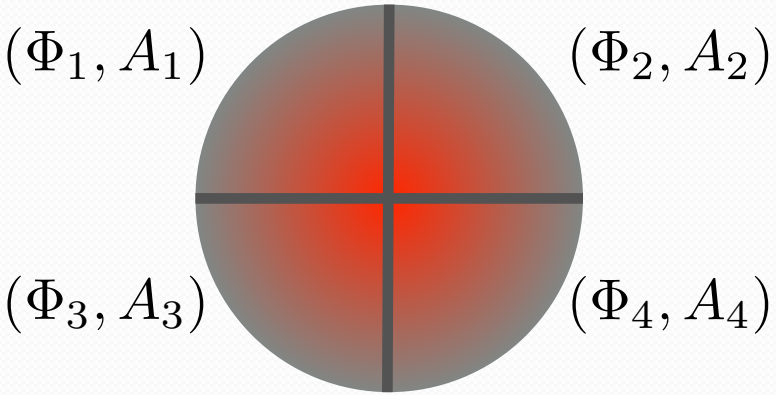

$$\Phi_{\mathrm{LPF}} = \mathrm{Arg}\left(\sum_{n=1}^{4} A_n \exp(i\Phi_n)\right)$$

$$\Phi_{\mathrm{AP}} = \frac{1}{4}\left(\sum_{n=1}^{4} \Phi_n\right)$$

$$\Phi_{\mathrm{WAP}} = \frac{A_1\Phi_1 + A_2\Phi_2 + A_3\Phi_3 + A_4\Phi_4}{A_1 + A_2 + A_3 + A_4}$$

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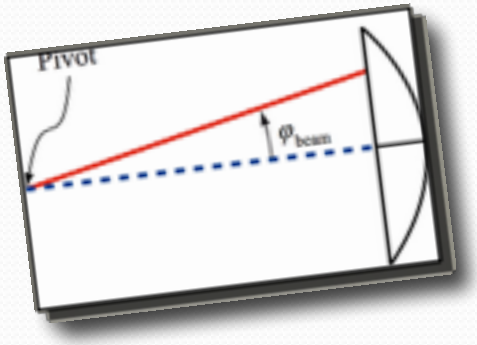

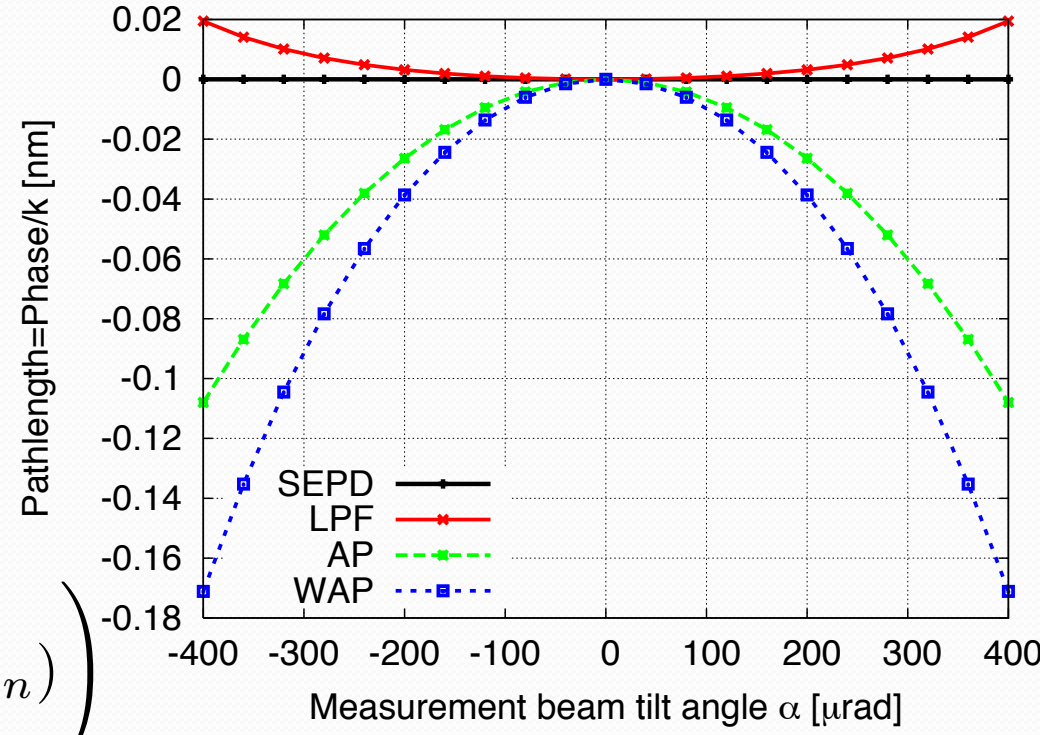

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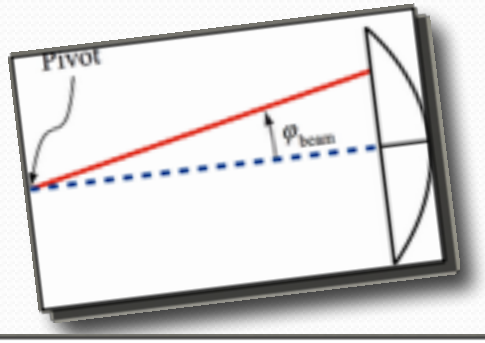

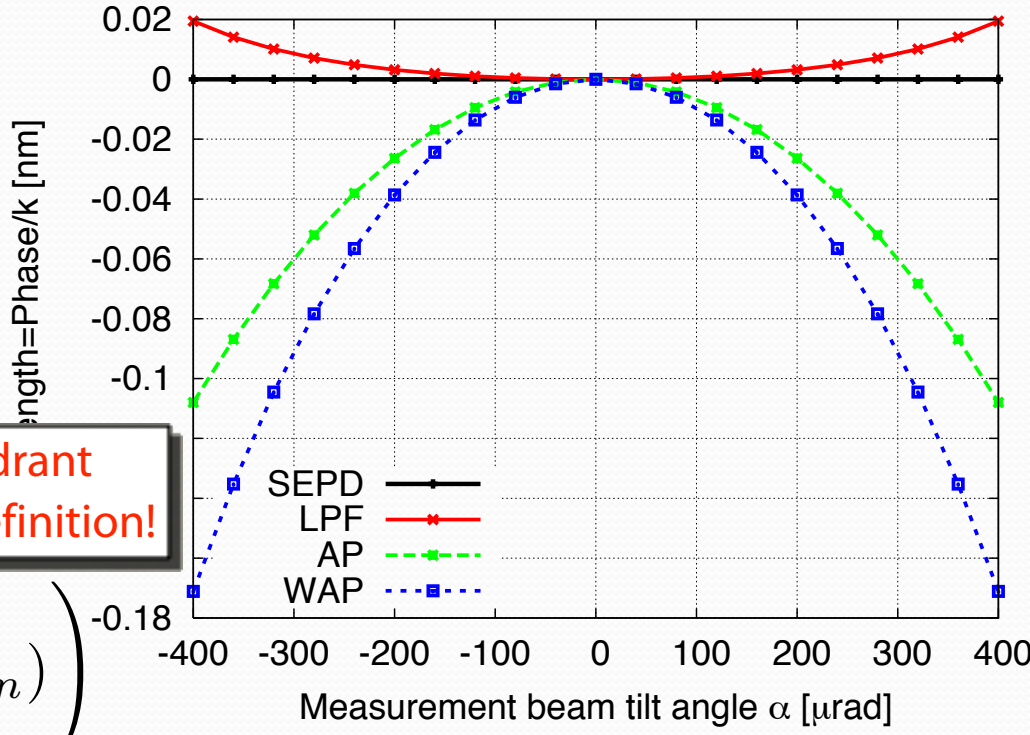

The amount of phase noise on our quadrant diode depends on the chosen phase definition!

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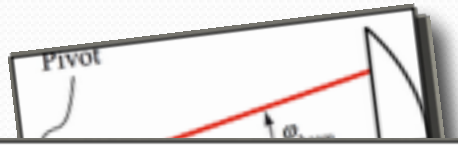
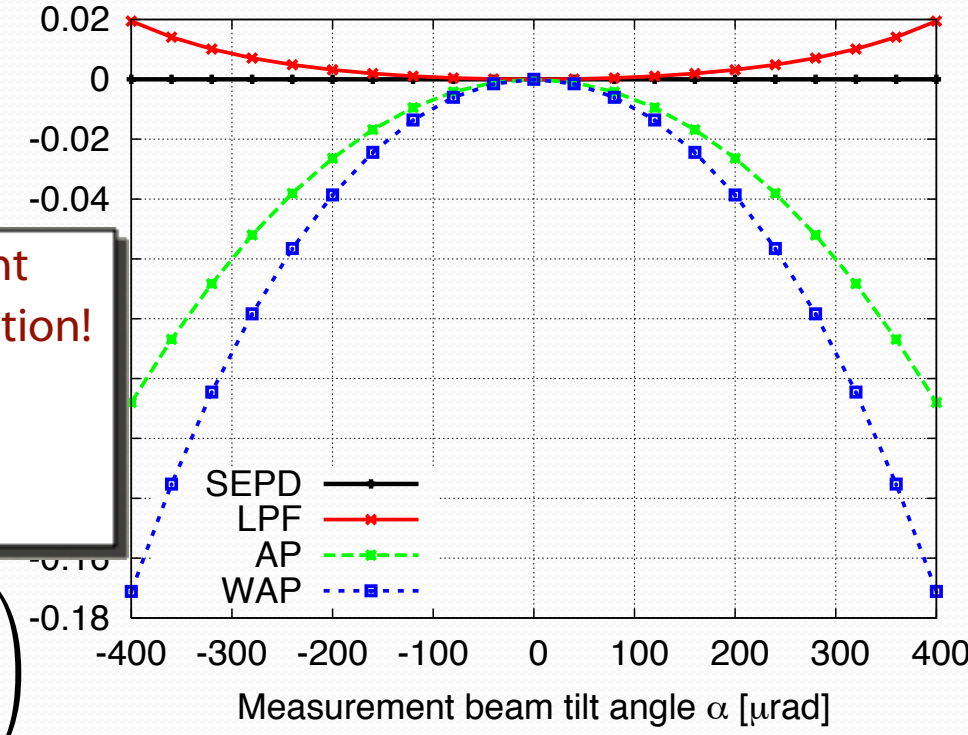


The amount of phase noise on our quadrant diode depends on the chosen phase definition!
This is an **example.**
We do not know - yet - ….
which definition is best for LISA



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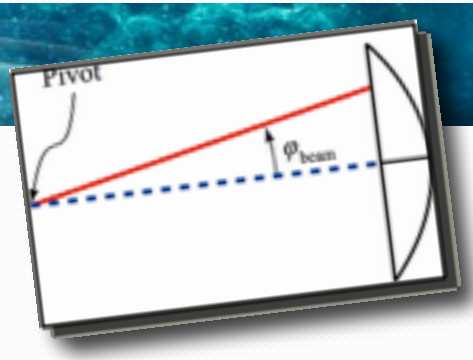
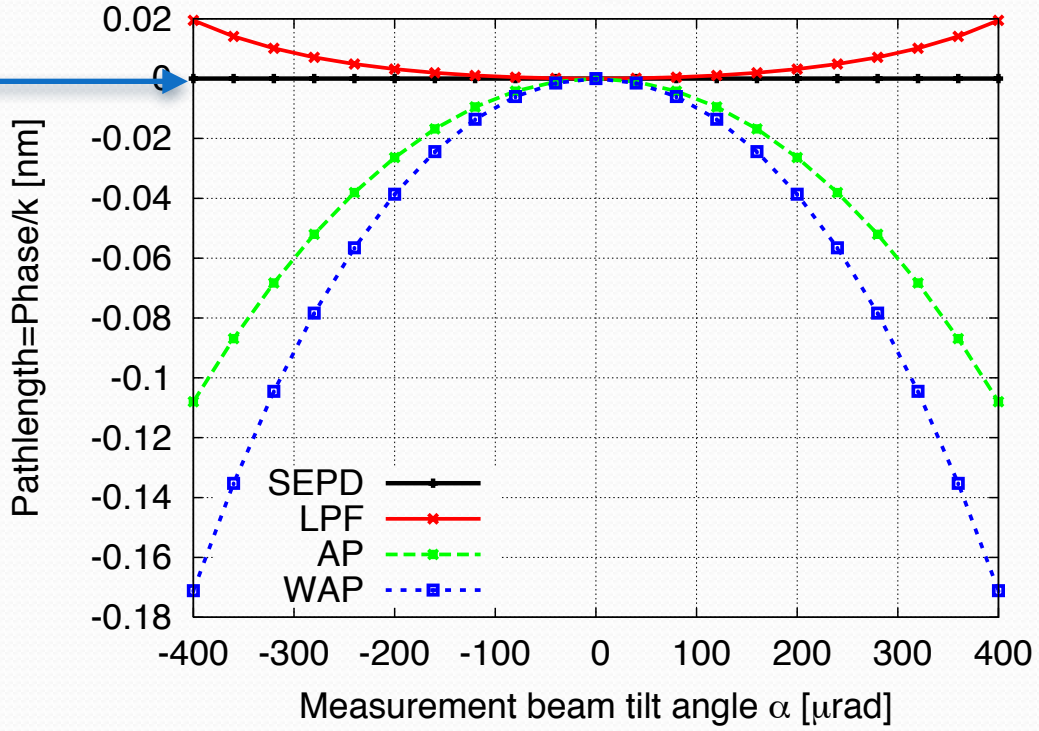


cross coupling vanishes
on a large single element detector
- provided that
the beam parameters are matched

this is not intuitive…



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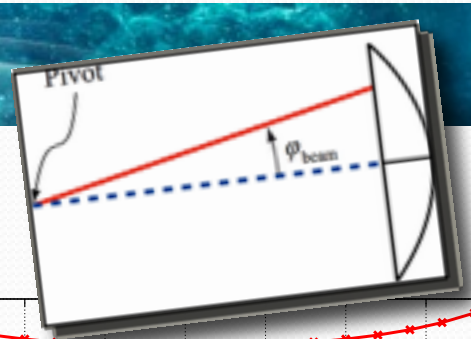

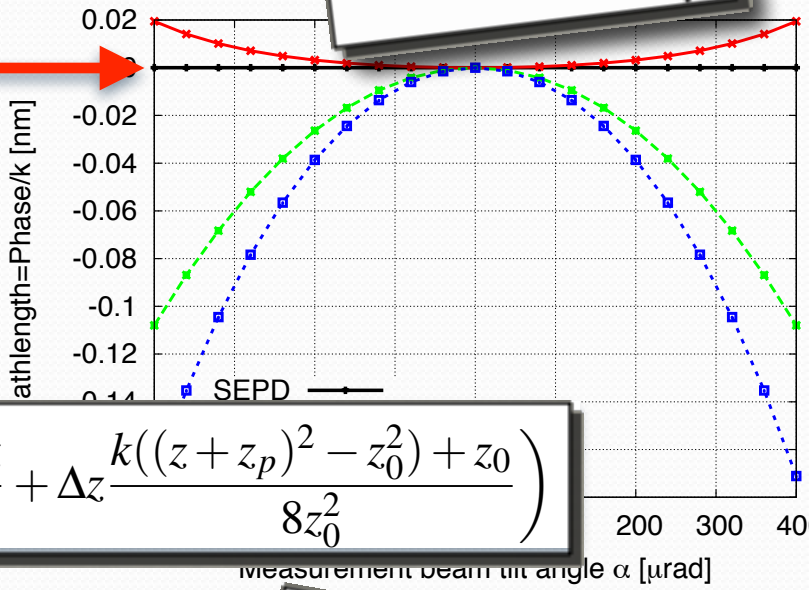

http://dx.doi.org/10.1364/AO.53.003043

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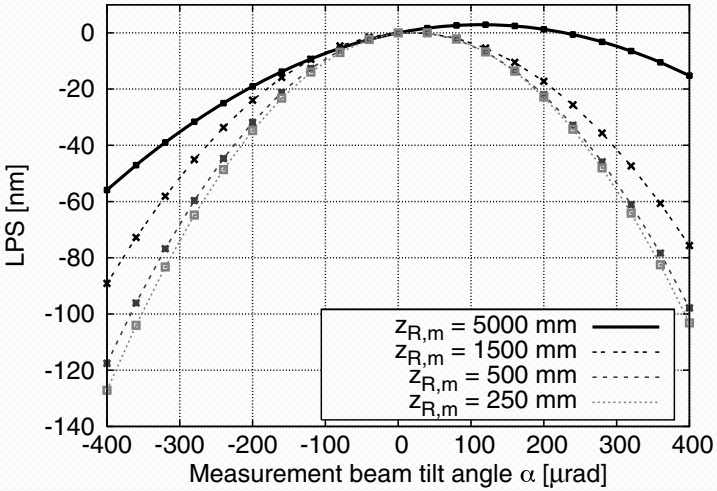

Analytical description of interference between two misaligned and mismatched complete Gaussian beams

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*Corresponding author: gudrun.wanner@aei.mpg.de

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OCIS codes: (120.3180) Interferometry; (000.3860) Mathematical methods in physics; (200.1130) Algebraic optical processing.
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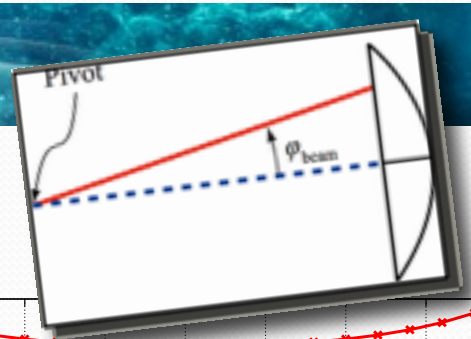

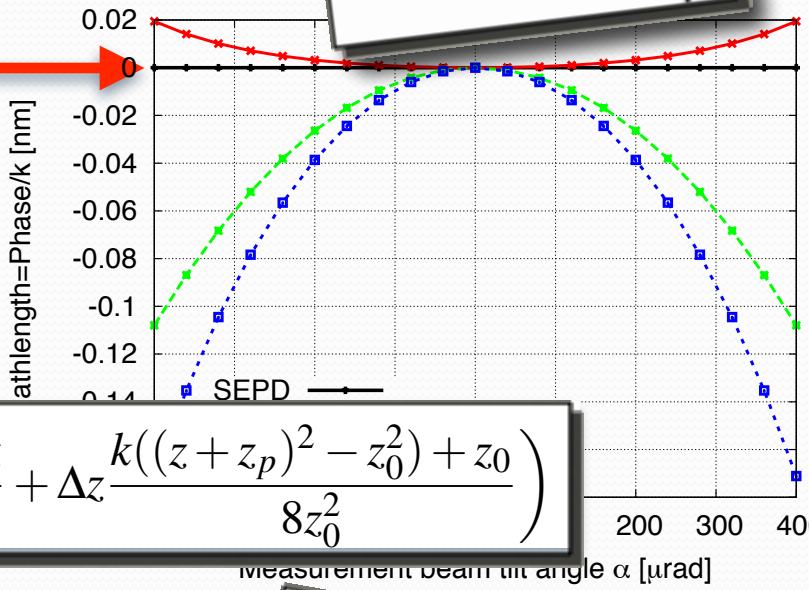

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$$\approx 10^{-7}$$

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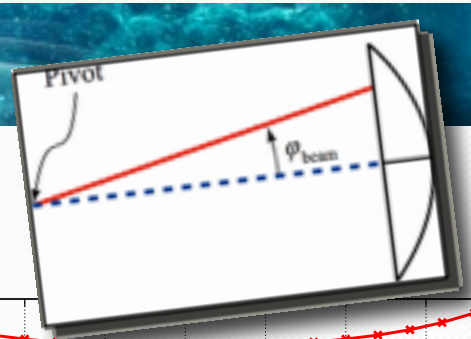

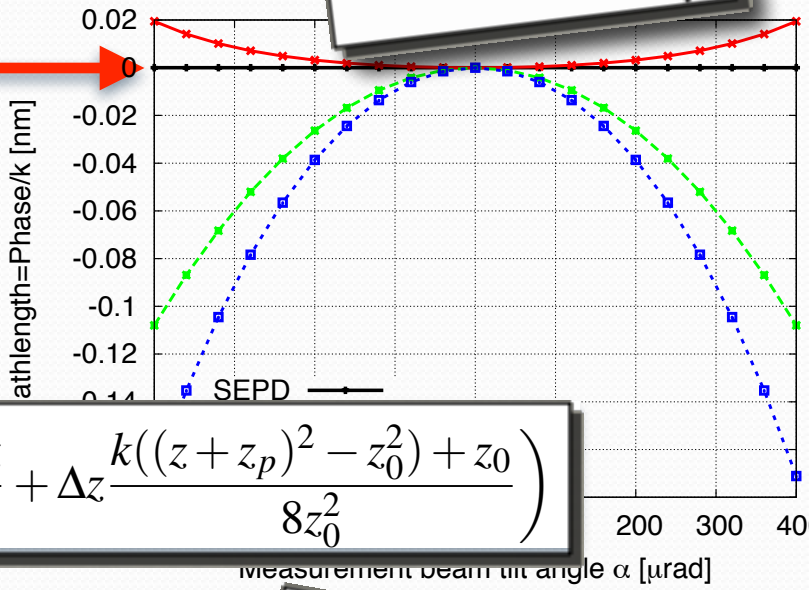

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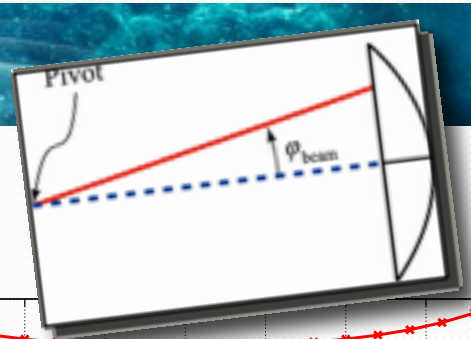

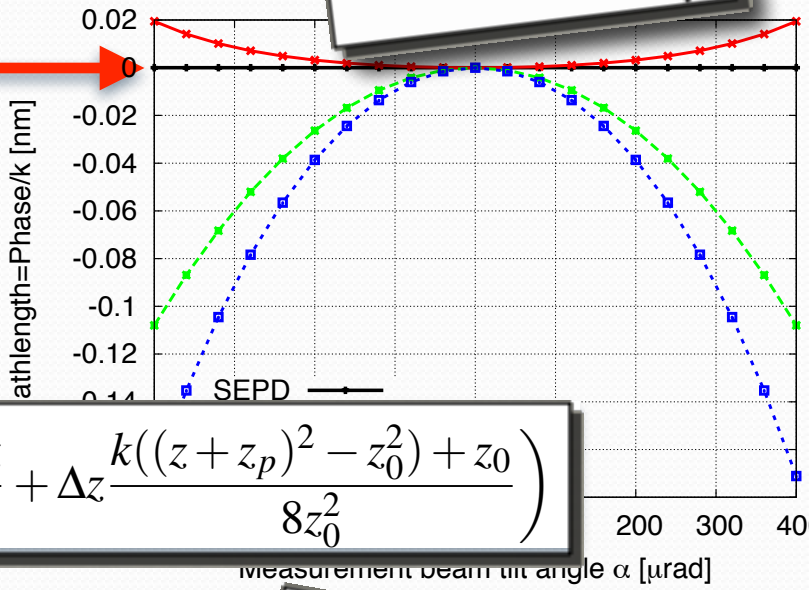

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$$\approx 10^{-14}$$

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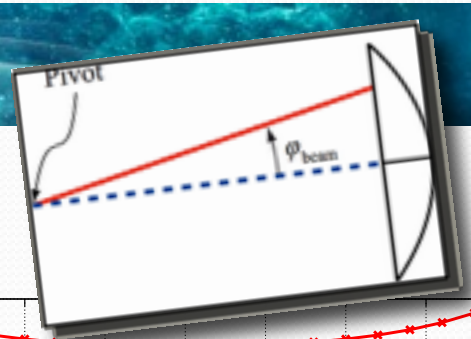

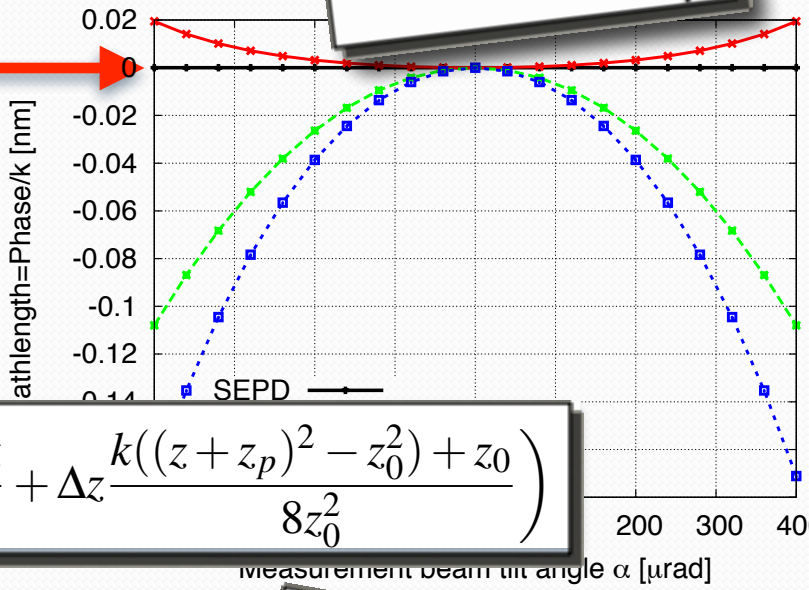

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≈ 1

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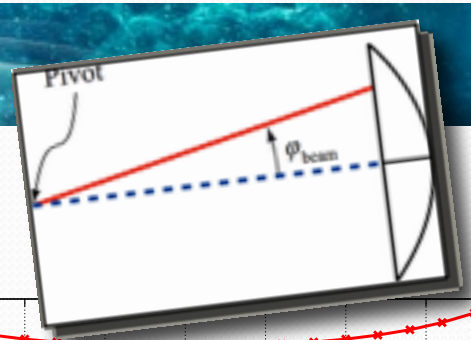

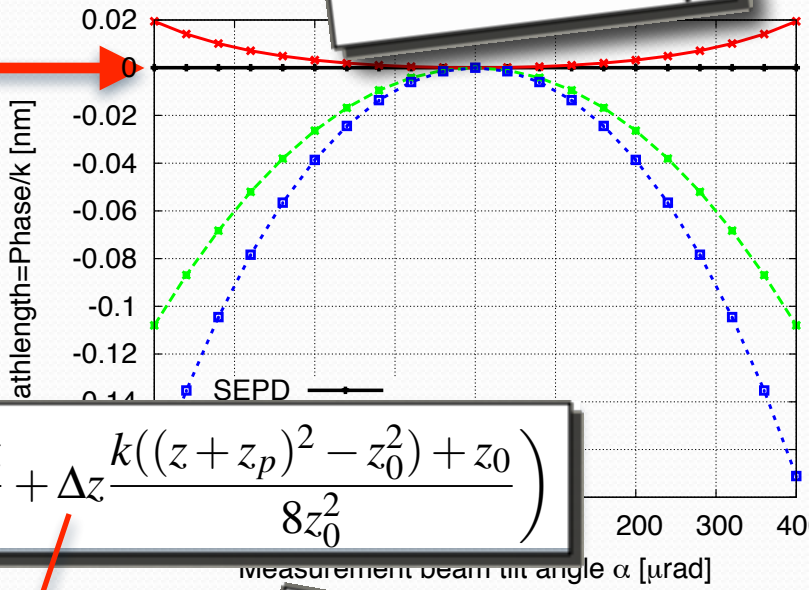

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$\approx 10^{-14}$

≈ 1

Rayleigh range mismatch

waist position mismatch

Analytical description of interference between two misaligned and mismatched complete Gaussian beams

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A typical application for laser interferometers is a precision measurement of length changes that results in interferometric phase shifts. Such phase changes are typically predicted numerically, due to the complexity of the overlap integral that needs to be solved. In this paper we will derive analytical representations of the interferometric phase and contrast (aka fringe visibility) for two beam interferometers, both homodyne and heterodyne. The fundamental Gaussian beams can be arbitrarily misaligned and mismatched to each other. A limitation of the analytical result is that both beams must be detected completely, which can experimentally be realized by a sufficiently large single-element photodetector.

OCIS codes: (120.3180) Interferometry; (000.3860) Mathematical methods in physics; (200.1130) Algebraic optical processing.
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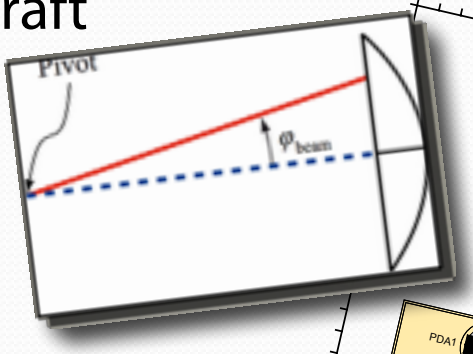
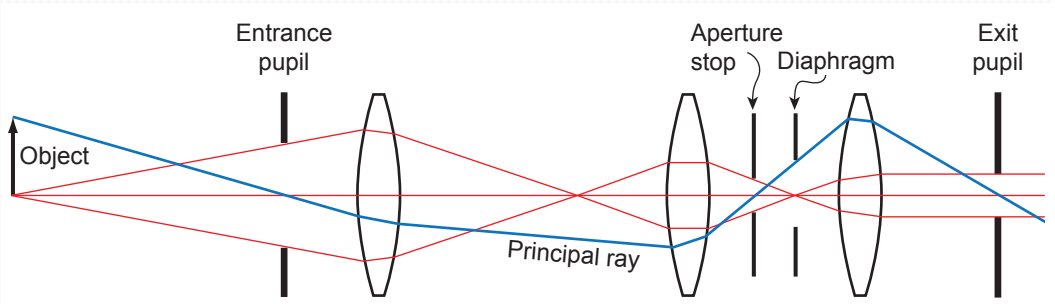
Tilt to Length Coupling

- Angular jitter of Test Mass or Spacecraft
  → beam walk on photodiode
  → pathlength noise

- LISA Pathfinder:
  noise reduction by subtraction

- (e)LISA:
  - subtraction AND
  - imaging of critical planes to PD planes





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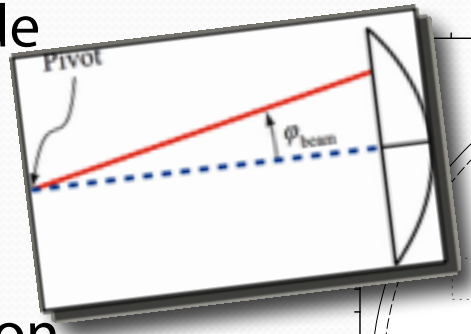

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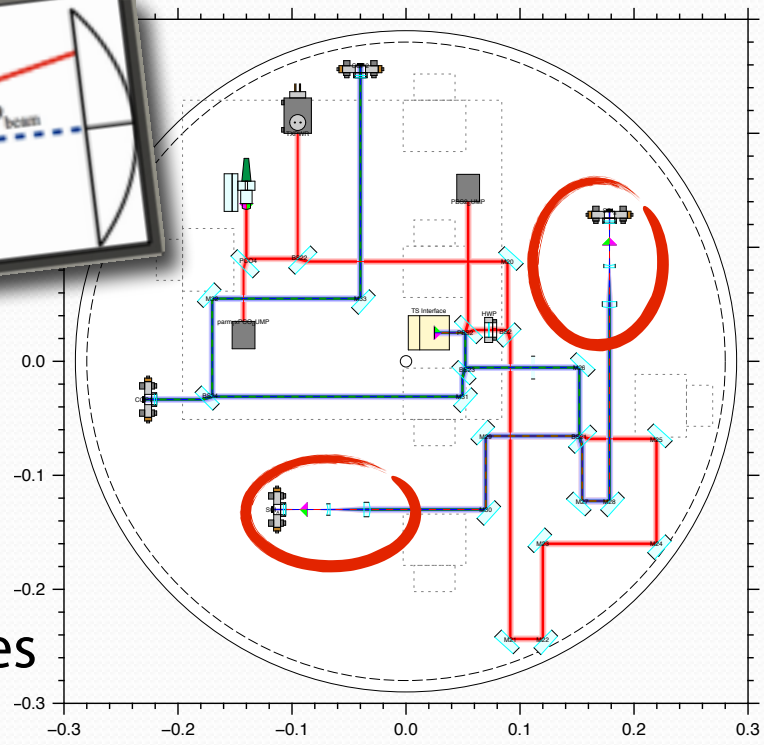

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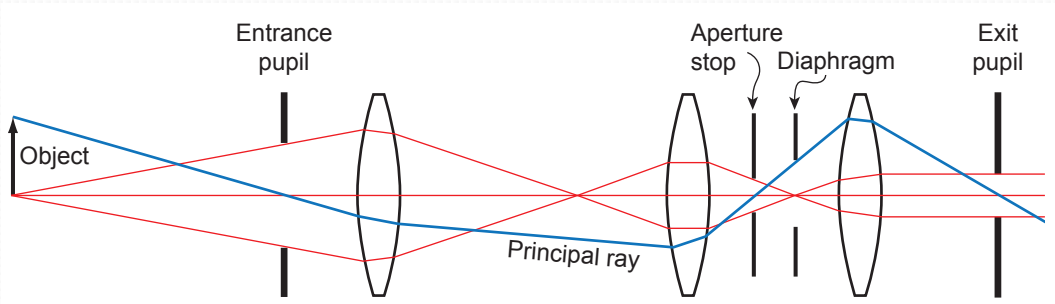
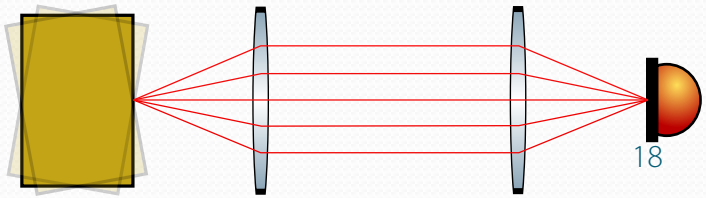
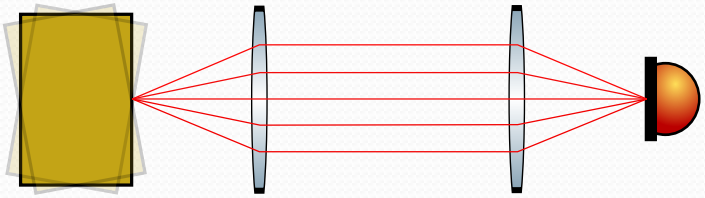


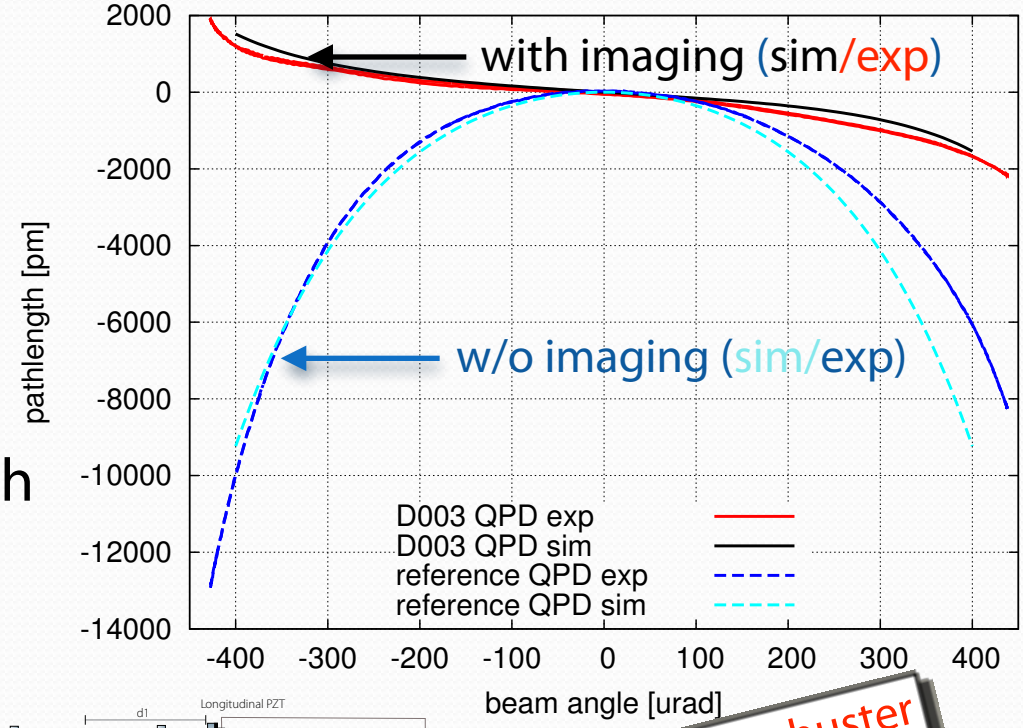


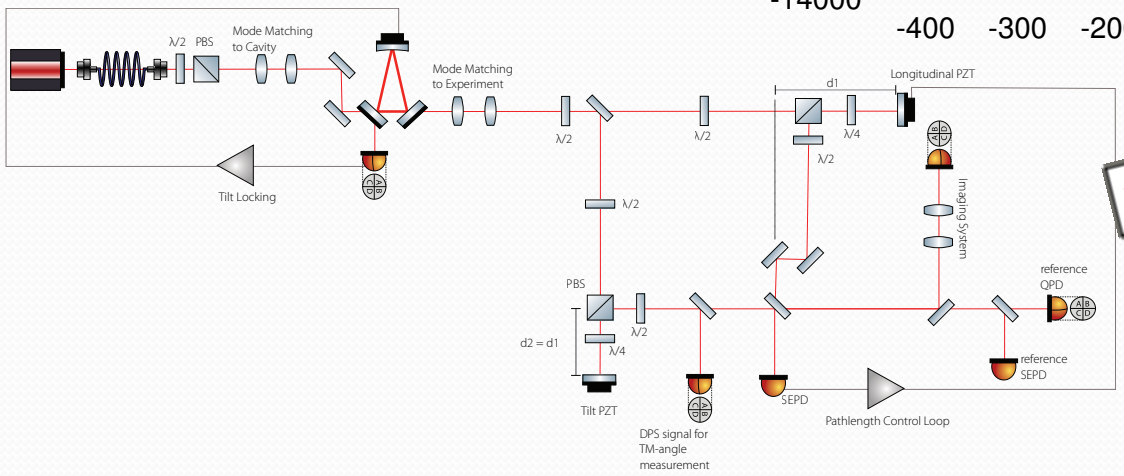
Tilt to Length Coupling Reduction

- Imaging → reduction of phase noise?



- issues:
  - alignment
  - beam parameter mismatch

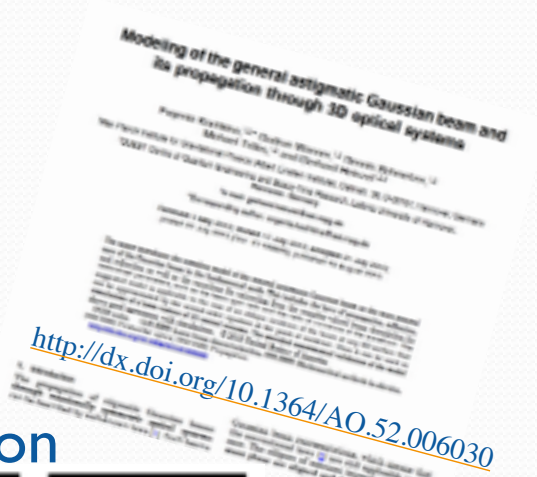




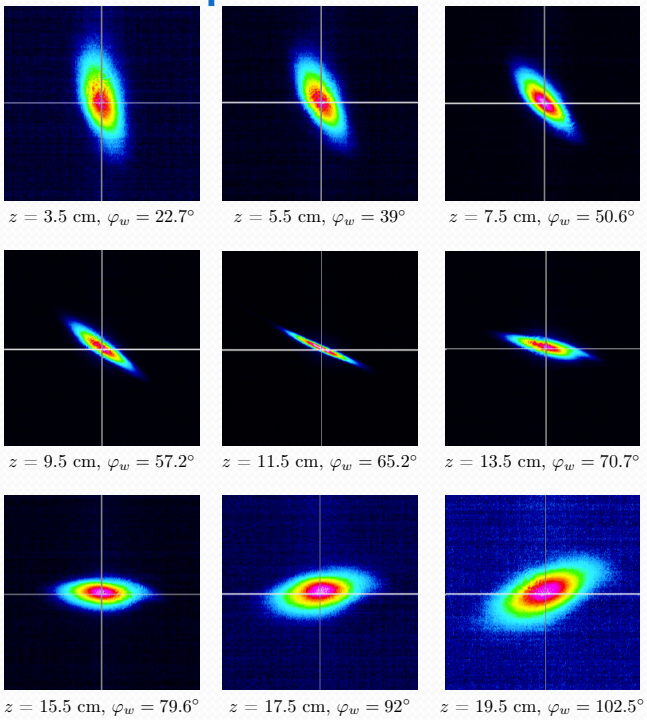
poster of Sönke Schuster

Beam propagation

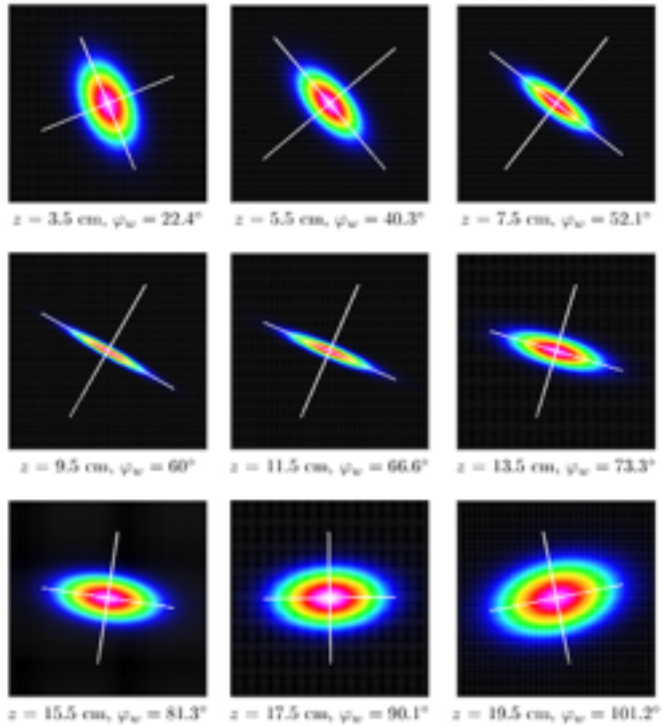
- Ray tracing
- **Gaussian beams**
  - q-parameter propagation
  - **general astigmatism** accounted for



experiment



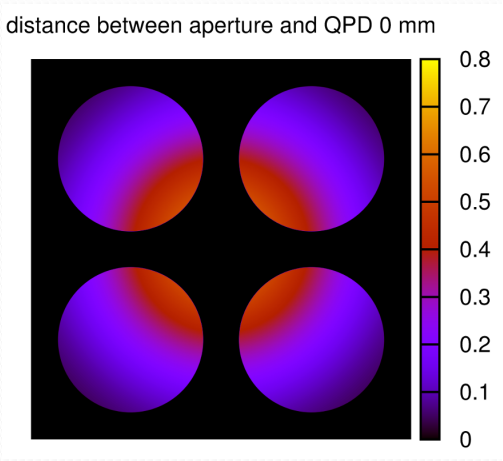
simulation

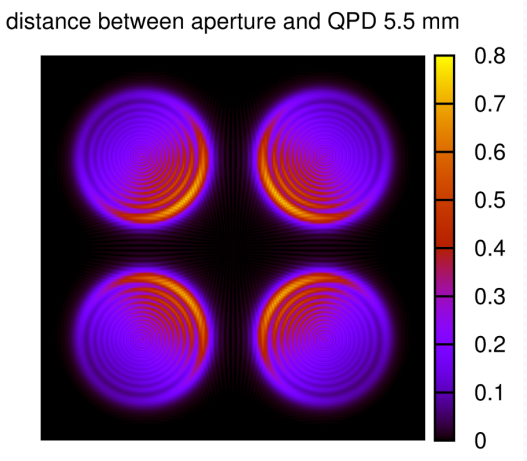


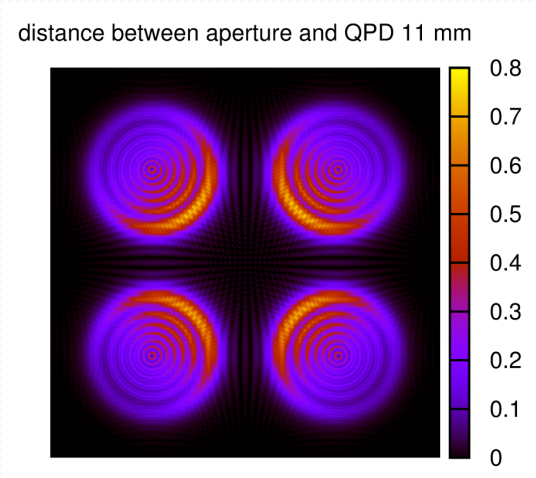
courtesy: Evgenia Kochkina

Beam propagation with defraction

- Non-Gaussian beams: **Top Hats, Fibre modes, …**
  - **FFT-propagation**



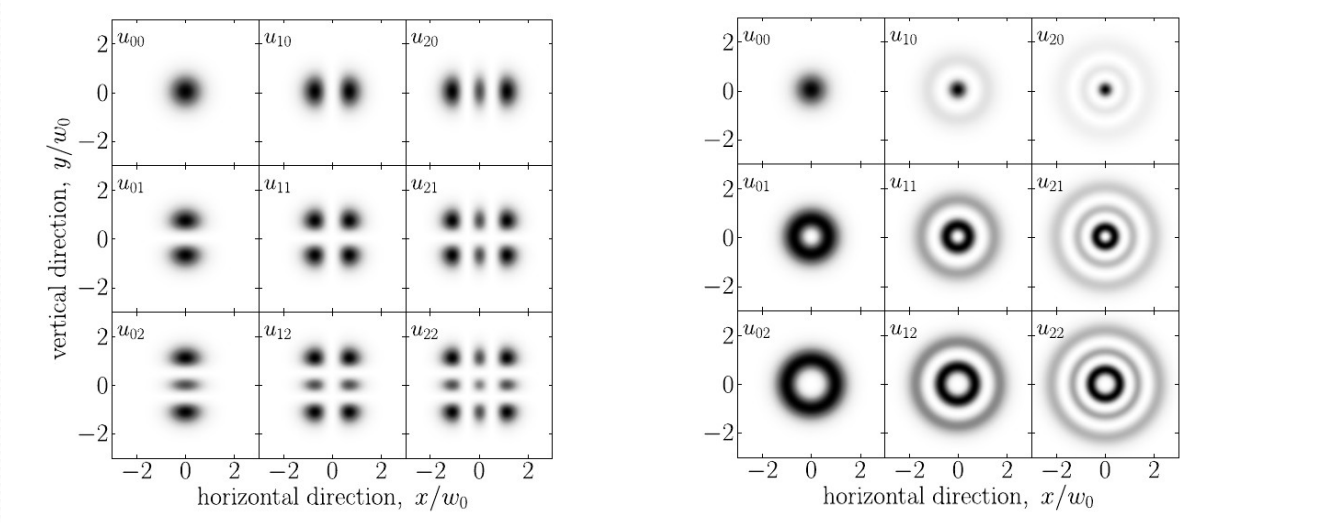




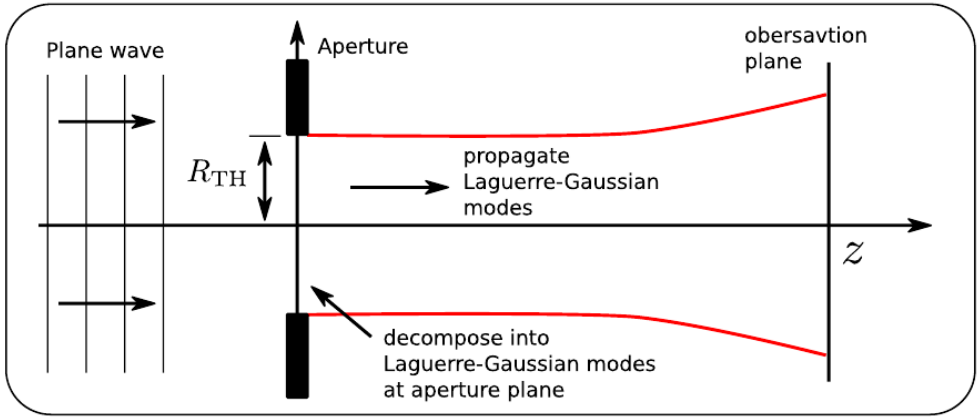
courtesy: Vitali Müller, Sönke Schuster

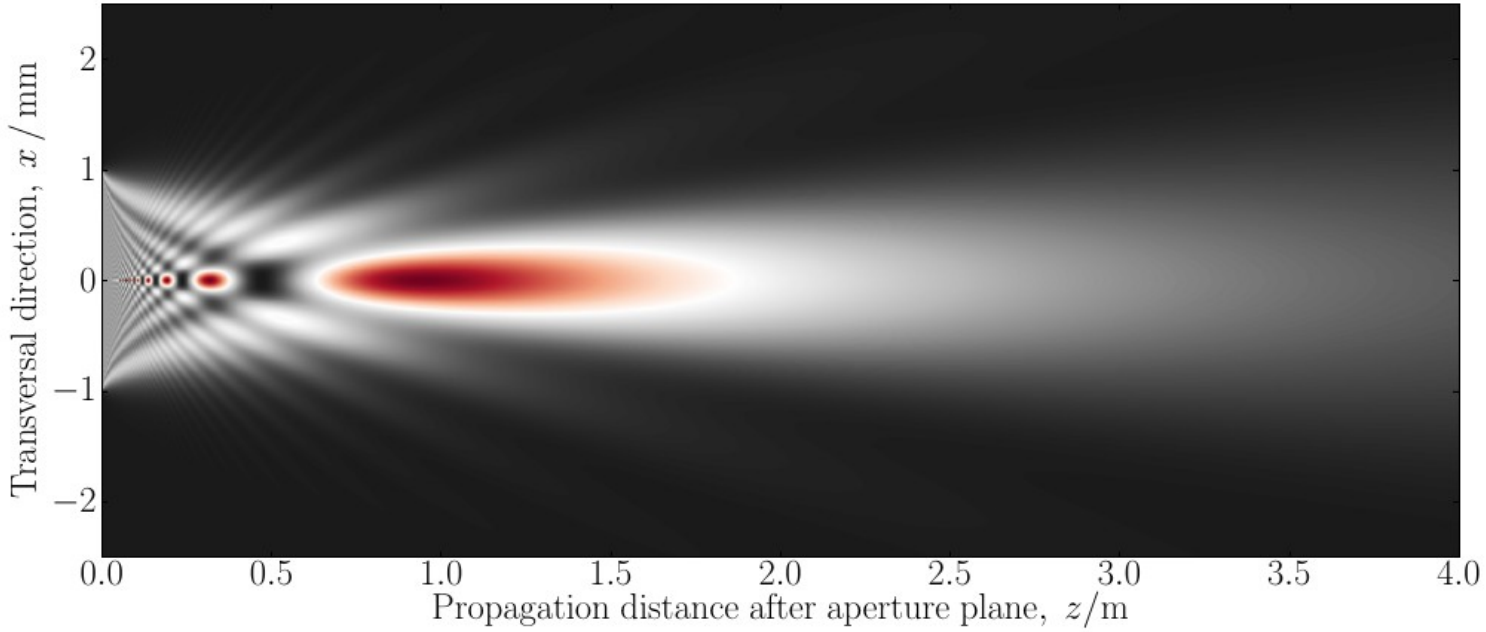
Beam propagation

- Non-Gaussian beams: **Top Hats**, **Fibre modes, …**
  - **FFT-propagation**
  - **Mode Expansion Method:**
    Expansion of fields in higher order
    Hermite-Gaussian or **Laguerre-Gaussian** modes



Top Hat beam

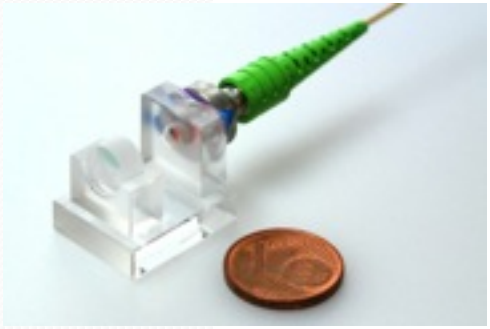


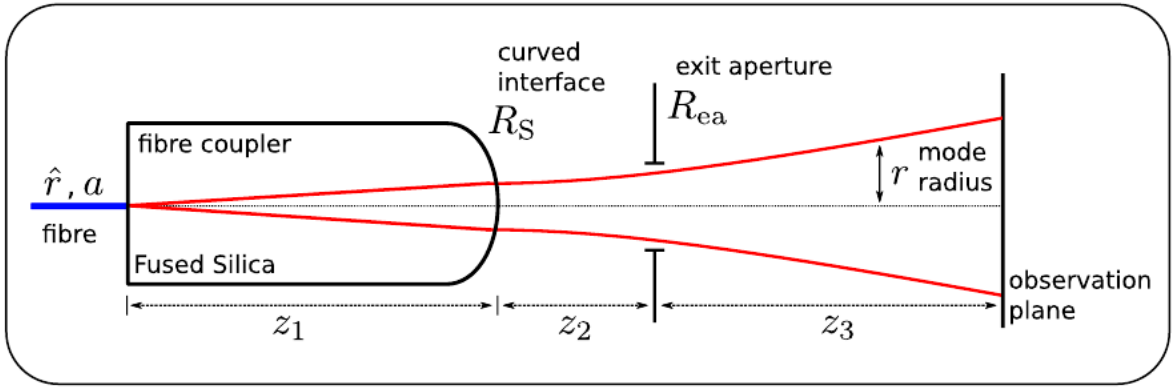


courtesy: Christoph Mahrdt

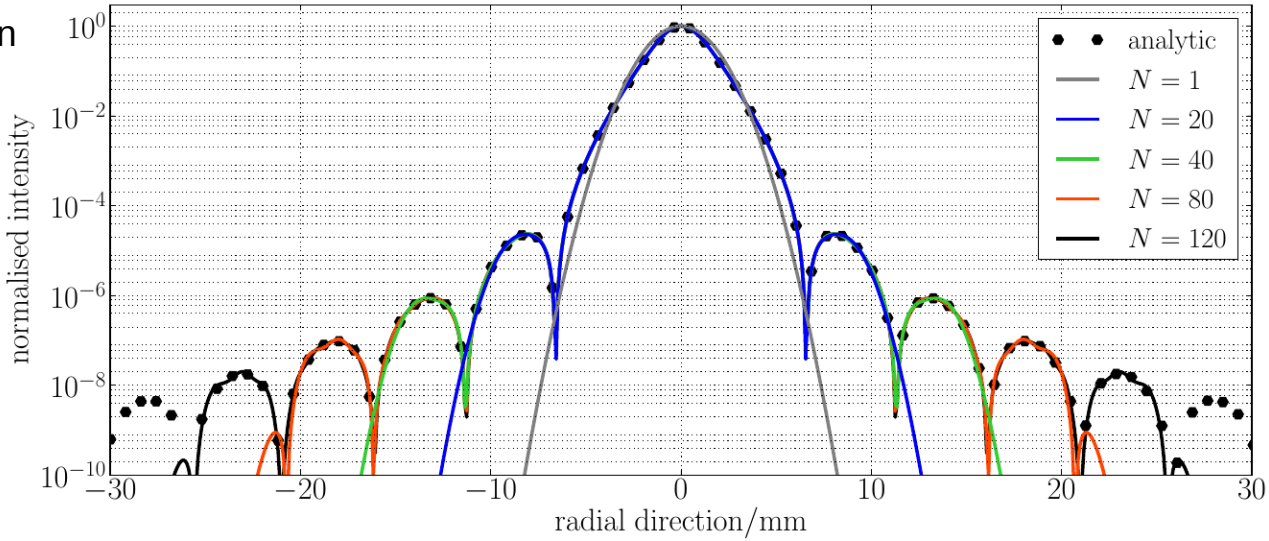
Fibre mode propagation

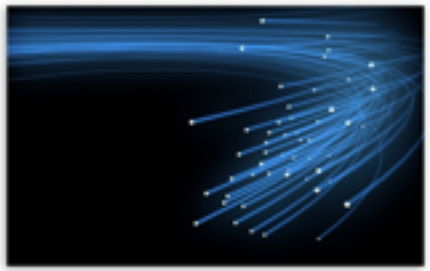
Propagation of Fibre Mode LP_{01}





Intensity pattern at the end-face of the curved Interface

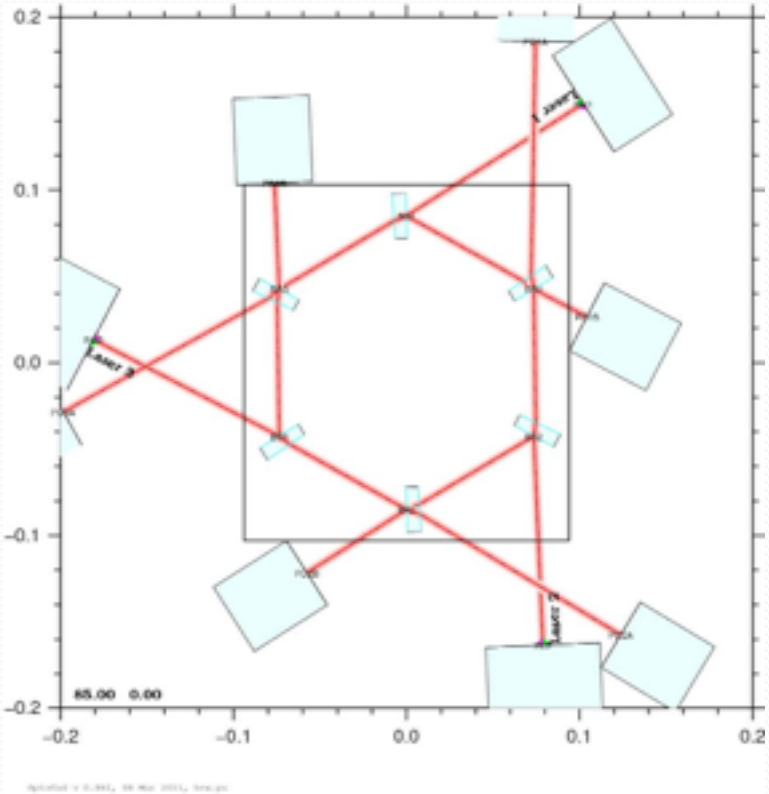




courtesy: Christoph Mahrdt

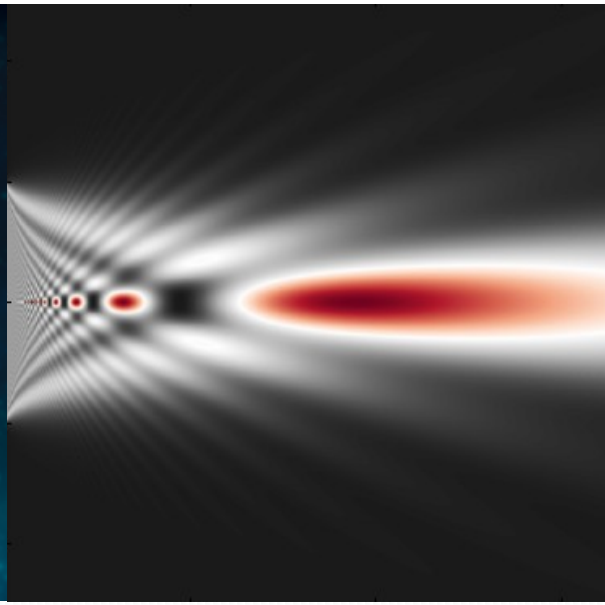
Optimizing interferometers

- Optimize
  - **optimal beam overlap** on beam combiners and photo diodes
  - **least clipping**
  - **equal arm lengths**
  - **low stray light**
  - **space**
  - …
- Example: Hexagon Interferometer
  - optimization with **wedged beam splitters**
  - **auto-alignment**



Thank You

for your Attention



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