Bayesian Statistics to calibrate the LISA Pathfinder experiment

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lisa pathfinder

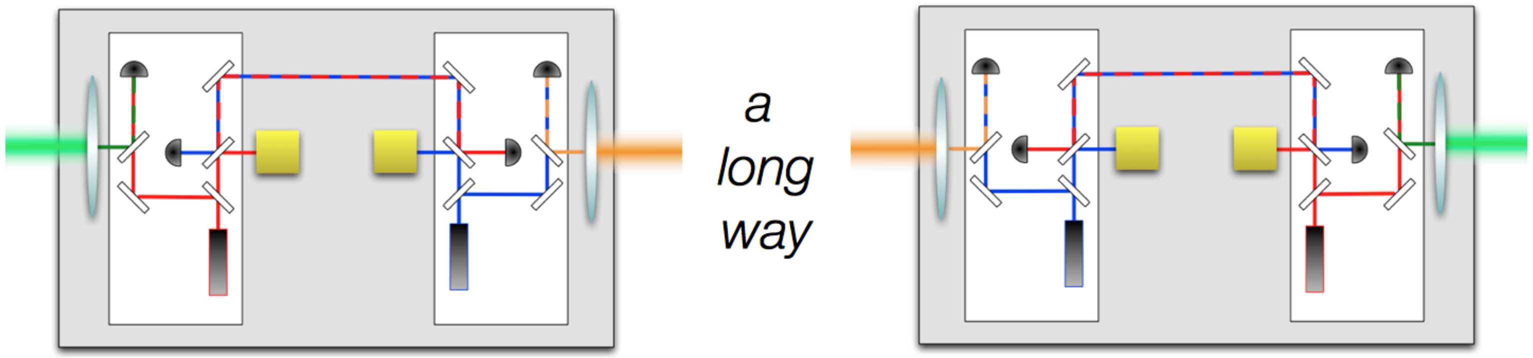
CSIC
Consejo Superior de Investigaciones Científicas

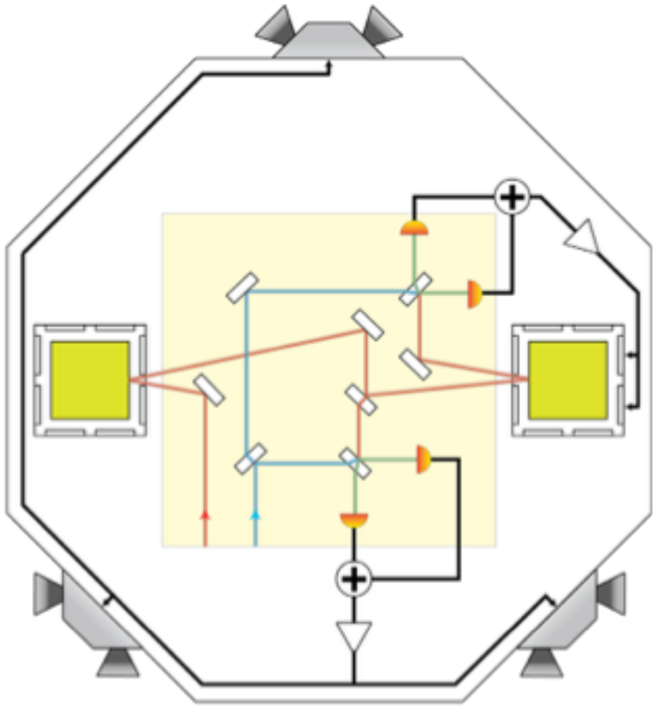
Outline

- LPF System Overview
- LPF System Identification Experiments
- Data Analysis framework
- Applications to Simulated Data
- The Pipeline Design for on-line analysis

eLISA to LISA Pathfinder

* Squeeze two eLISA SCs into one SC.

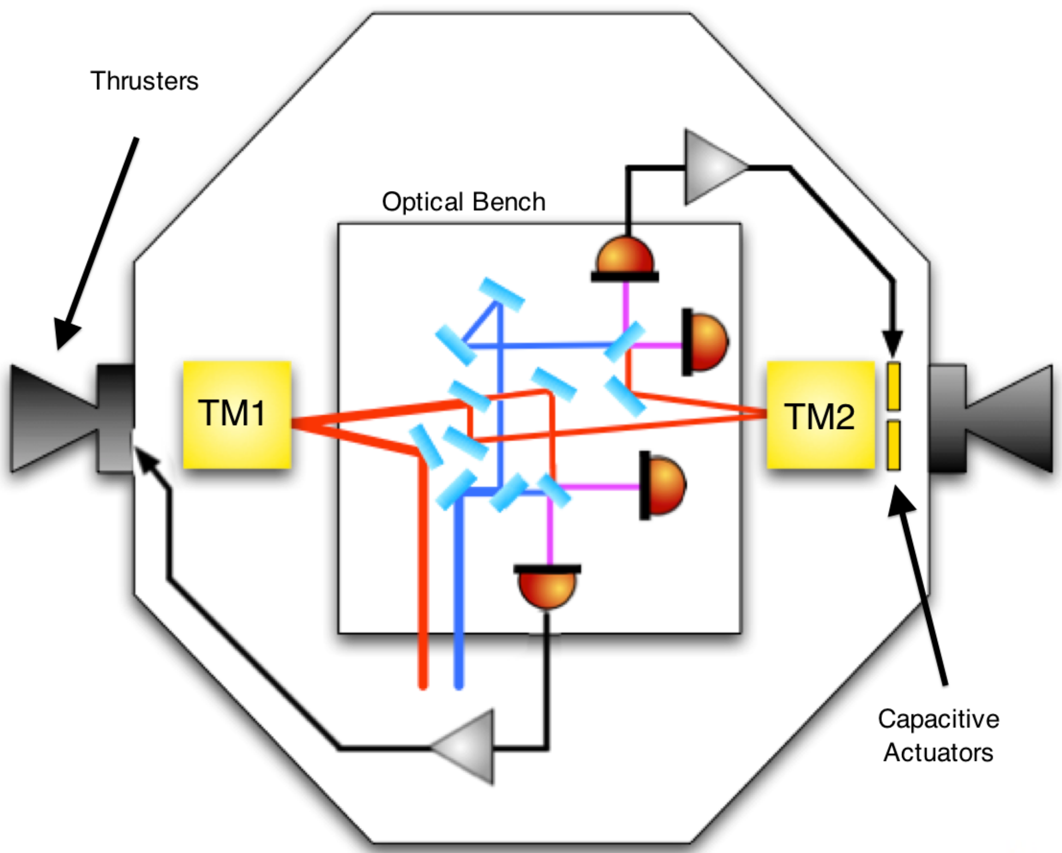




- Prove geodesic motion by monitoring the relative acceleration of the two test masses.

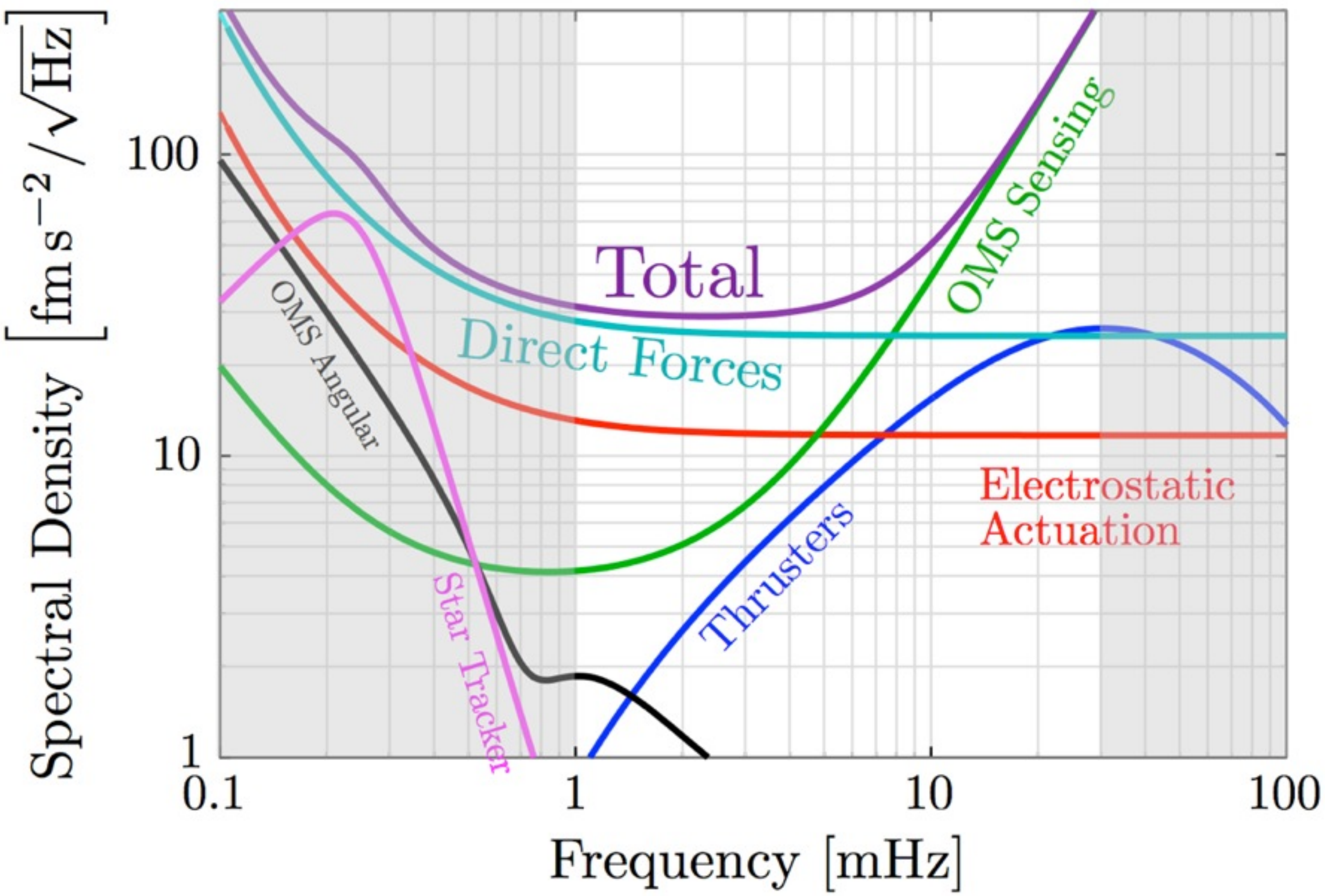
- Characterise all noise sources of the instrument, build accurate noise models.

- Test all key technologies for the future space-based gravitational-wave detectors.



> Science Mode: TM2 is following TM1 through capacitive actuators. (simplified 1D case)

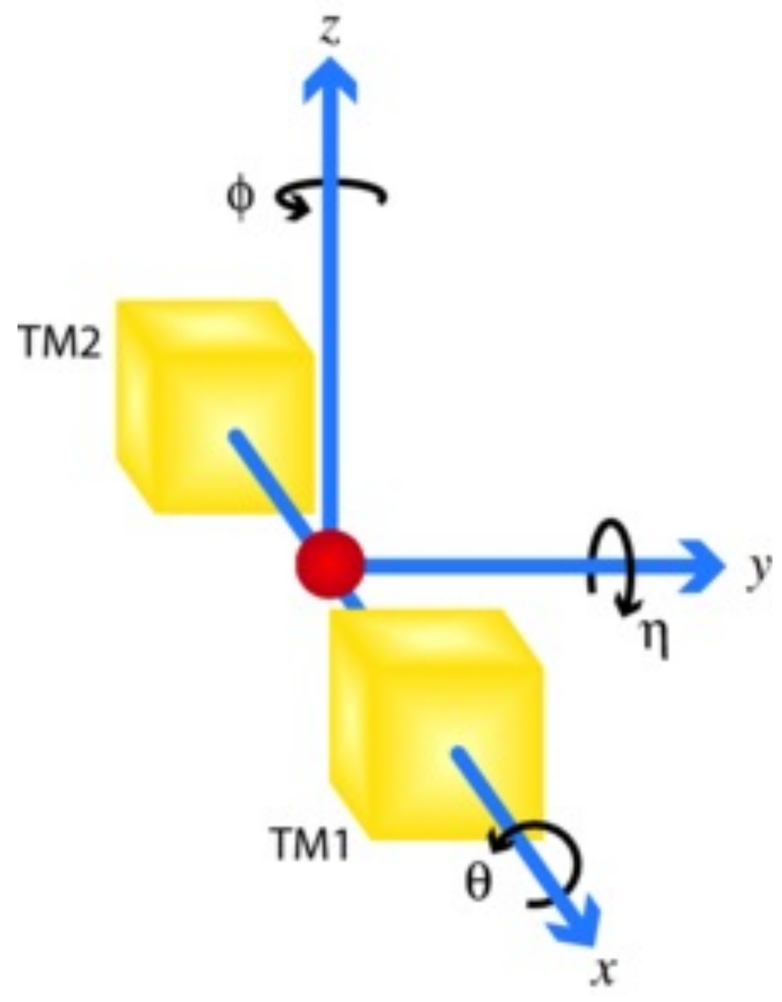
> The LPF noise budget*:



*Antonucci et al, CQG, 29-124014

System Identification

- "Kick" the system, measure the response, get the system parameters.
- Two main dynamics sys-id experiment families:
  - X-axis
  - cross-talk



Data Analysis & Parameter Estimation

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|                                                      |
|          Welcome to the LTPDA Toolbox                |
|                                                      |
|                 Version: 2.8.dev                     |
|                 Release: (R2013a)                    |
|                    Date: 30-04-14                    |
|                                                      |
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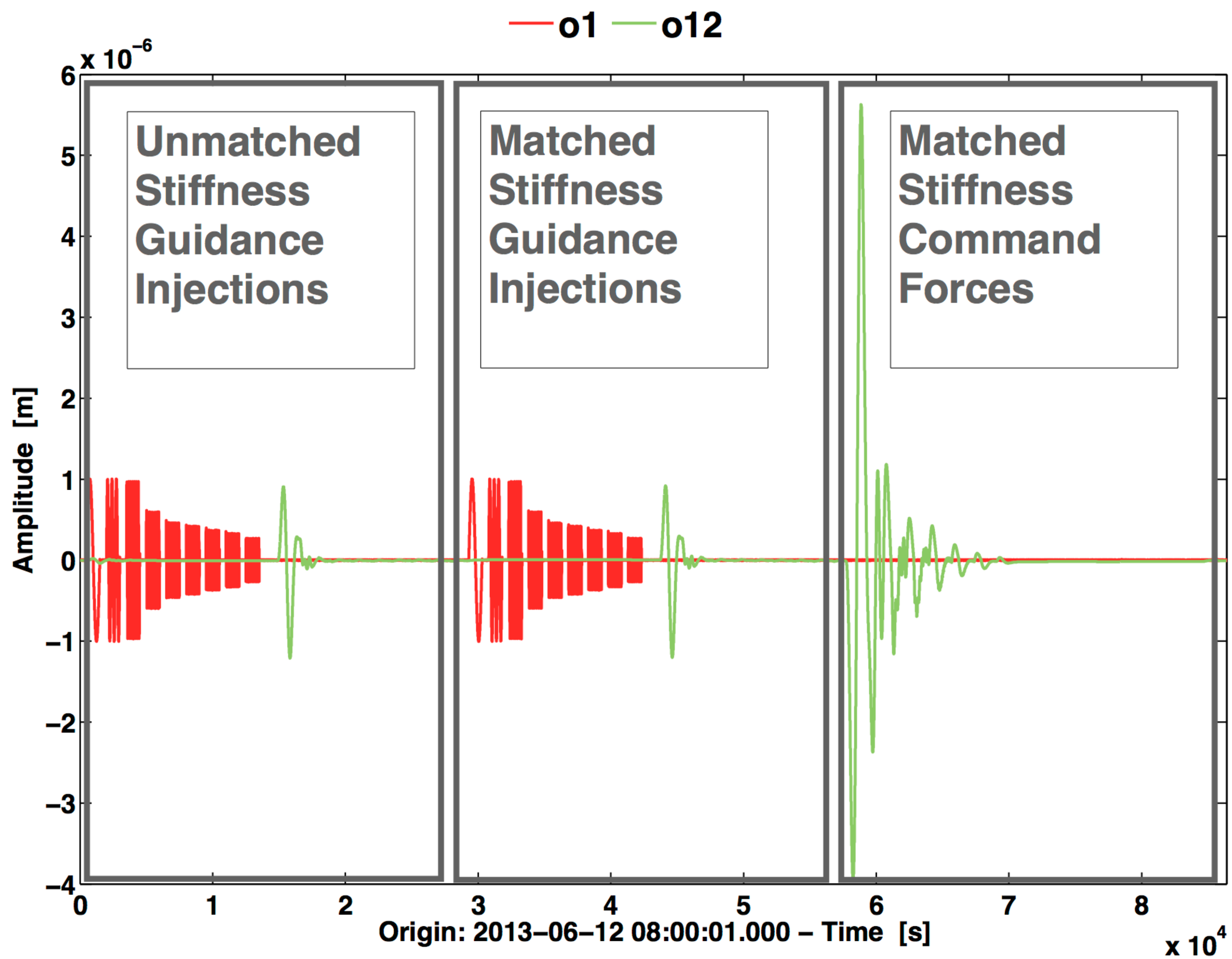
- LTPDA toolbox!
- It's there:

  http://www.lisa.aei-hannover.de/ltpda/

- All the data analysis tools presented here are available in the toolbox with the proper documentation!

Sensitive x-axis system identification

- Command along the “sensitive” x-axis between the two test-masses
- Large signal-to-noise ratio, satisfactory recovery of the parameters.
- Three experiments:
  1. “fake displacement”, unmatched stiffness.
  2. “fake displacement”, matched stiffness.
  3. Out-of-loop forces injections to the three bodies of the system.



Data Analysis & Parameter Estimation

- For the parameter estimation, the standard approach:

A. Assume that $\vec{\mathbf{d}} = \vec{\mathbf{h}} + \vec{\mathbf{n}}$

B. then $\pi(\vec{\mathbf{d}}|\vec{\theta}) = C \times exp[-1/2 \times < \vec{\mathbf{d}} - \vec{\mathbf{h}}(\vec{\theta})|\vec{\mathbf{d}} - \vec{\mathbf{h}}(\vec{\theta}) >]$

where, $< \vec{a}|\vec{b} >= 2 \int_0^\infty \left[\tilde{a}^*(f)\tilde{b}(f) + \tilde{a}(f)\tilde{b}^*(f)\right] / S_n(f)$

and $\chi^2 \equiv < \vec{\mathbf{d}} - \vec{\mathbf{h}}(\vec{\theta})|\vec{\mathbf{d}} - \vec{\mathbf{h}}(\vec{\theta}) >$

C. Perform the fit using MCMC* methods.

D. Also use linear•, or non-linear† methods.

* PRD82, 122002, (2010), •CQG, 28 094006 (2011), †PRD85, 122004, (2012)

System identification along the x-axis

- Perform the fit in the “acceleration” domain.
- The model now, looks like:

$$\Delta a = \sum_{j=1}^{N_g} \Delta g_j(\vec{\theta}) + \Delta g_{noise}$$

- And in particular, for the differential acceleration (x-axis):

$$\Delta a = \left[\frac{d^2}{dt^2} + \omega_2^2\right] x_{12}(t-\tau) + \left(\omega_2^2 - \omega_1^2\right) x_1(t-\tau) - AF_{cmd,TM2}$$

System identification along the x-axis: -iterative chi^2 method

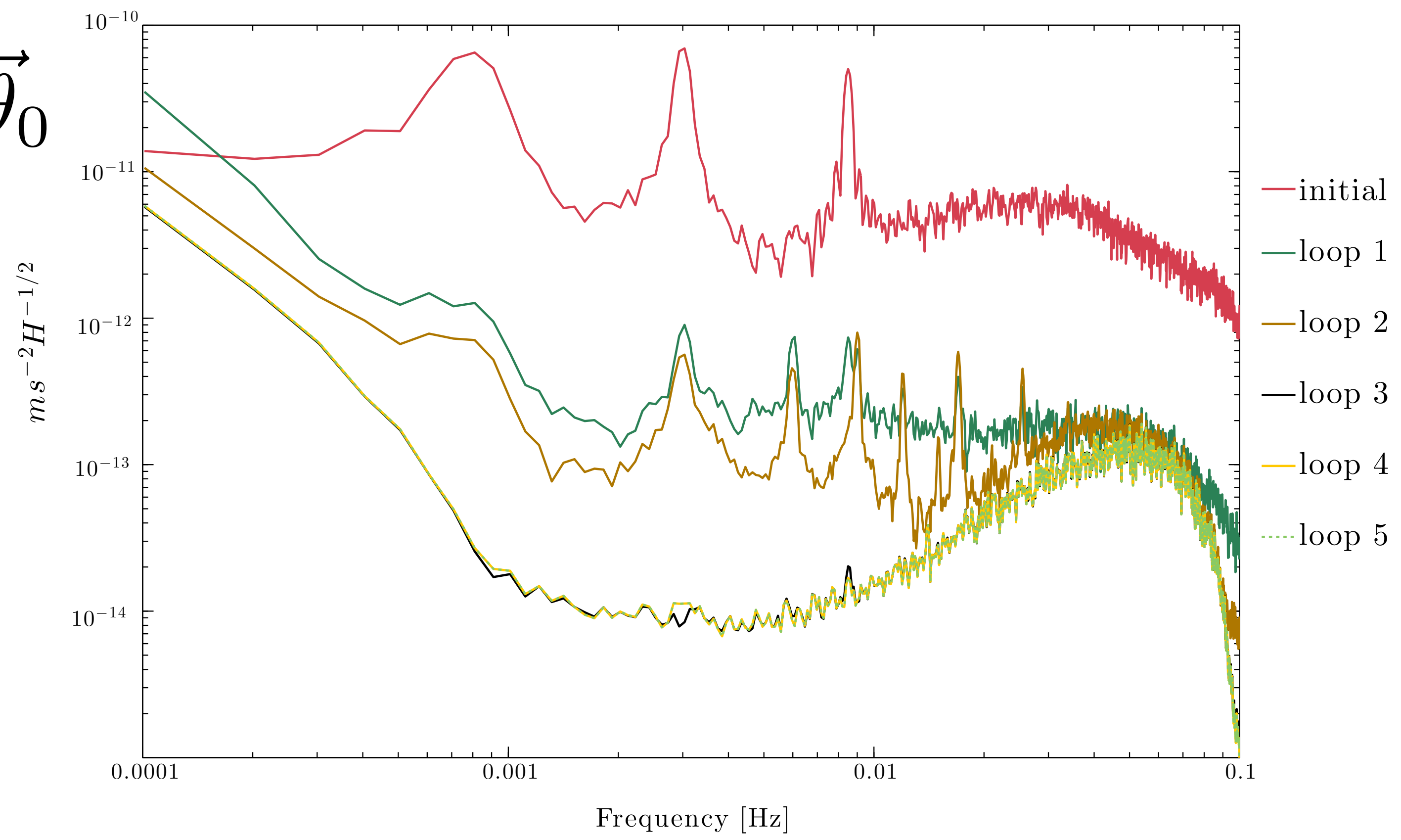
- Given this equation we can now minimise the log-likelihood by following the following recipes:

A. Iterative chi2 minimisation:

1. define initial set of parameters $\vec{\theta}_0$
2. estimate $S_n(\vec{\theta})$
3. minimise the log-likelihood

$$log(\pi(\vec{\mathbf{d}}|\vec{\theta})) = -\chi^2/2$$

4. get an estimate of $\vec{\theta}_{new}$
5. set $\vec{\theta}_0 = \vec{\theta}_{new}$, repeat from 2, until equilibrium.

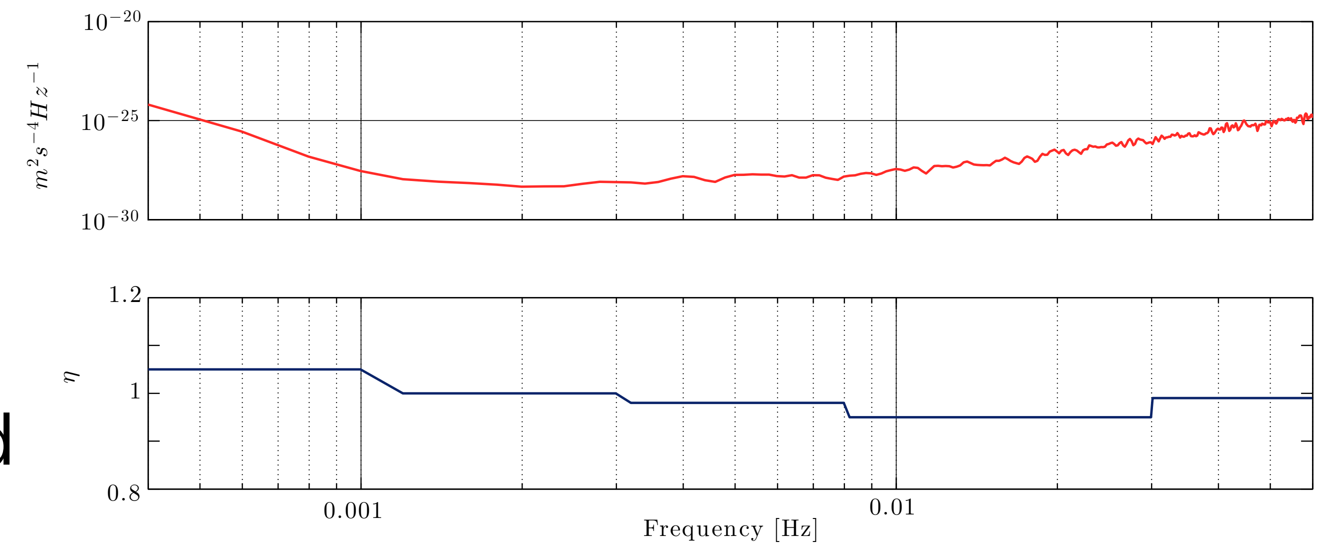


System identification along the x-axis:
- By modelling the noise

B. Assume that the noise can be written as*

$$S_{n,i} \rightarrow \eta_j S_{n,i}, \quad i_j < i \leq i_{j+1}$$

$$i \rightarrow \ bin, \ j \rightarrow segment$$

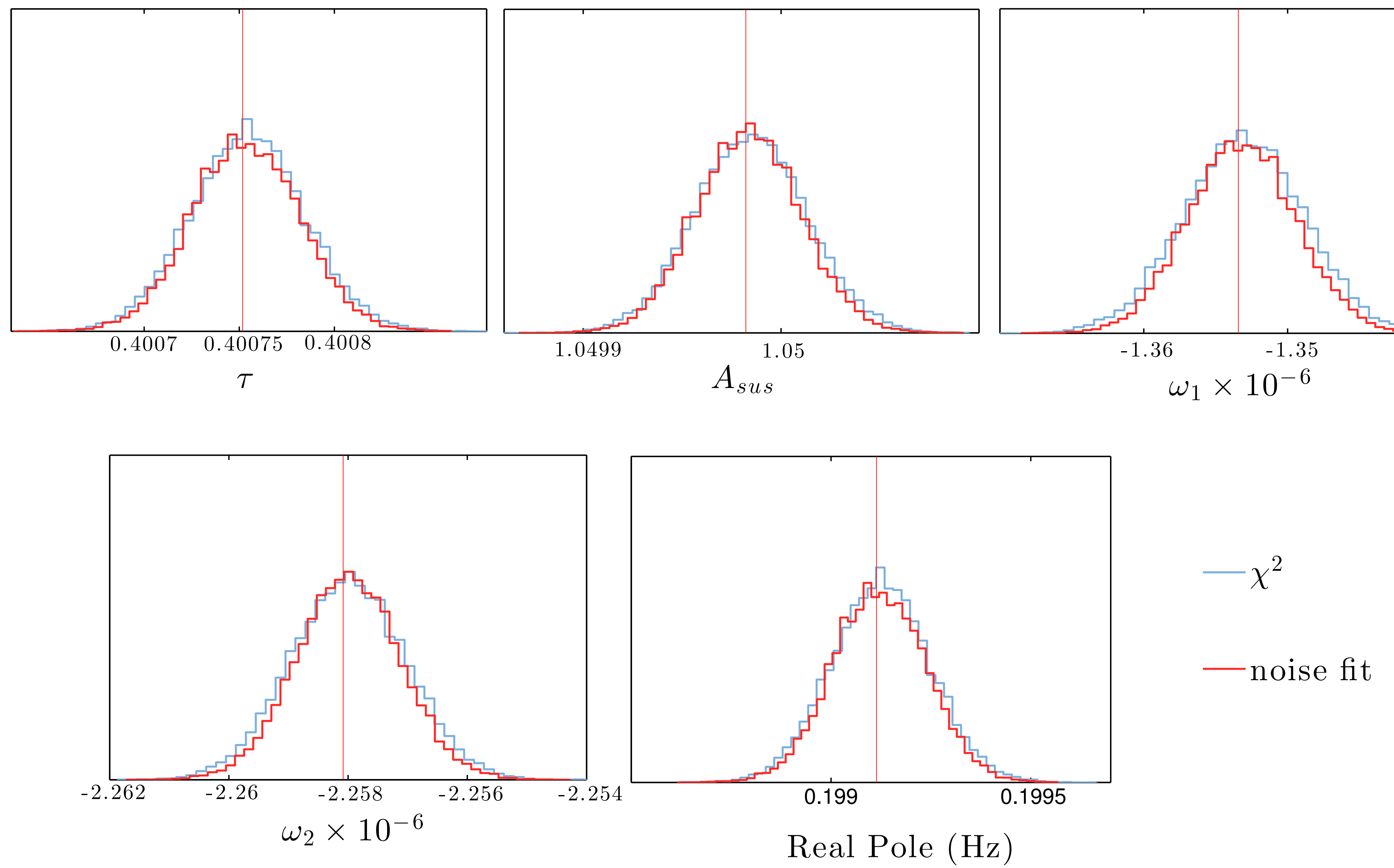


1. then define the log-likelihood function as

$$log(\pi(\vec{\mathbf{d}}|\theta)) = -1/2 \left(\chi^2 + \sum_j N_j log(\eta_j) \right) + C$$

2. assign priors, sample the posterior.

* Littenberg et al, PRD80, 063007, 2009



comparison of the iterative chi^2 and the noise modelled log-likelihood resulting parameter estimates.

System identification along the x-axis:
- Assuming unknown and unmodeled noise

C. Assume that all noise sources zero-mean and Gaussian. Also taking into account the spectral window properties, one can marginalise over the noise parameters†.

- the log-likelihood then turns into

$$log(\pi(\vec{\theta}|\vec{\mathbf{d}})) = -N_s \sum_{k\in Q} \log\left(\overline{\left|\tilde{n}\left[k,\vec{\theta}\right]\right|^2}\right)$$

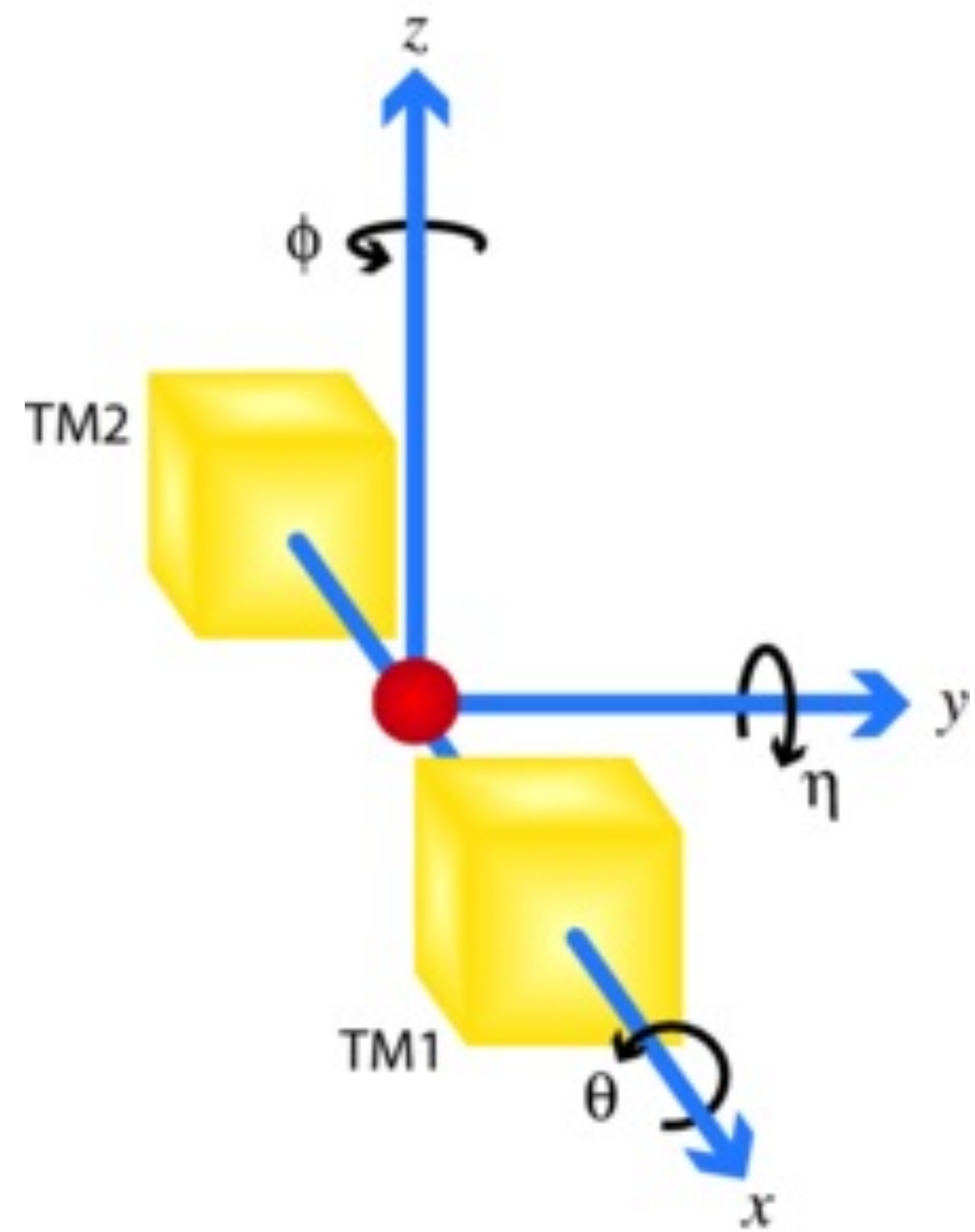
where, Q is the set of DFT coefficients and $\tilde{n}$ the residuals.

*** See next talk from D. Vetrungo, for a more detailed explanation!

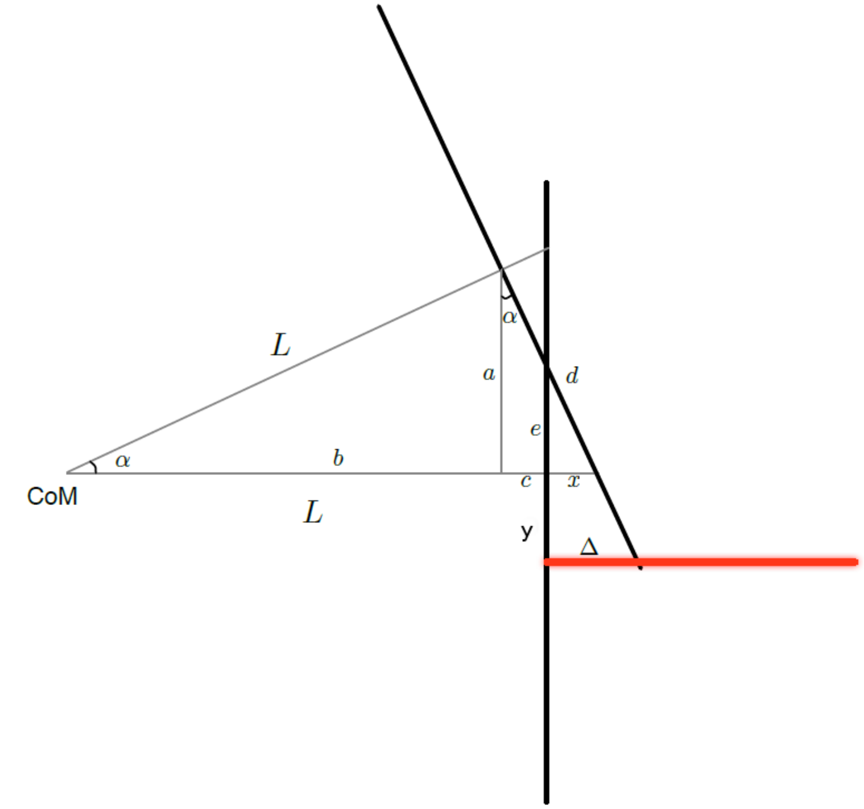
† Vitale et al, arXiv:1404.4792, submitted to PRD

Cross-talk system identification

- Command forces and torques in different degrees of freedom (ϕ1, ϕ2, y1, y2, Φ).
- measure with the sensitive differential channel (o12).
- estimate cross-talk/cross-coupling coefficients.
- Lower resulting SNR.



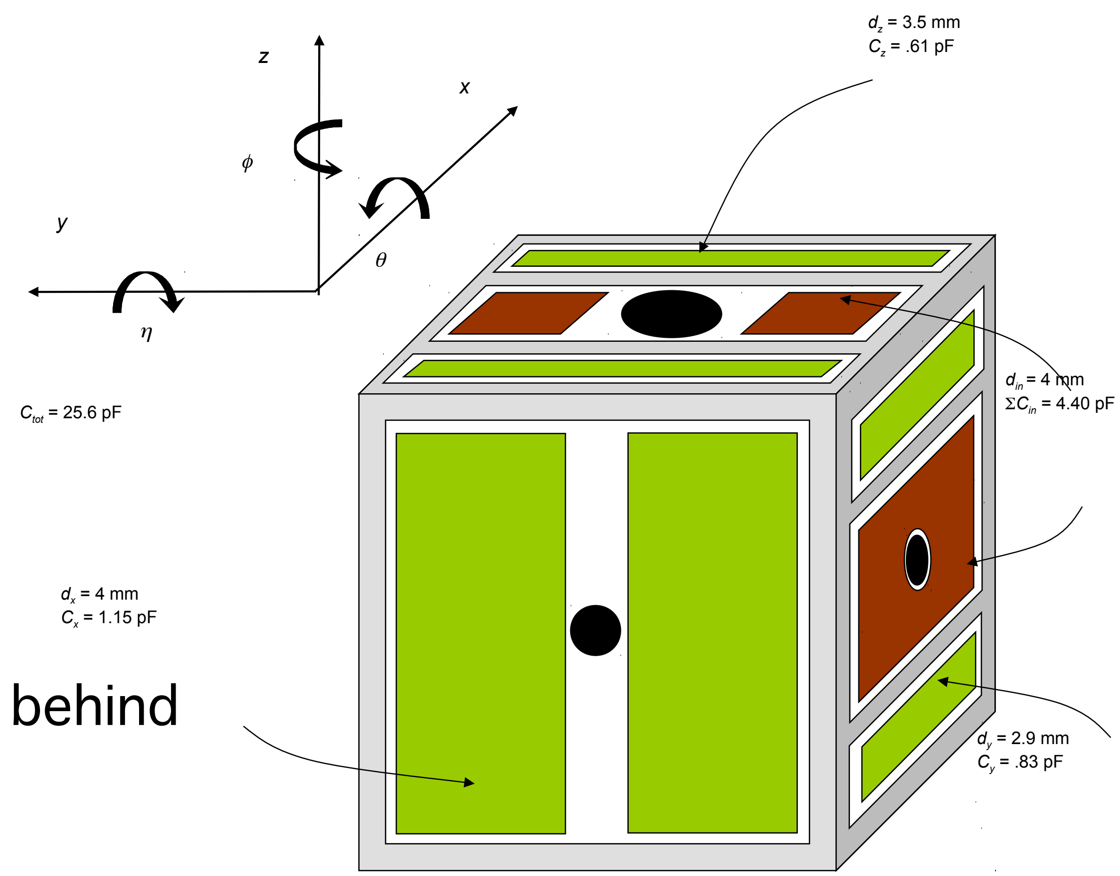
Cross-talk System Identification



- The parameters to estimate in this case are:

    1. system parameters (gains, delays)

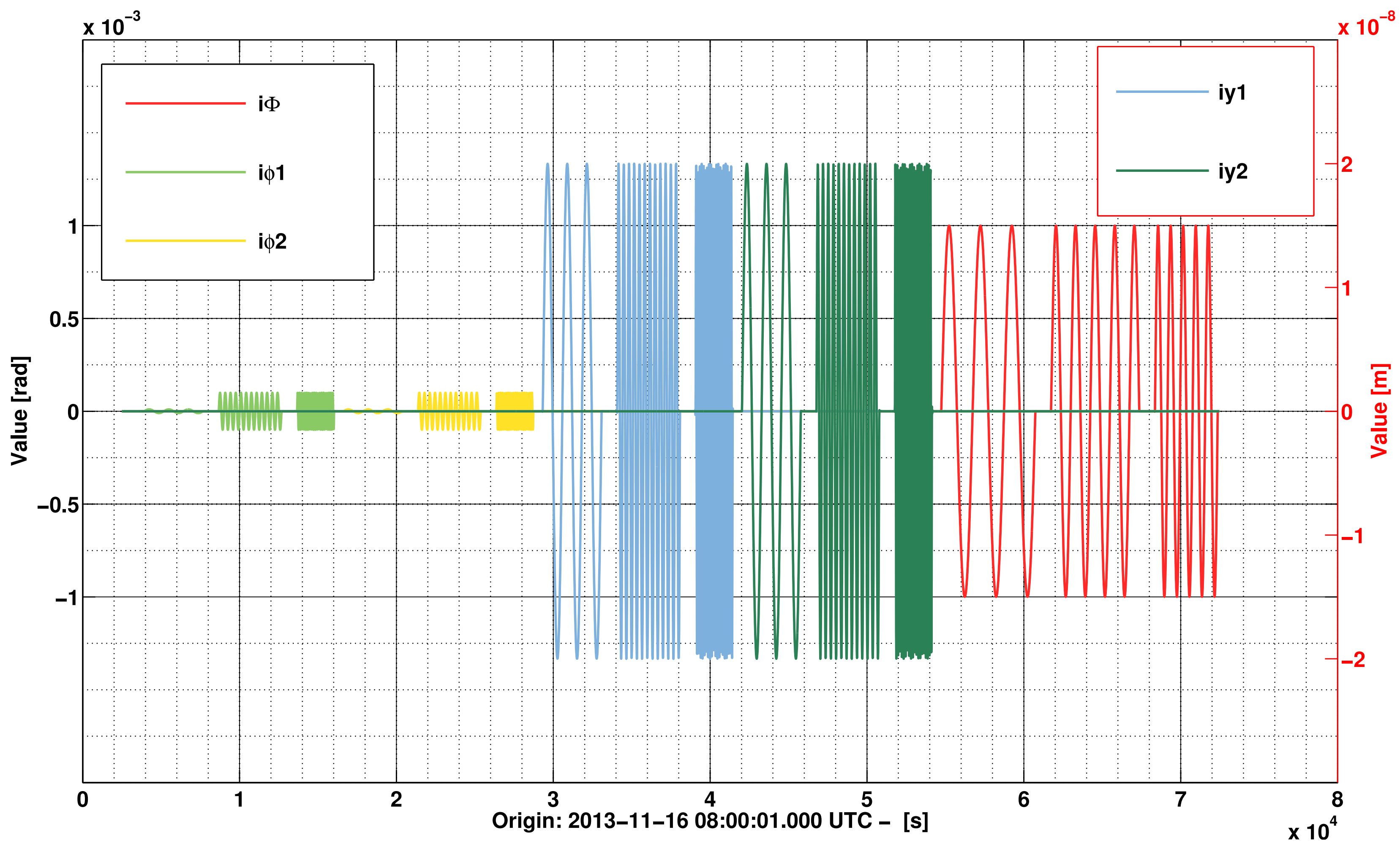
    2. cross-talk terms (piston effects, mechanical imperfections, cross-stiffness, secondary effects)



* See also, next talk by D. Vetrungo for the physics behind

Cross-talk System Identification

- Analysis of each experiment separately, or
- define a model that describes all the cross-coupling terms for all the cross-talk experiments

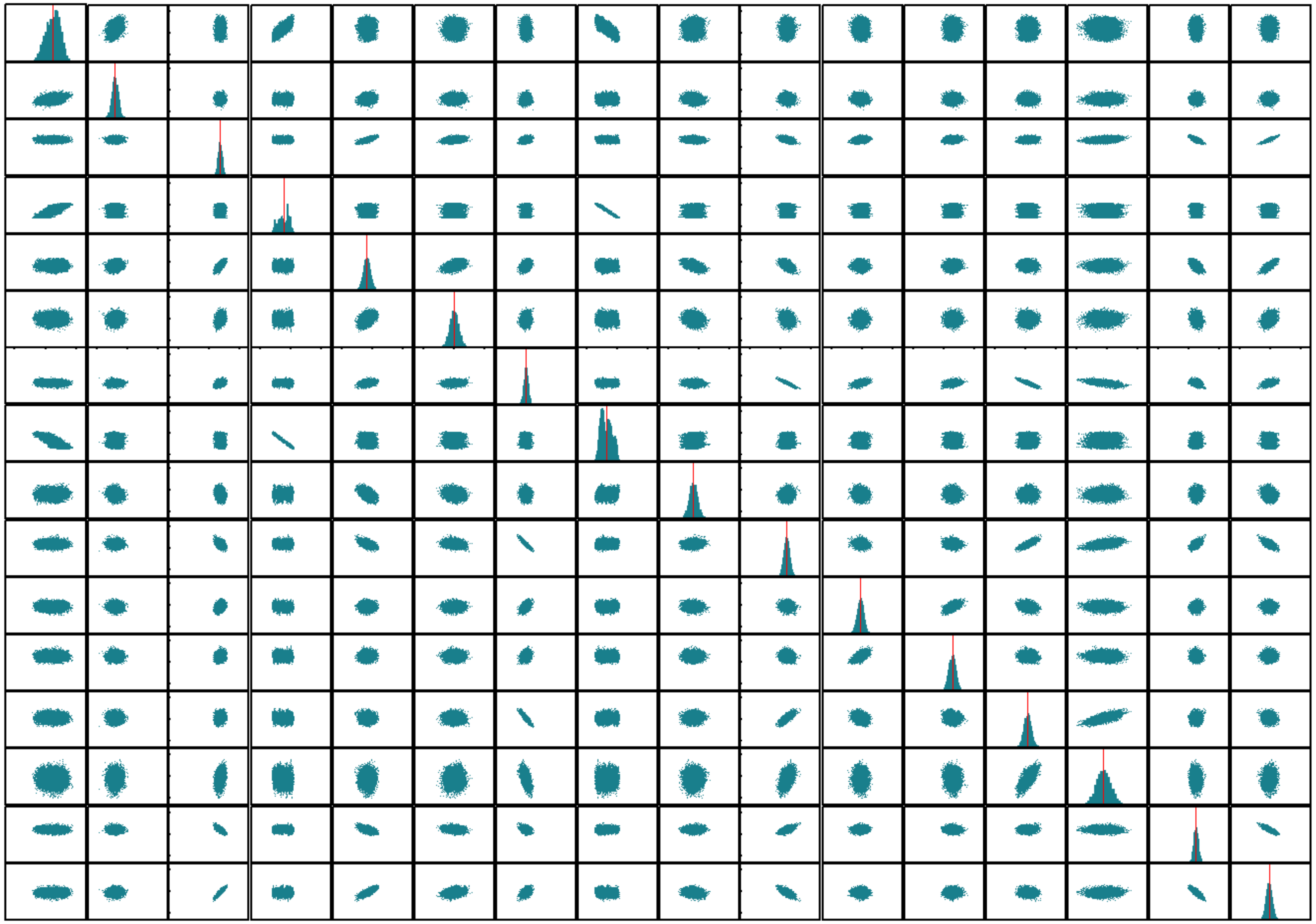


Cross-talk System Identification

- Analyse each experiment separately: good understanding of physics for each injection case.
- Analyse the joint experiments: verify that all cross-talk terms are included in the dynamics model → subtract total produced acceleration and reach the noise level.
- An example of the joint analysis model could be:

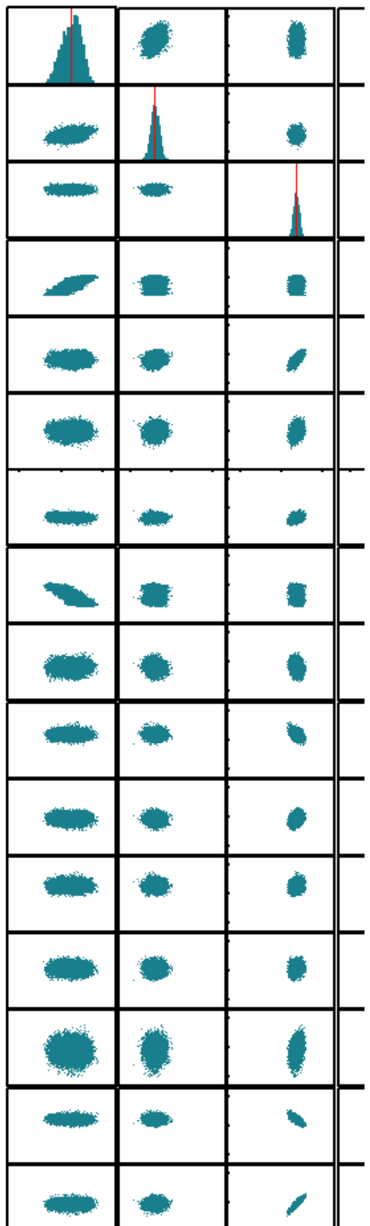
$$\begin{aligned}
a_{12,ct} = &-\delta_{\ddot{\phi}_1}\ddot{\phi}_1 - \delta_{\phi_1}\phi_1 + \delta_{\ddot{\phi}_1^2}\ddot{\phi}_1^2 \\
&+ \delta_{\Delta N\phi}\big(N_{cmd,\phi_1}(t-\tau) - N_{cmd,\phi_2}(t-\tau)\big) \\
&- \delta_{\ddot{\phi}_2}\ddot{\phi}_2 - \delta_{\phi_2}\phi_2 + \delta_{\ddot{\phi}_2^2}\ddot{\phi}_2^2 \\
&- \delta_{\ddot{y}_1}\ddot{y}_1 - \delta_{y_1}y_1 - \delta_{\ddot{y}_2}\ddot{y}_2 - \delta_{y_2}y_2. \\
&+ \delta_{\Delta N\theta}\big(N_{cmd,\theta_1}(t-\tau) - N_{cmd,\theta_2}(t-\tau)\big) \\
&+ \delta_{\Delta N\eta}\big(N_{cmd,\eta_1}(t-\tau) - N_{cmd,\eta_2}(t-\tau)\big) \\
&- \omega_2^2(x_{12} + x_1) + \omega_1^2 x_1 + A_{sus}F_{cmd,x_2}(t-\tau).
\end{aligned}$$

Cross-talk System Identification

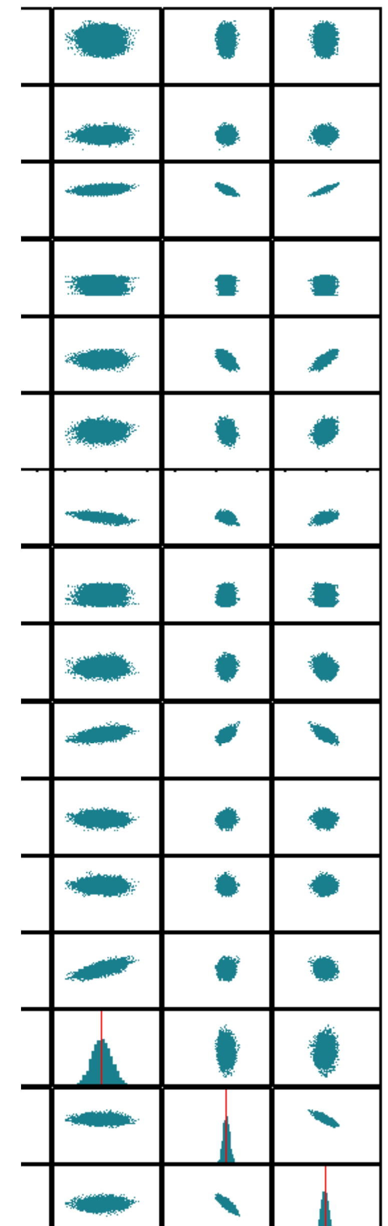


Sample the posterior with MCMC methods. Extract covariance/correlation matrices from the chains.

Cross-talk System Identification



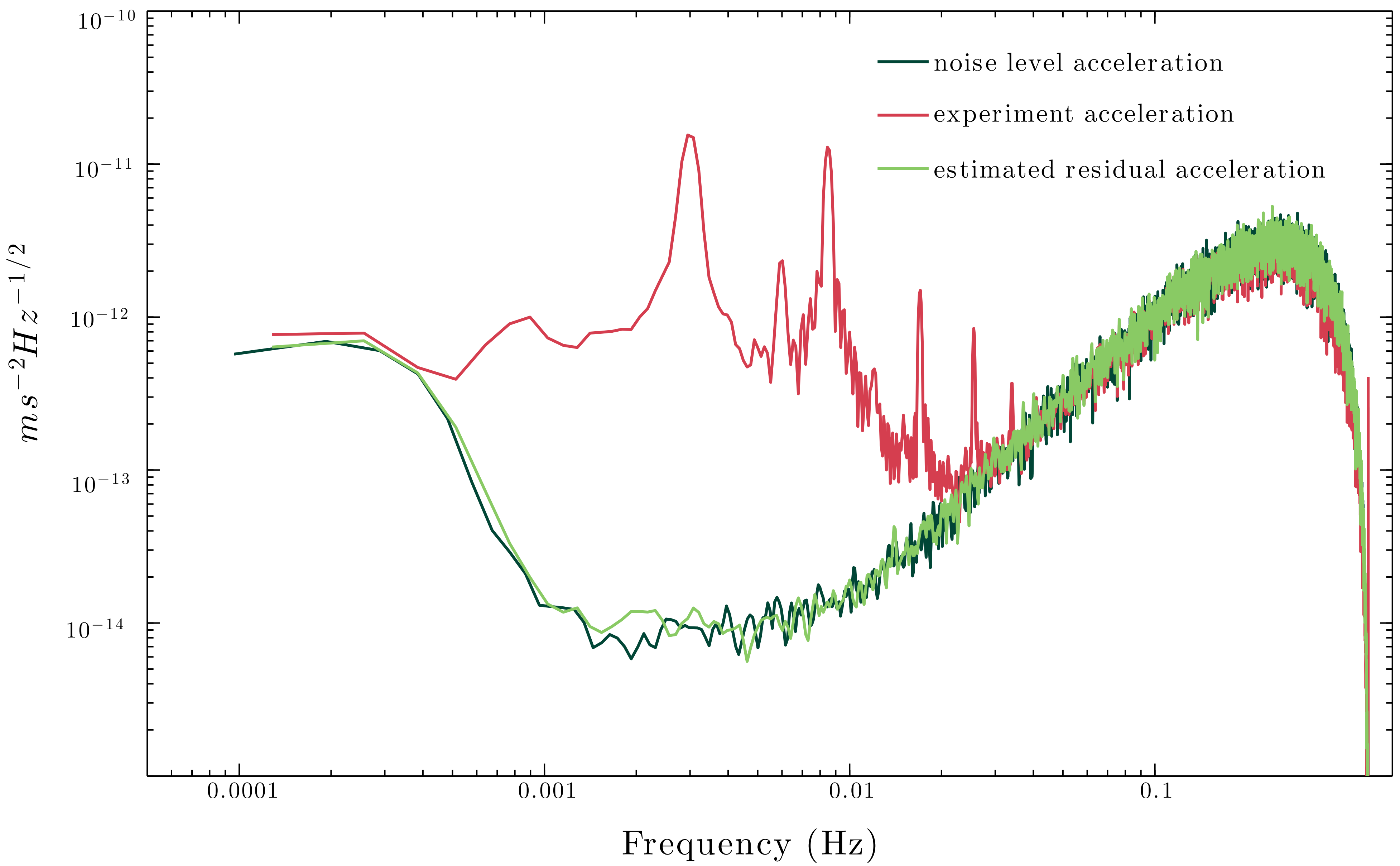
param	initial guess value	estimated $\pm\sigma$
$\delta_{\ddot{\phi}_1}$	50×10^{-6}	$(1.370 \pm 0.002) \times 10^{-4}$
δ_{ϕ_1}	5×10^{-10}	$(-7.0 \pm 0.5) \times 10^{-10}$
$\delta_{\ddot{\phi}_1^2}$	0.19	-0.190072 ± 10^{-6}
$\delta_{\Delta N\phi}$	18×10^{-6}	$(-1.7 \pm 2) \times 10^{-7}$
ω_2^2	-2.42×10^{-6}	$-(2.1 \pm 0.1) \times 10^{-6}$
ω_1^2	-2.42×10^{-6}	$-(2.0 \pm 0.3) \times 10^{-6}$
τ	0.001	0.395 ± 0.002
$\delta_{\ddot{\phi}_2}$	0.19	$-(1.293 \pm 0.002) \times 10^{-4}$
δ_{ϕ_2}	-5×10^{-7}	$-(5.2 \pm 0.2) \times 10^{-10}$
$\delta_{\ddot{\phi}_2^2}$	0.1	-0.2301888 ± 10^{-7}
$\delta_{\ddot{y}_1}$	10^{-4}	$-(0.9 \pm 2) \times 10^{-5}$
δ_{y_1}	10^{-4}	$-(1.2 \pm 0.1) \times 10^{-7}$
$\delta_{\ddot{y}_2}$	5×10^{-5}	$(0.2 \pm 0.6) \times 10^{-4}$
δ_{y_2}	10^{-4}	$-(0.7 \pm 1) \times 10^{-7}$
$\delta_{\Delta N\theta}$	10^{-6}	$-(7.5 \pm 0.8) \times 10^{-5}$
$\delta_{\Delta N\eta}$	10^{-6}	$-(0.8 \pm 0.5) \times 10^{-5}$



Sample the posterior … e/correlation matrices from the chains.

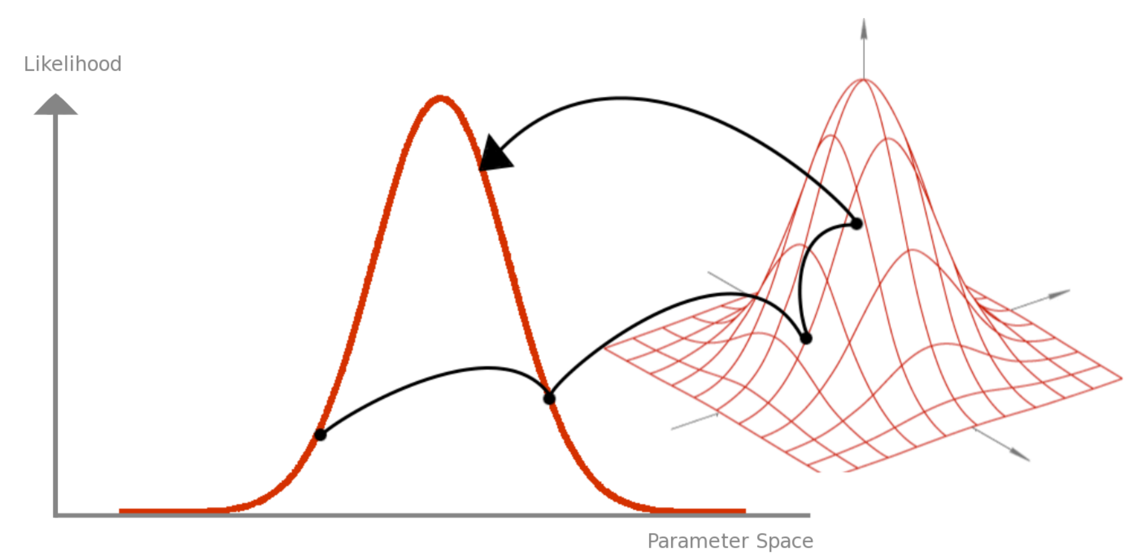
Cross-talk System Identification

- For the given simulated data-set, the results are quite satisfying!



Cross-talk System Identification

- Since the cross-talk experiment requires a high dimensionality model,
- and many physical effects contribute with very low SNR…
- we can apply other Bayesian techniques like the Reversible Jump MCMC to perform model selection*.
    - A generalised MCMC: allows transdimensional moves.
    - Directly calculates the Bayes factor (ratio of the "evidences" of the models)
    - Will most probably be used off-line. Other approximations (like the Laplace) can be put to use during operations.



* Karnesis et al, PRD89, 062001, 2014

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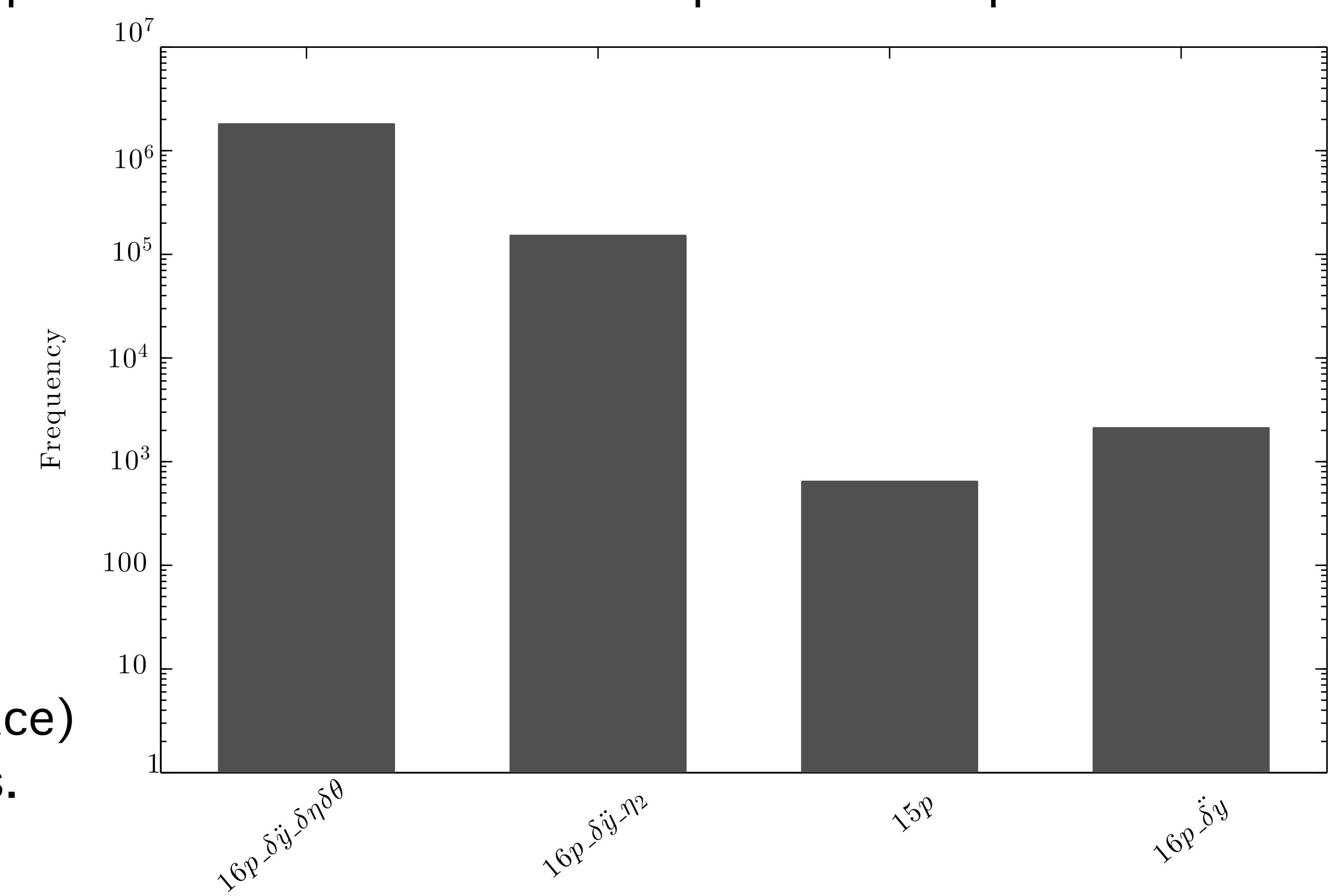
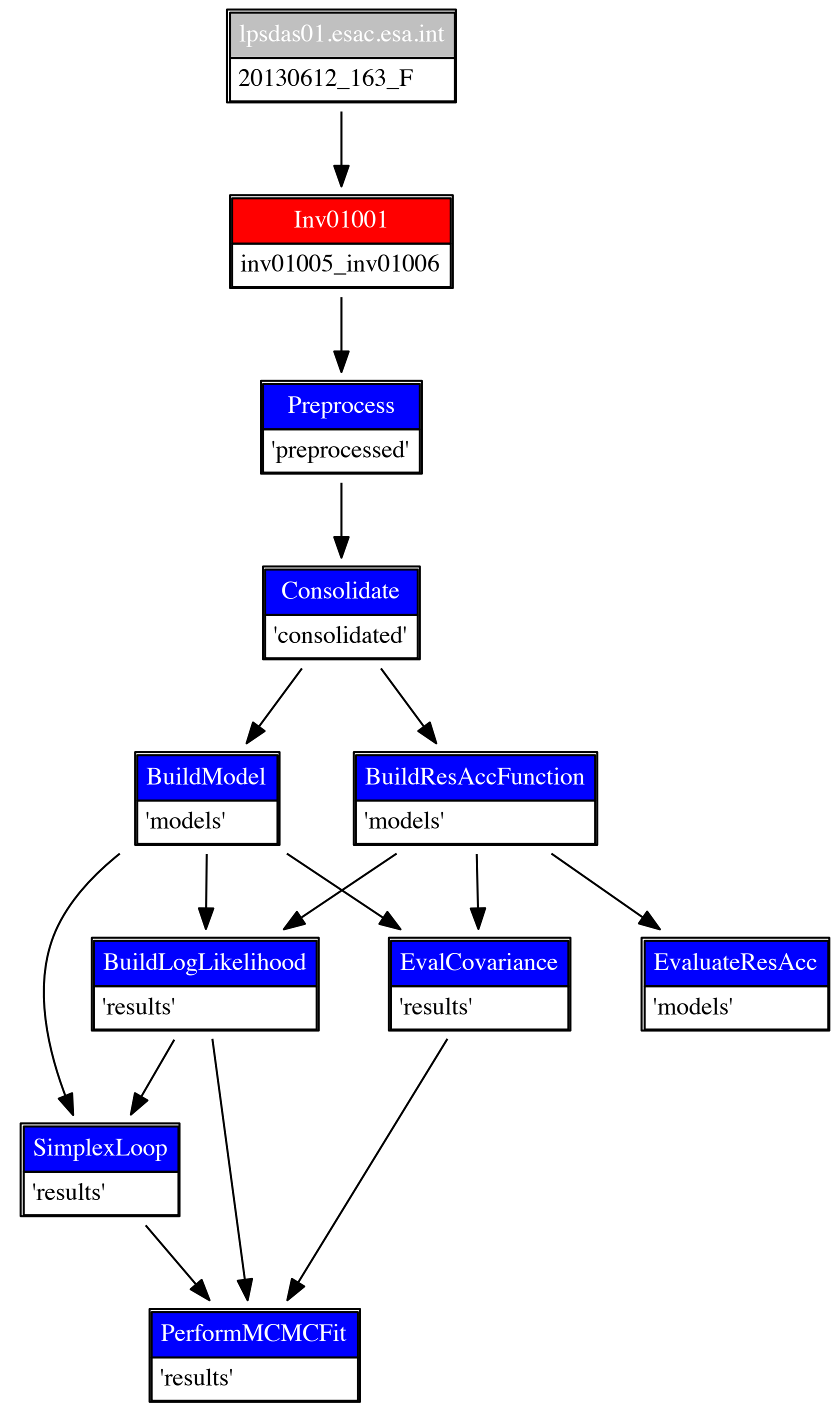


* Karnesis et al, PRD89, 062001, 2014

The Sys-ID Pipeline for on-line analysis

- This work presented here, is integrated in data analysis pipelines.
- The pipeline includes all analysis steps, from downloading the telemetry, to the submission of the analysis results.
- Can be modified/tuned/ configured by the user.
- Flexibility to adjust the model symbolic equation on the spot.



Summary

- We have developed a Bayesian tool to perform system identification/Model Selection for the LPF experiments.
- It has been integrated to data analysis pipelines.
- Already being tested systematically in numerous Simulations.
- Always improving/enhancing
- Getting ready for the launch!!!

Thank you!
Questions?