External force measurement in controlled dynamical systems with unknown noise: the case of LISA Pathfinder

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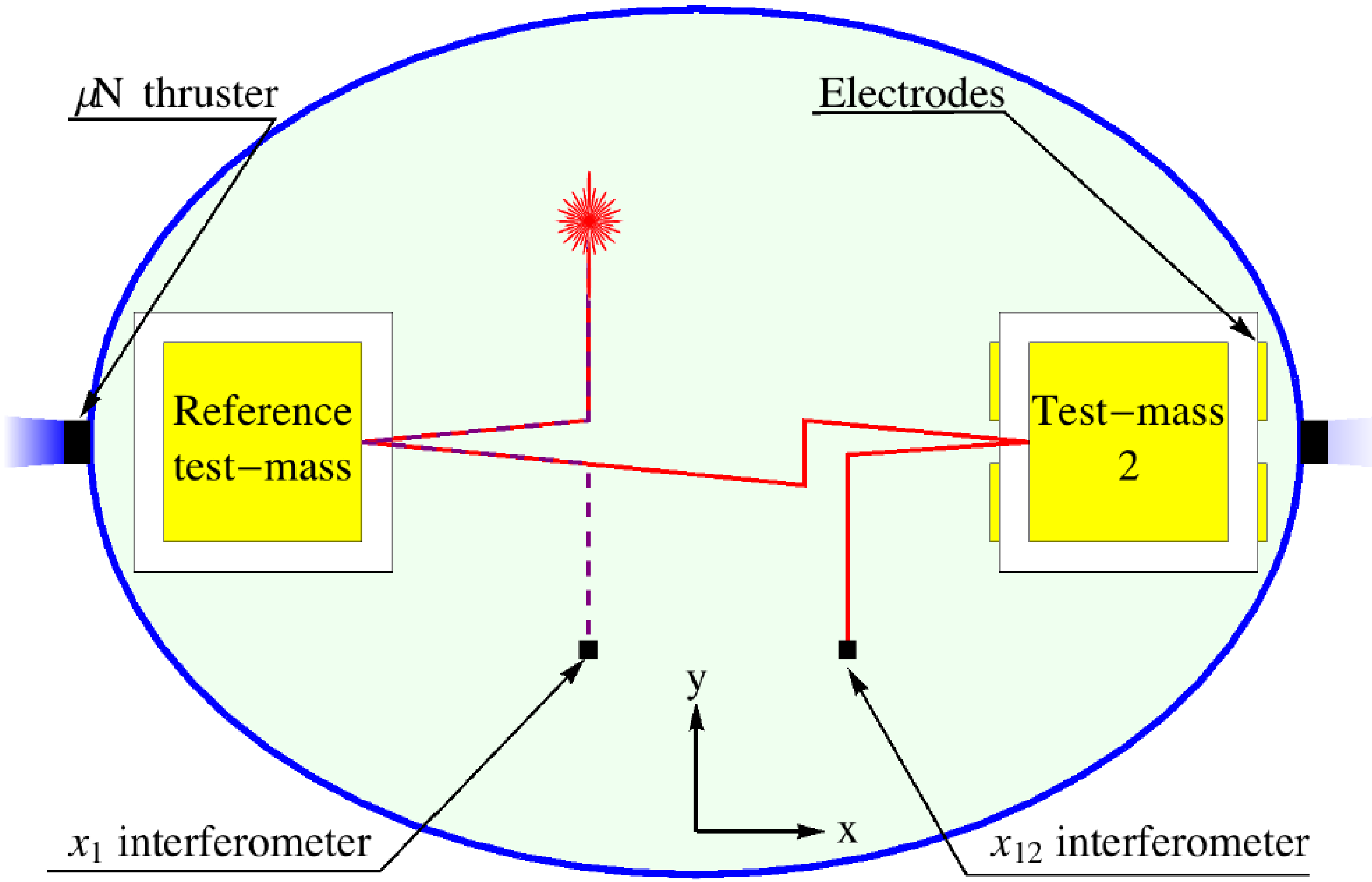
LISA Pathfinder (LPF)



LPF is a dynamical controlled system

- Control loops should make the system to behave linear with a good approximation

$$\Delta a = \sum_j \frac{d^2}{dt^2}\left(S_j \ast x_j\right) - \sum_j \omega_j^2 \ast x_j + \sum_j A_j \ast g_j^c + \Delta g$$

- x_j : signals like x_1 or x_{12} (but also other dof)
- S_j : pickup
- ω_j^2 : stiffness and cross-stiffness
- g_j^c : commanded forces by the control loops
- A_j : converts g_j^c into ‘‘true’’ forces
- Δg : external forces (also the noise of commanded forces)

The basics of LISA Pathfinder (LPF)

Main Goal: Δg

Requirements: the square root of PSD(Δg) better than

$$\leq 3 \times 10^{-14} \sqrt{1 + \left(\frac{f}{3\,\mathrm{mHz}}\right)^2}\ \mathrm{ms}^{-2}/\sqrt{\mathrm{Hz}}$$

over a frequency bandwidth of 1-30 mHz

- We need to:
  - characterize the disturbance noises of the instrument
  - subract the relative timeseries

Conventional method

- Frequency domain
- Solve the equation of motion
- Derive the transfer function

$$x_i(\omega) = H_{ij}(\omega)\Delta g_j(\omega)(+x_{i,free-evolution}(\omega))$$

- Invert and get $\Delta g_j(\omega)$

Conventional method

- Drawbacks:
  - We need the solution of the dynamics $\ddot{x} = g$
  - We need to know very well the control law

$$\ddot{x} = g + g_{feedback}$$

  - Free-evolution term

$$x(t) = \int_0^\infty t' g(t - t') dt' + v_0 t + x_0$$

The case of LPF

- We want to measure Δg at low frequency (1-30 mHz)

- We have a controlled dynamical system

- We do not need to solve the equation of motion but just to fit it!

- We have all the timeseries of position and commanded force (from the telemetry)

Fitting the acceleration

- Solution of dynamics not required
- No transients from the free-evolution (annoying at low frequencies)
- We can subtract forces directly (read or measured)
- Spourious force spikes are spikes within the force dataseries

Fitting the acceleration

- We should have an a priori estimation of the noise;
- However, in signal-dominated experiment (as eLISA) it is not so easy;
- In this case, noise is one of the output of a global signal search procedure;
- In LPF, noise is the main scientific goal!

Applying to the case of LPF

- The residual could be written as

$$\Delta\tilde{a}\left[k\right] - \sum_{j=1}^{N_g} \Delta\tilde{g}_j\left[k, \vec{\theta}\right] = \Delta\tilde{g}_{noise}\left[k\right]$$

- ~ indicates Discrete Fourier Transform (DFT) that means

$$\Delta\tilde{g}_i\left[k\right] = \frac{1}{\sqrt{N_d}} \sum_{n=0}^{N_d-1} \Delta g_i\left[n\right] w\left[n\right] e^{-i\ n\ k \frac{2\pi}{N_d}}$$

- w is the spectral window (Blackmann-Harris)
- N_d is the number of data

The new loglikelihood

- Instead of maximizing

$$\Lambda\left(\vec{\theta}\right) = -N_s \sum_{k \in Q} \frac{\overline{\left|\Delta\tilde{g}_{noise}\left[k, \vec{\theta}\right]\right|^2}}{s_k} + C'$$

we can maximize (after noise marginalization)

$$\Lambda\left(\vec{\theta}\right) = -N_s \sum_{k \in Q} \log\left(\overline{\left|\Delta\tilde{g}_{noise}\left[k, \vec{\theta}\right]\right|^2}\right) + C$$

arXiv:1404.4792v1, S. Vitale, D.V. et al.

The Crosstalk Experiment

What are we talking about?

- What we think is pure noise, actually contains some signal we could try to subtract

$$\Delta\tilde{a}\left[k\right] = A\tilde{g}_2^c\left[k,\tau\right] - \omega_2^2\tilde{x}_{12}\left[k\right] - \left(\omega_1^2 - \omega_2^2\right)\tilde{x}_1\left[k\right] + \Delta\tilde{g}_{noise}\left[k\right]$$

Pick-up

Actuation

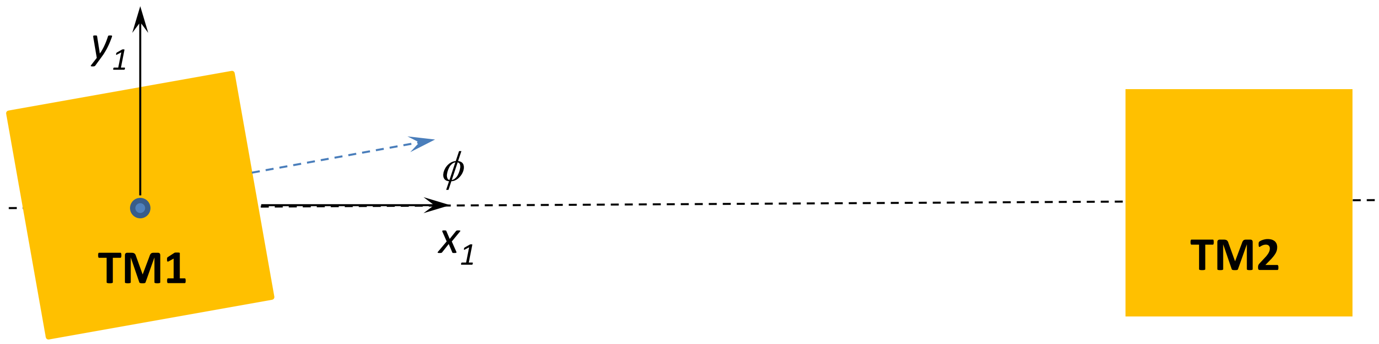
Cross-stiffness

Other effects

The pick-up

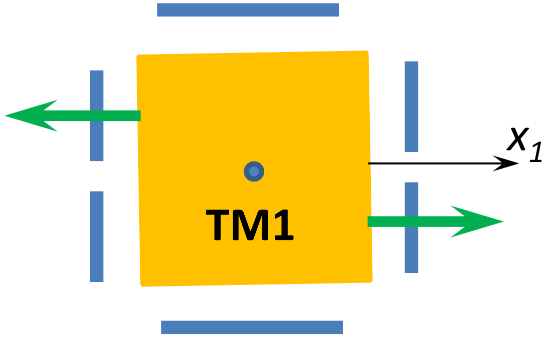
The piston effect (linear and non linear)

$$\delta_{\ddot{\phi}^2{}_{1/2}} \ddot{\phi}^2{}_{1/2} + \delta_{\ddot{\phi}_{1/2}} \ddot{\phi}_{1/2}$$



The actuation crosstalk

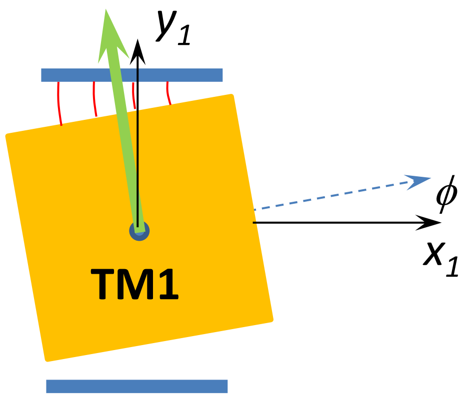
The torque-into-acceleration effect



$$a_x = \frac{\delta F}{F_0}\frac{1}{6}\frac{\ell^2}{\Delta r}N$$

The cross-stiffness

Projection of force gradients from other dof



$$\ddot{o}_{12} = \frac{1}{m}\frac{\partial F_x}{\partial \phi}\phi = \delta_\phi \phi$$

where $\frac{\partial F_x}{\partial \phi} = F_{act(y)}$

The crosstalk on the xy-plane

- Inject a guidance signal into the control loop of **ϕ_1**
- Measure the readout of the differential interferometer
- Fit the our data to the equation of crosstalk for the ϕ_1 injection

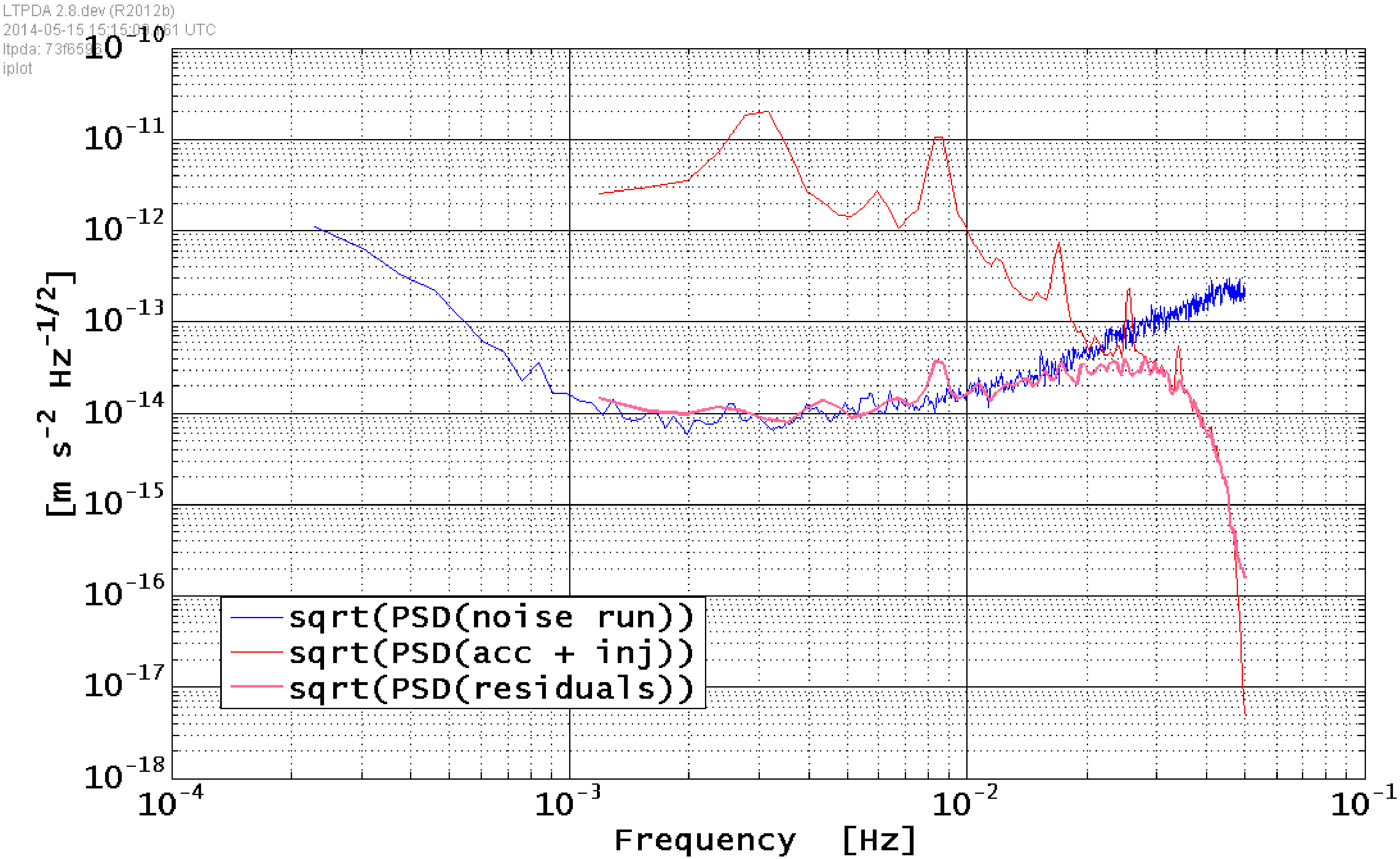
$$\Delta a = -\delta_{\ddot{\phi}_1}\ddot{\phi}_1 + \delta_{\phi_1}\phi_1 + \delta_{\ddot{\phi}_1^2}\ddot{\phi}_1^2 - \delta_{N_{cmd,\phi 1}} N_{\phi_1} - \omega_2^2(x_{12} + x_1) +$$

$$+\omega_1^2 x_1 - A_{sus} f_{cmd,x_2} + \delta_{\ddot{\delta} y}(\ddot{y}_1 - \ddot{y}_2) + \Delta g$$

ϕ_1 injections – Results

Param	Units	Noise	Unknown Noise	mechanism
$\delta_{\ddot{\phi}_1}$	m/rad	$(1.37 \pm 0.003) \times 10^{-4}$	$(1.37 \pm 0.003) \times 10^{-4}$	Linear Piston
δ_{ϕ_1}	$m/s^2 rad$	$(-6.5 \pm 0.1) \times 10^{-10}$	$(-6.6 \pm 0.2) \times 10^{-10}$	Cross-stiffness $\omega^2_{\phi x}$
$\delta_{\ddot{\phi}_1^2}$	m/rad^2	$-0.19 \pm 0.24 \times 10^{-6}$	$-0.19 \pm 0.06 \times 10^{-6}$	Non-linear piston
$\delta_{N_{cmd,\phi 1}}$	m	$(-2 \pm 3) \times 10^{-7}$	$(-2 \pm 3) \times 10^{-7}$	Torque imbalance
ω_2^2	$1/s^2$	$(-2.14 \pm 0.06) \times 10^{-6}$	$(-2.14 \pm 0.06) \times 10^{-6}$	Stiffness TM2
ω_1^2	$1/s^2$	$(-2.01 \pm 0.37) \times 10^{-6}$	$(-2.01 \pm 0.37) \times 10^{-6}$	Stiffness TM1
A_{sus}	no units	1.05	1.05	Gain of the suspension
τ	s	0.226 ± 0.002	$0.222 \pm 1.141 \times 10^{-10}$	delay of the OBC
$\delta_{\ddot{\delta}y}$	rad	-0.00035 ± 0.00004	-0.00035 ± 0.00004	non-orthogonality of TM faces

The PSD of the residual noise



The search for GW with eLISA

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Space-borne gravitational-wave detectors as time-delayed differential dynamometers

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- eLISA will scan the GWs sky at very low frequency (0.1 mHz – 1 Hz)

- It is signal dominated (we hope!)

- Data analysis should be performed in this framework (fit in acceleration with unknown noise)

Conclusions

- Fit in acceleration to subtract force noise
- Use the new likelihood to fit with a priori unknown noise
- The procedure was tested with LPF Simulator and Torsion pendulum: it works!
- GWs as tidal forces allows to use in eLISA data analysis