



# Noise in Pulsar Timing Arrays

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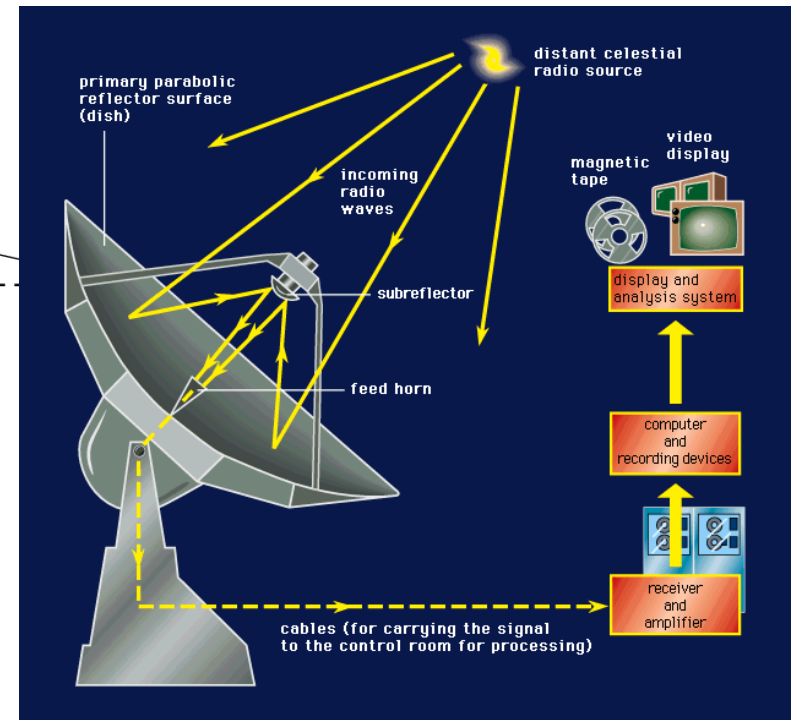
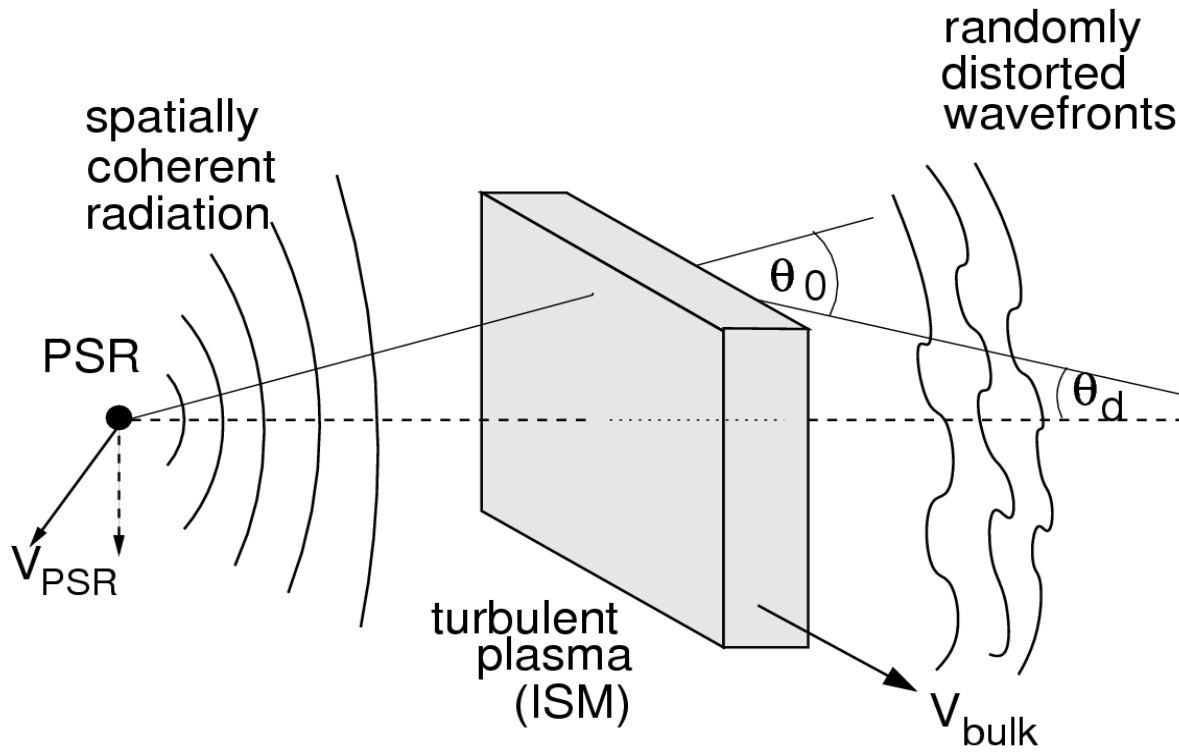
*present for*  
**NANOGrav collaboration**

May 21, 2014  
LISA symposium X, Gainesville, Florida

# Outline

- End to end noise model of pulsar timing  
*timing noise, radiometer noise, jitter noise*
- Noise budget working group  
*tests on whiteness and Gaussianity for NANOGrav 5 yr data*
- J1713+0747 24 hr global observation

# End to end noise model



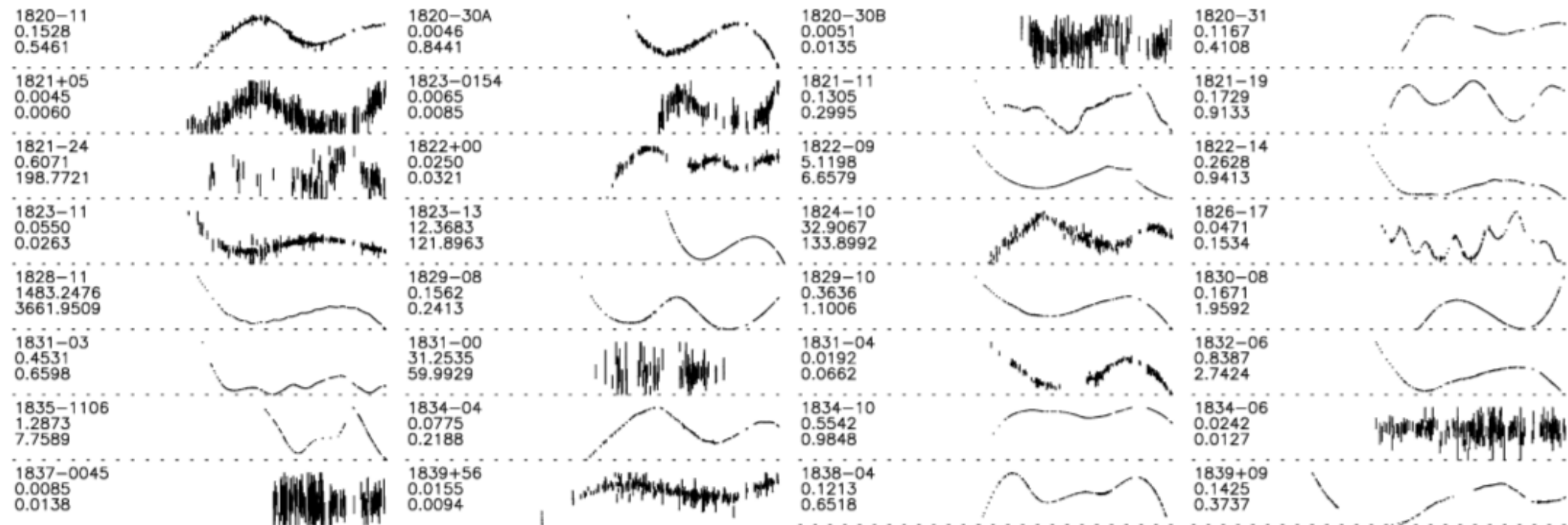
$$t_e^{\text{psr}} = t_a^{\text{obs}} - \Delta_{\odot} - \Delta_{\text{IS}} - \Delta_{\text{B}}$$

$$\Delta t_{S,A} = \Delta t_{\text{J}} + \Delta t_{\text{AST}} + \Delta t_{\text{S/N}} + \Delta t_{\text{pol}}$$

# Selected timing effects

Term	Type <sup>a</sup>	Mean Part		Stochastic Part		Achromatic or Chromatic <sup>b</sup>
		Symbol	Value	Symbol	Value	
Spin rate	A	$t_{\text{spin}}$	yr	$\Delta t_{\text{spin}}$	$\mu\text{s} - \text{s}$	a
Magnetosphere:						
Pulse Shape	A, T	$t_{\text{p}}$	$\mu\text{s} - \text{ms}$	—	—	c
Pulse Jitter	A, T	—	—	$\Delta t_{\text{J}}$	$< \mu\text{s} - \text{ms}$	c
Orbital	A	$t_{\text{orb}}$	hr	$\Delta t_{\text{orb}}$	$< \text{ms}$	a
Dispersion	A, T	$t_{\text{DM}}$	$\lesssim \text{s}$	$\Delta t_{\text{DM}}$	$\lesssim 100\mu\text{s}$	C
Faraday Rotation	A, T	$t_{\text{RM}}$	$\lesssim \mu\text{s}$	$\Delta t_{\text{RM}}$	$\lesssim \text{ns}$	C
Interstellar Turbulence						
Pulse Broadening	A, T	$t_{\text{PBF}}$	$\text{ns} - \text{s}$	$\Delta t_{\text{PBF}}$	$< \text{ns} - \text{ms}$	C
DISS	A, T	—	—	$\Delta t_{\delta\text{PBF}}$	$\lesssim \mu\text{s}$	C
RISS	A, T	$t_{\text{PBF,RISS}}$	$\lesssim \mu\text{s}$	$\Delta t_{\delta\text{PBF,RISS}}$	$\lesssim \mu\text{s}$	C
Angle of Arrival	A, T	—	—	$\Delta t_{\text{AOA}}$	$\lesssim \mu\text{s}$	C
Angle of Arrival	A, T	—	—	$\Delta t_{\text{AOA,SSBC}}$	$\lesssim \mu\text{s}$	C
Multipath averaging	A, T	—	—	$\Delta t_{\text{DM},\nu}$	$\lesssim 0.1\mu\text{s}$	C
Astrometric <sup>e</sup>	T	$t_{\text{AST}}$	—	$\Delta t_{\text{AST}}$	—	a
Newtonian solar perturbations	T	—	—	$\Delta t_{\text{Newt,SSBC}}$	—	a
Radiometer Noise	T	—	—	$\Delta t_{\text{S/N}}$	$< \mu\text{s} - \text{ms}$	c→C
Polarization	T	—	—	$\Delta t_{\text{pol}}$	—	c
Gravitational Lensing	A	$t_{\text{GL}}$	—	$\Delta t_{\text{GL}}$	—	a
Cosmic Strings	A	$t_{\text{STR}}$	—	—	—	a
Gravitational Waves	A	—	—	$\Delta t_{\text{GW}}$	$\lesssim 100 \text{ ns}$	a

# Timing Noise



J1823-3021A	B1820-30A	183.823	-114.336	0.522(4)	J1832-1021	B1829-10	3.027	-38.493	0.247(3)
J1823-3021B	B1820-30B	2.641	-0.225	0.0031(5)	J1833-0338	B1831-03	1.456	-88.139	0.104(5)
J1823-3106	B1820-31	3.520	-36.277	-0.134(5)	J1833-0827	B1830-08	11.725	-1260.890	-1.357(9)
J1824-1118	B1821-11	2.295	-18.716	0.059(5)	J1834-0010	B1831-00	1.920	-0.039	0.0008(53)
J1824-1945	B1821-19	5.282	-145.939	-0.152(19)	J1834-0426	B1831-04	3.447	-0.854	0.0208(6)
J1824-2452	B1821-24	327.406	-173.519	0.047(7)	J1835-0643	B1832-06	3.270	-432.507	0.790(9)
J1825+0004	B1822+00	1.284	-1.445	-0.0076(5)	J1835-1106	—	6.027	-748.776	9.4(3)
J1825-0935	B1822-09	1.300	-88.369	1.541(16)	J1836-0436	B1834-04	2.823	-13.239	-0.066(3)
J1825-1446	B1822-14	3.582	-290.947	0.190(4)	J1836-1008	B1834-10	1.777	-37.278	-0.024(9)
J1826-1131	B1823-11	0.478	-1.121	0.00454(13)	J1837-0045	—	1.621	-4.424	-0.0033(15)
J1826-1334	B1823-13	9.856	-7293.991	138.2(4)					
J1827-0958	B1824-10	4.069	-16.597	-0.0399(15)					
J1829-1751	B1826-17	3.256	-58.852	0.048(3)					
J1830-1059	B1828-11	2.469	-365.843	0.865(3)					
J1832-0827	B1829-08	1.545	-152.462	-0.011(4)					

Hobbs, Lyne, and Kramer (2010)

# Timing Noise cont...

Scaling law:

$$\hat{\sigma}_{\text{TN},2} = C_2 \nu^\alpha |\dot{\nu}|^\beta T^\gamma$$

Relative TN parameter:

$$\zeta = \frac{\sigma_{\text{TN},2}(T)}{\hat{\sigma}_{\text{TN},2}(T)} = \frac{\sigma_{\text{TN},2}(T)}{C_2 \nu^\alpha |\dot{\nu}|^\beta T^\gamma}$$

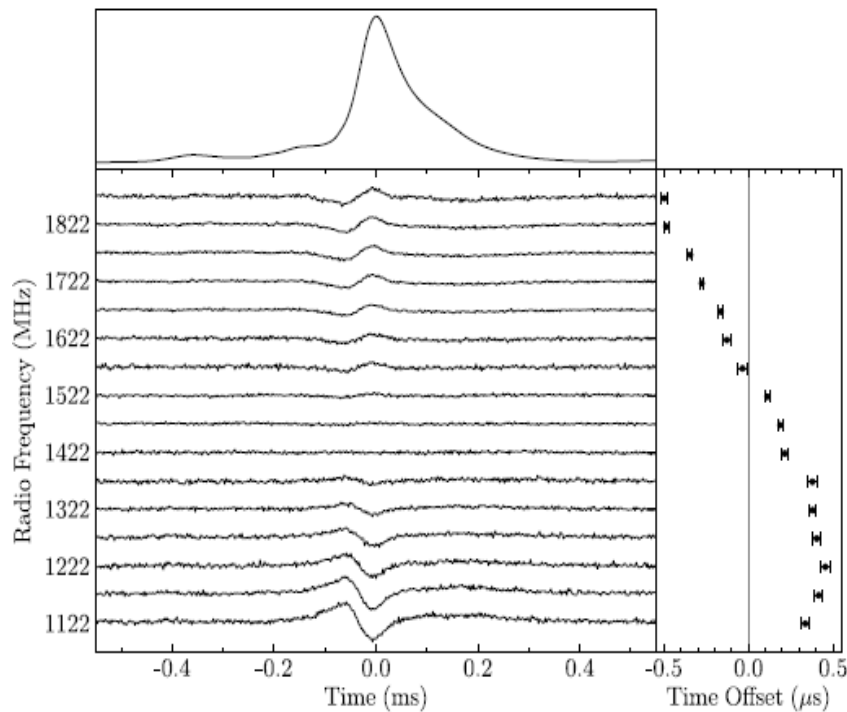
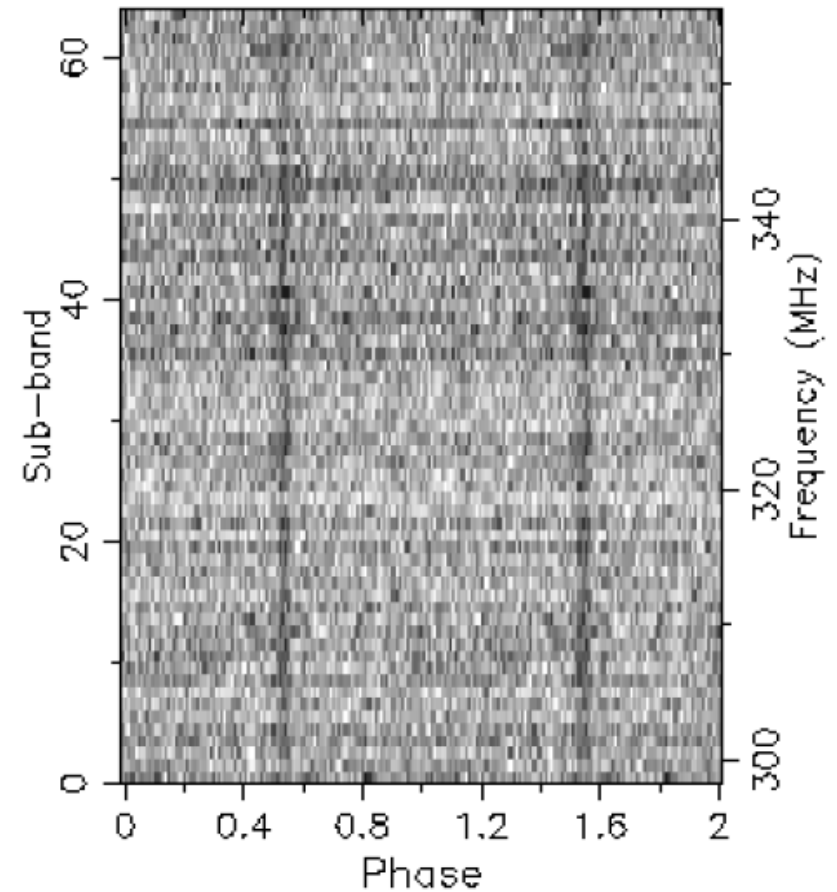
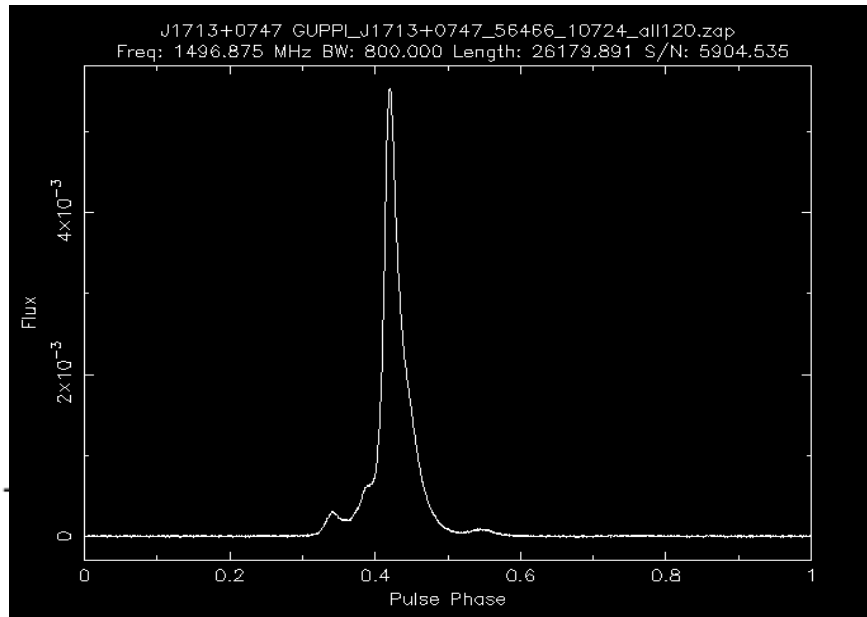
**Table 1**  
Best-fit Parameters

Fit	$\ln(C_2)$	$\alpha$	$\beta$	$\gamma$	$\delta$	$N_D(N_{\text{UL}})$
CP	$2.0 \pm 0.4$	$-0.9 \pm 0.2$	$1.00 \pm 0.05$	$1.9 \pm 0.2$	$1.6 \pm 0.1$	563 (470)
MSP	$-20 \pm 20$	$1 \pm 2$	$2 \pm 1$	$2.4 \pm 0.6$	$1.2 \pm 0.5$	12 (147)
CP+MSP	$1.6 \pm 0.4$	$-1.4 \pm 0.1$	$1.1 \pm 0.1$	$2.0 \pm 0.2$	$1.6 \pm 0.1$	575 (617)
MAG	$3 \pm 7$	$-1 \pm 3$	$1.5 \pm 0.6$	$3 \pm 1$	$2.1 \pm 0.7$	15 (7)
CP+MAG	$2.4 \pm 0.5$	$-1.4 \pm 0.2$	$1.13 \pm 0.07$	$1.7 \pm 0.2$	$1.7 \pm 0.2$	578 (477)
ALL	$2.2 \pm 0.4$	$-1.5 \pm 0.1$	$1.2 \pm 0.1$	$1.8 \pm 0.1$	$1.7 \pm 0.1$	590 (624)

**Notes.** Best-fit parameters and  $\pm 2\sigma$  confidence limits for different populations of pulsars.  $N_D$  is the number of time series with detected TN used in the fit.  $N_{\text{UL}}$  is the number of time series with upper limits of TN used in the fit.

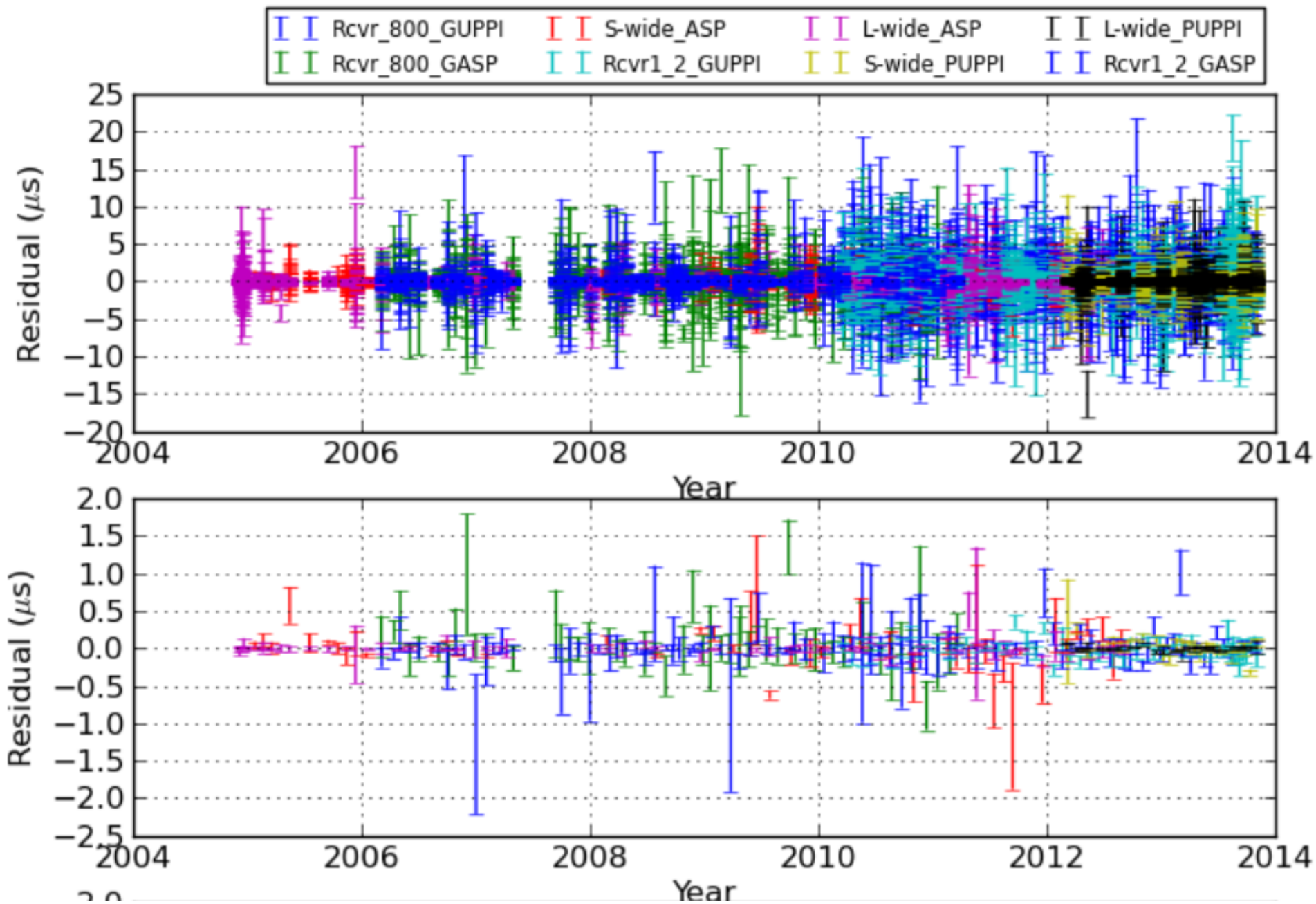
Shannon and Cordes (2010)

# Radiometer noise



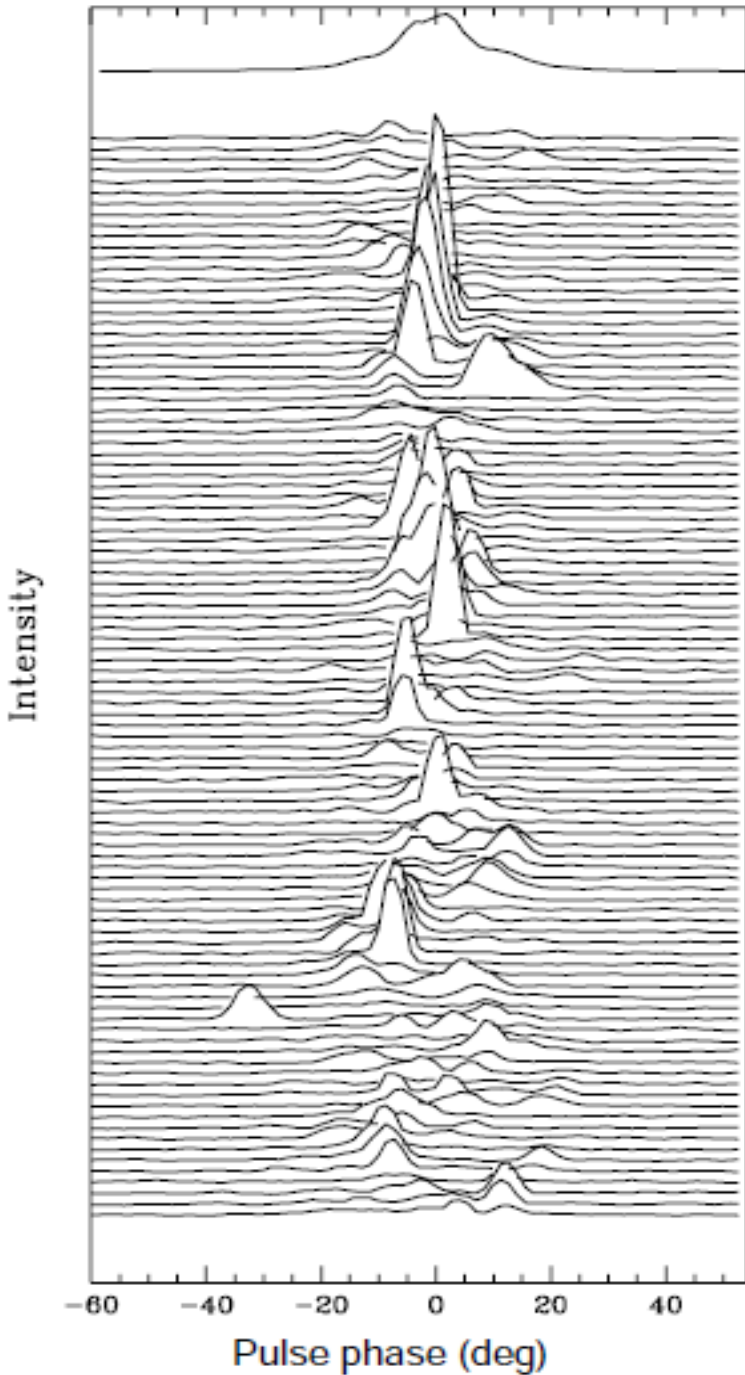
$$\Delta t_{S/N} = \frac{W_{\text{eff}}}{\text{SNR}_1 \sqrt{N}} \left( \frac{\Delta}{W_{\text{eff}}} \right)^{1/2}$$

$$= \frac{S_{\text{sys}}}{S_{\text{peak}} \sqrt{2B}} \left( \frac{W_{\text{eff}}}{N} \right)^{1/2}$$





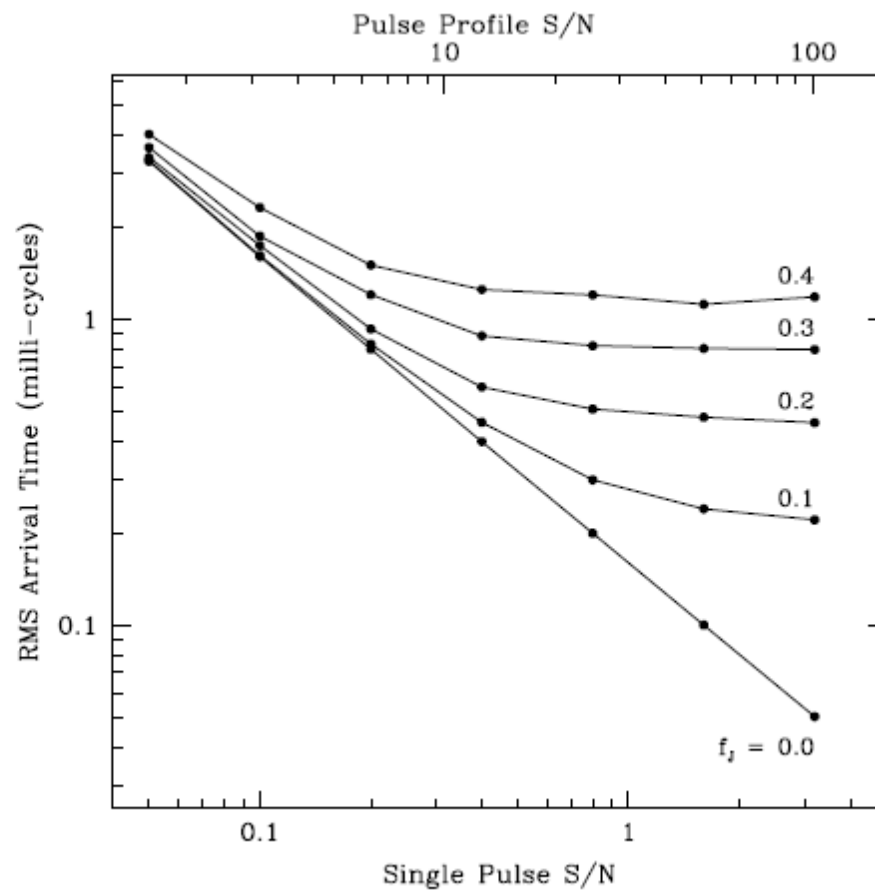
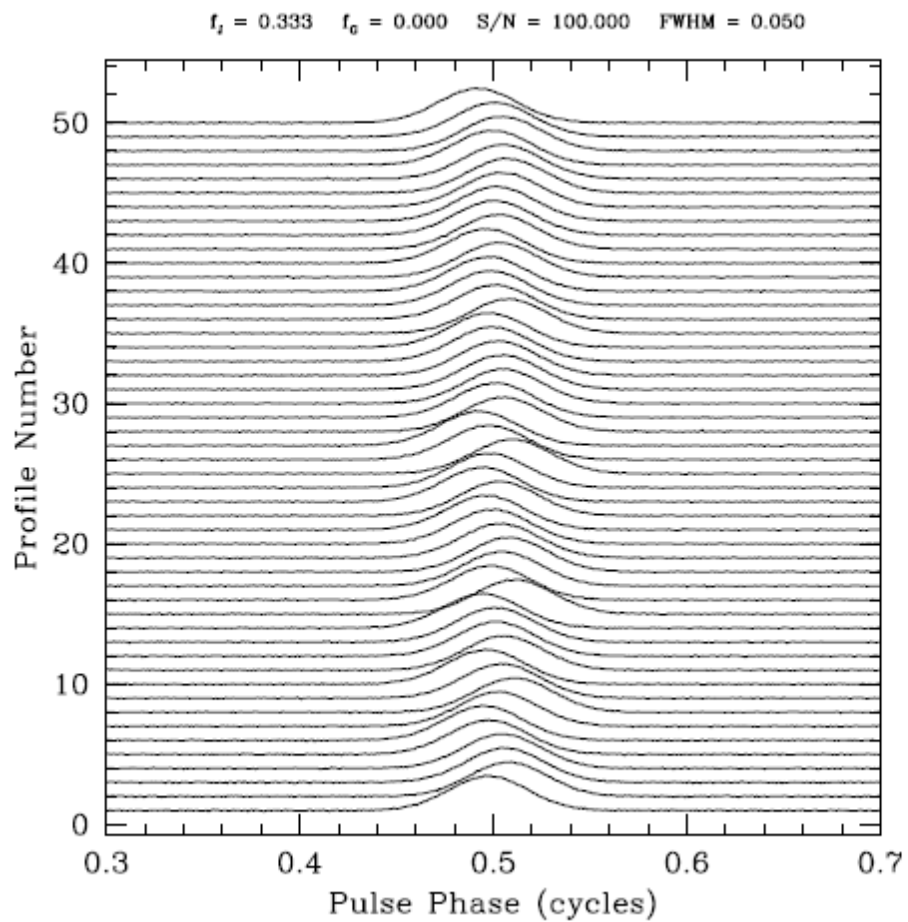
# Jitter noise



$$\Delta t_J = 0.28 \mu s W_{i,ms} N_6^{-1/2} \left( \frac{f_J}{1/3} \right) \left( \frac{1 + m_I^2}{2} \right)^{1/2}$$

PSR J1740-1000  
McLaughlin et al. 2002

# Jitter noise, cont...



# Noise budget working group

## **NANOGrav Pulsar Timing Error Budget Analysis**

J. Cordes, S. Chatterjee, P. Demorest, T. Dolch, J. Ellis, S. Finn, R. Jenet, M. Lam, D. Madison, M. McLaughlin, D. Perrodin, J. Rankin, X. Siemens, M. Vallisneri, and Y. Wang

May 23, 2013

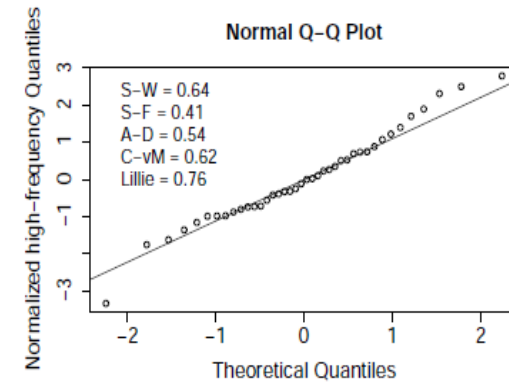
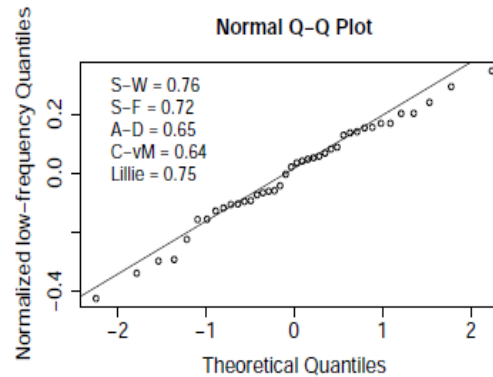
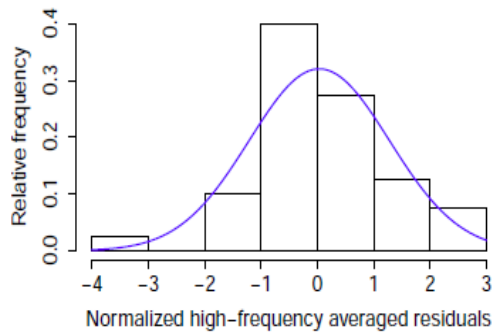
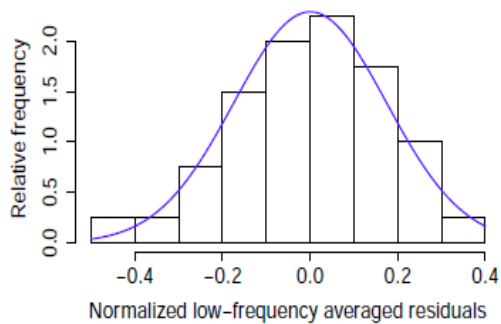
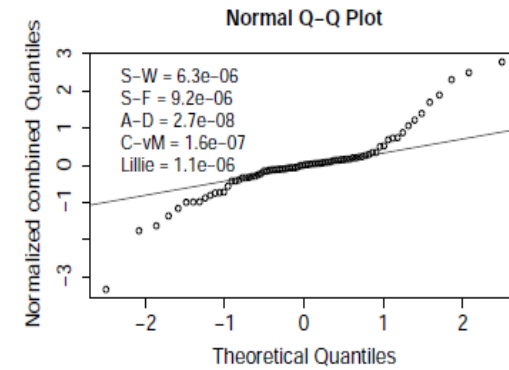
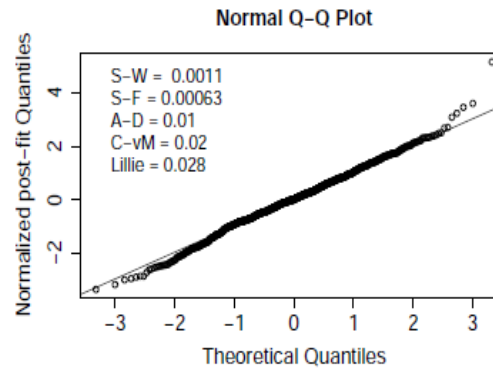
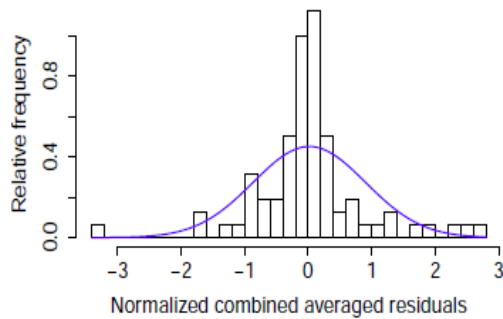
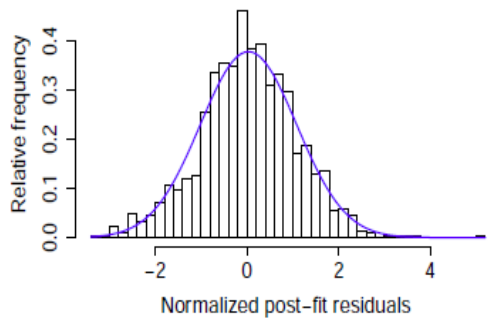
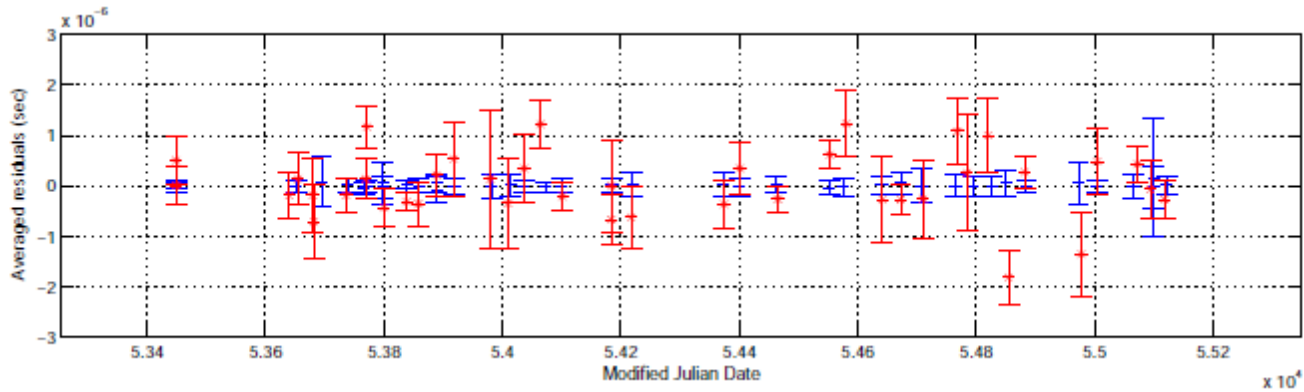
**Executive Summary:** Discussions in the NB Task Force telecons indicate that there are complementary ways to assess the constituents of timing residuals and their root causes.

This document describes how the various results can be merged into an overall assessment of each pulsar reported in Demorest et al. (2013).

So far the following kinds of analyses have been conducted to quantify timing residuals and investigate their consistency with white noise and with Gaussian statistics.

1. The rms residuals reported in Demorest et al. after correction for DM variations. Values are reported for each frequency individually and for the DM-removed TOA estimate.
2. White noise tests based on the autocorrelation function, on a comparison with the delta-function ACF expected from white noise, and on the number of zero crossings.
3. Investigations into non-Gaussianity of the residuals using histogram and moment tests and the role of diffractive interstellar scintillations in causing non-Gaussian statistics.
4. Likelihood estimation of combined parameters for white and red noise.

# Gaussianity for *PSR J0613-0200*



# Tests for Gaussianity

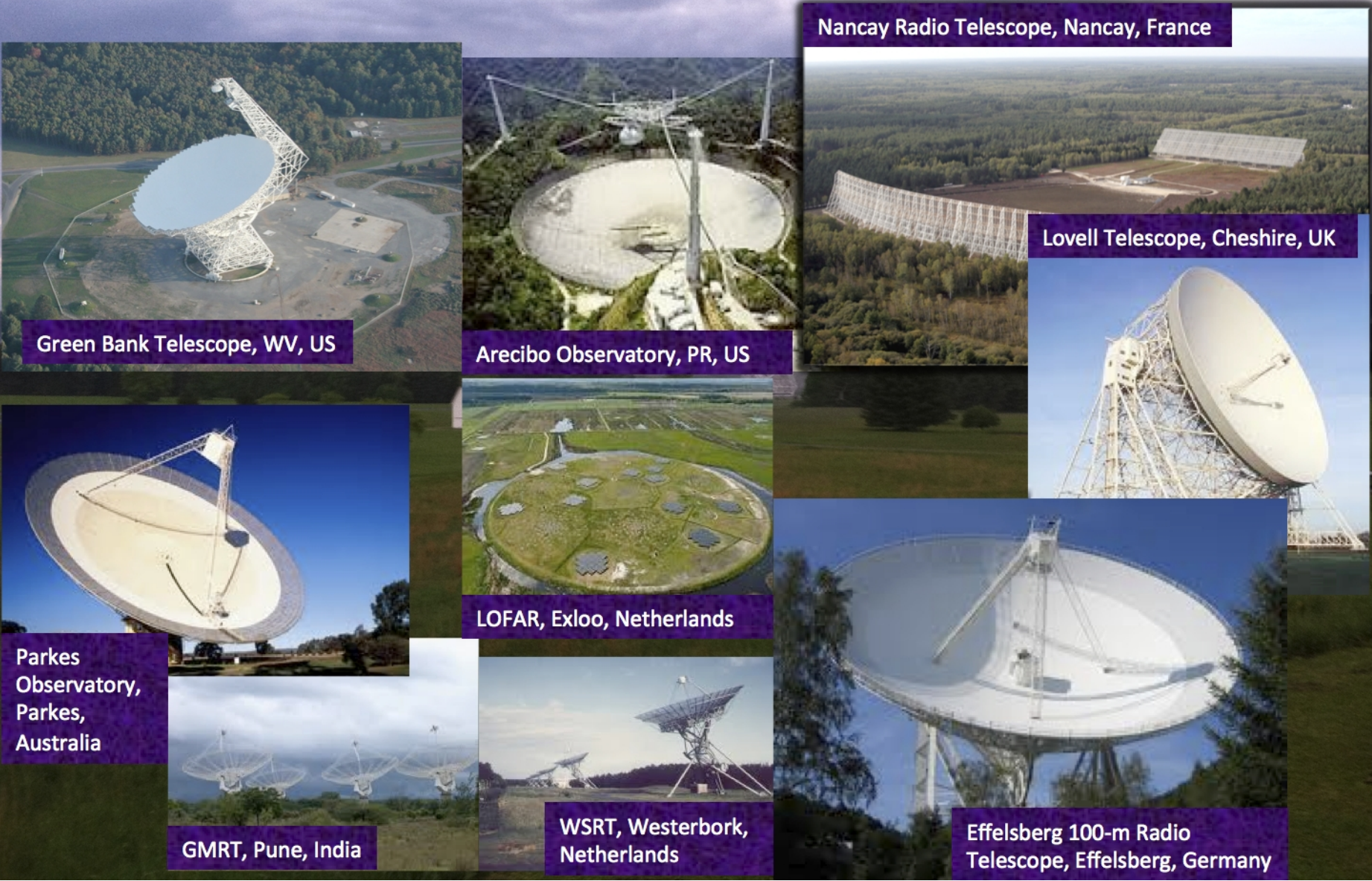
Source	$P$ (ms)	DM (pc cm <sup>-3</sup> )	Averaged timing residuals						Post-fit
			327 MHz	430 MHz	820 MHz	1.4 GHz	2.3 GHz	Comb.	
J0030+0451	4.87	4.33	-	24 n	-	26 N	-	50 N	545 Y
J0613-0200	3.06	38.78	-	-	40 Y	40 Y	-	80 N	1113 n
J1012+5307	5.26	9.02	-	-	47 N	63 n	-	110 N	1678 n
J1455-3330	7.99	13.57	-	-	41 n	45 Y	-	86 N	1100 n
J1600-3053	3.60	52.33	-	-	22 n	26 n	-	48 n	625 Y
J1640+2224	3.16	18.43	-	34 N	-	32 N	-	68 N	631 N
J1643-1224	4.62	62.42	-	-	47 N	50 Y	-	97 N	1266 N
J1713+0747	4.57	15.99	-	-	38 N	84 N	31 Y	153 N	2368 N
J1744-1134	4.07	3.14	-	-	48 N	60 Y	-	108 N	1617 N
J1853+1308	4.09	30.57	-	-	-	41 Y	2 NA	43 Y	497 Y
B1855+09	5.36	13.30	-	37 N	-	32 Y	-	69 n	702 N
J1909-3744	2.95	10.39	-	-	35 n	33 Y	-	68 N	1001 N
J1910+1256	4.98	34.48	-	-	-	31 Y	6 Y	37 Y	525 Y
J1918-0642	7.65	26.60	-	-	40 Y	54 n	-	94 N	1306 Y
B1953+29	6.13	104.50	-	-	-	23 Y	2 NA	25 Y	208 Y
J2145-0750	16.05	9.03	-	-	22 N	24 Y	-	46 n	675 n
J2317+1439	3.45	21.90	43 n	41 n	-	-	-	84 N	458 n

# Zero-crossing test for whiteness

Source	Low-frequency band					High-frequency band					Combined				
	$N_{\text{crs}}$	$\mu_{Z_W}$	$\Delta$	$\sigma_{Z_W}$	Y/n/N	$N_{\text{crs}}$	$\mu_{Z_W}$	$\Delta$	$\sigma_{Z_W}$	Y/n/N	$N_{\text{crs}}$	$\mu_{Z_W}$	$\Delta$	$\sigma_{Z_W}$	Y/n/N
J0030+0451	12	11.5	-0.5	2.4	Y	14	12.5	-1.5	2.5	Y	27	24.5	-2.5	3.5	Y
J0613-0200	20	19.5	-0.5	3.1	Y	19	19.5	0.5	3.1	Y	58	39.5	-18.5	4.4	N
J1012+5307	26	23	-3	3.4	Y	34	31	-3	3.9	Y	62	54.5	-7.5	5.2	Y
J1455-3330	22	20	-2	3.2	Y	23	22	-1	3.3	Y	58	42.5	-15.5	4.6	N
J1600-3053	13	10.5	-2.5	2.3	Y	13	12.5	-0.5	2.5	Y	33	23.5	-9.5	3.4	n
J1640+2224	18	16.5	-1.5	2.9	Y	14	15.5	1.5	2.8	Y	35	32.5	-2.5	4.0	Y
J1643-1224	26	23	-3	3.4	Y	22	24.5	2.5	3.5	Y	62	48	-14	4.9	n
J1713+0747	-	-	-	-	-	-	-	-	-	-	90	77	-13	6.2	n
J1744-1134	25	23.5	-1.5	3.4	Y	33	29.5	-3.5	3.8	Y	70	53.5	-16.5	5.2	N
J1853+1308	21	20	-1	3.2	Y	-	-	-	-	-	19	21	2	3.2	Y
B1855+09	17	18	1	3.0	Y	17	15.5	-1.5	2.8	Y	46	34	-12	4.1	n
J1909-3744	17	17	0	2.9	Y	17	16	-1	2.8	Y	52	33.5	-18.5	4.1	N
J1910+1256	13	15	2	2.7	Y	3	2.5	-0.5	1.1	Y	21	18	-3	3	Y
J1918-0642	20	19.5	-0.5	3.1	Y	29	26.5	-2.5	3.6	Y	64	46.5	-17.5	4.8	N
B1953+29	14	11	-3	2.3	Y	1	0.5	-0.5	0.5	Y	16	12	-4	2.4	Y
J2145-0750	15	10.5	-4.5	2.3	Y	18	11.5	-6.5	2.4	n	31	22.5	-8.5	3.4	n
J2317+1439	26	21	-5	3.2	Y	27	20	-7	3.2	n	56	41.5	-14.5	4.6	N

$$Z_W \sim \mathcal{N}(\mu_{Z_W}, \sigma_{Z_W}^2) \quad \mu_{Z_W} = (N - 1)/2 \quad \sigma_{Z_W} = \sqrt{N - 1}/2$$

# PSR J1713+0747 24 hr global observation



Green Bank Telescope, WV, US

Arecibo Observatory, PR, US

Nancay Radio Telescope, Nancay, France

Lovell Telescope, Cheshire, UK

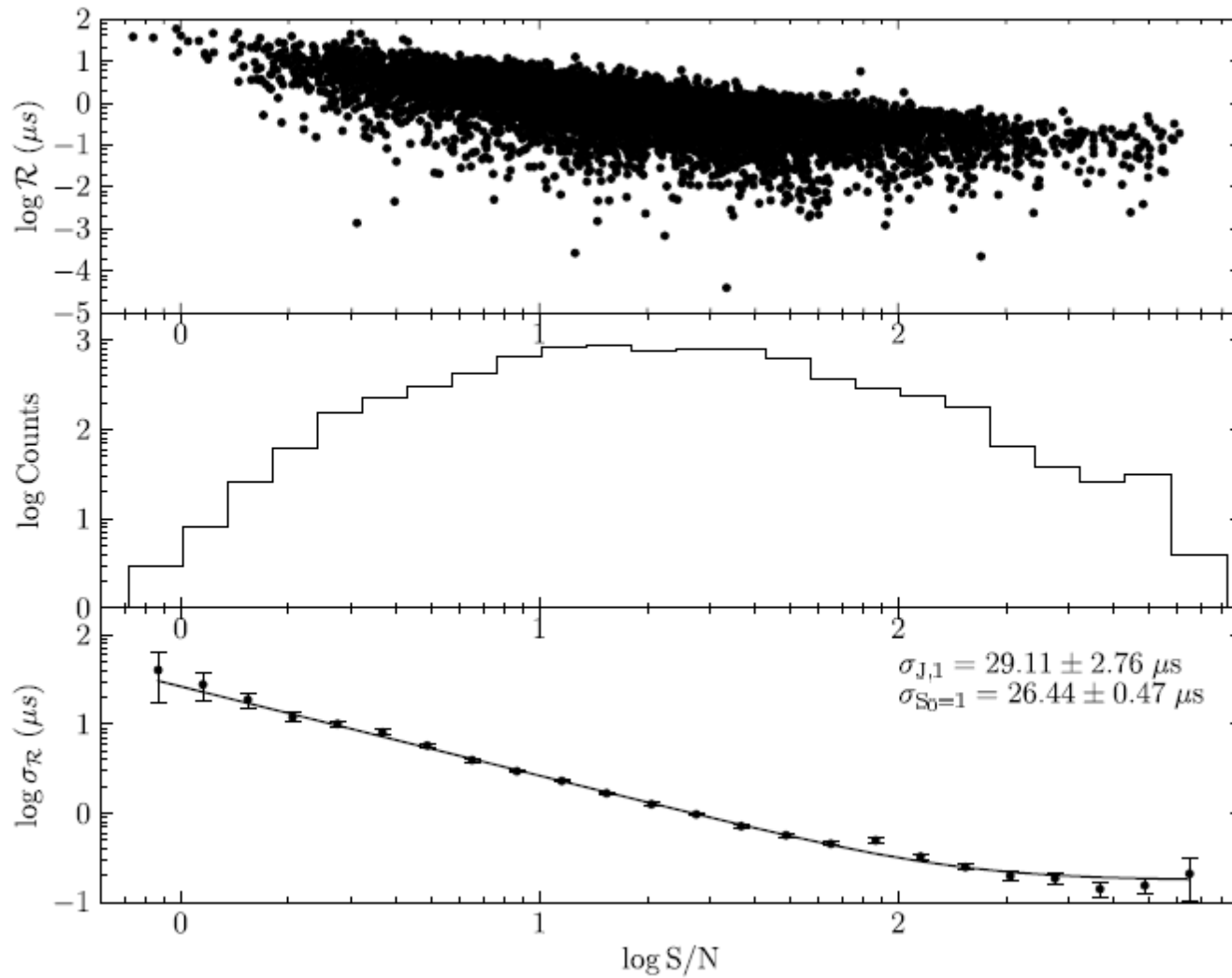
Parkes Observatory, Parkes, Australia

LOFAR, Exloo, Netherlands

GMRT, Pune, India

WSRT, Westerbork, Netherlands

Effelsberg 100-m Radio Telescope, Effelsberg, Germany



$$\sigma_{S/N} = \frac{W_{\text{eff}}}{S\sqrt{N_\phi}}$$

$$\sigma_J \propto W_{\text{eff}}/\sqrt{N}$$

$$\sigma_R = \sqrt{\sigma_J^2 + (\sigma_{S_0}(S_0/S))^2}$$



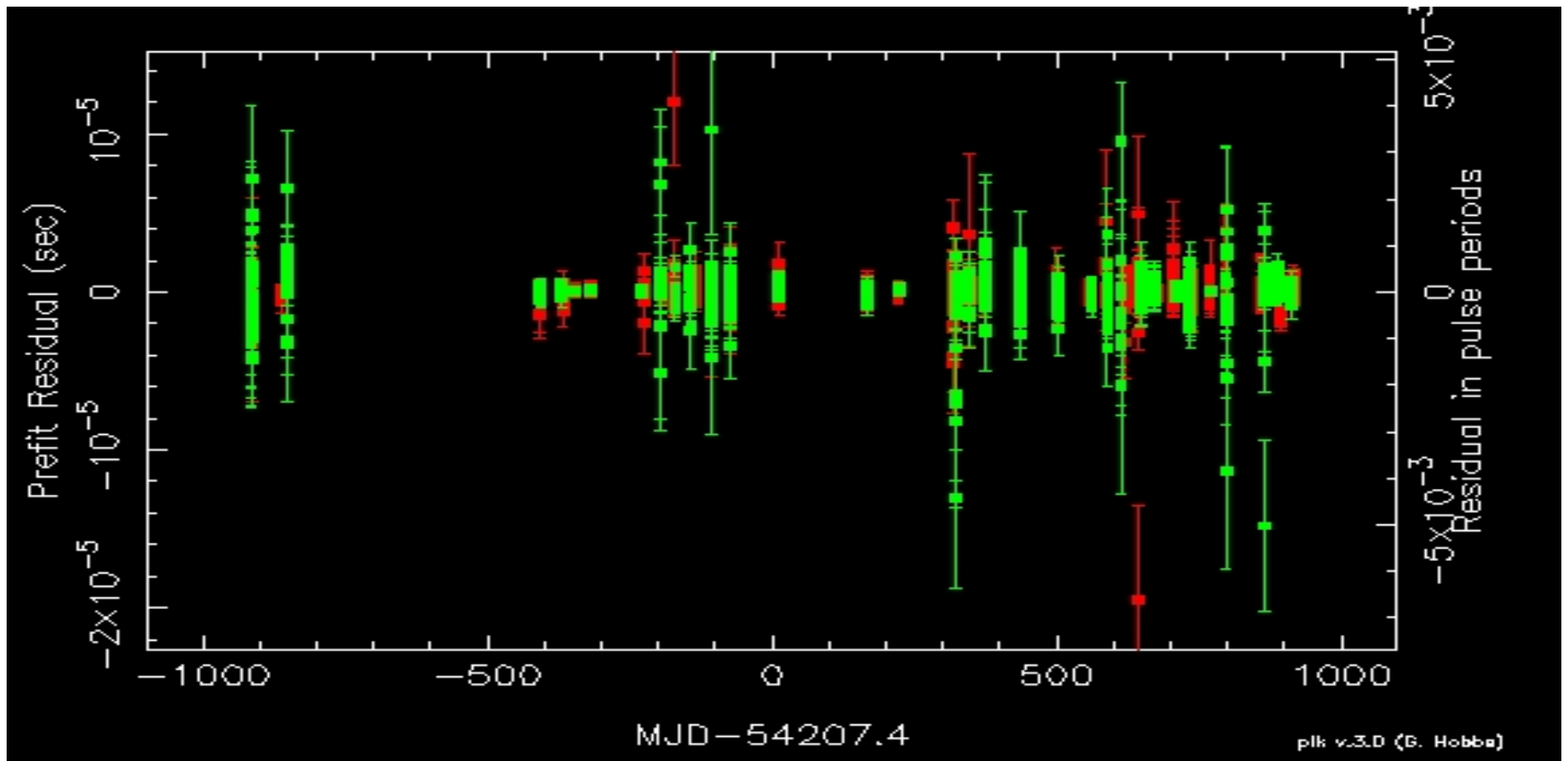
# Backup slides

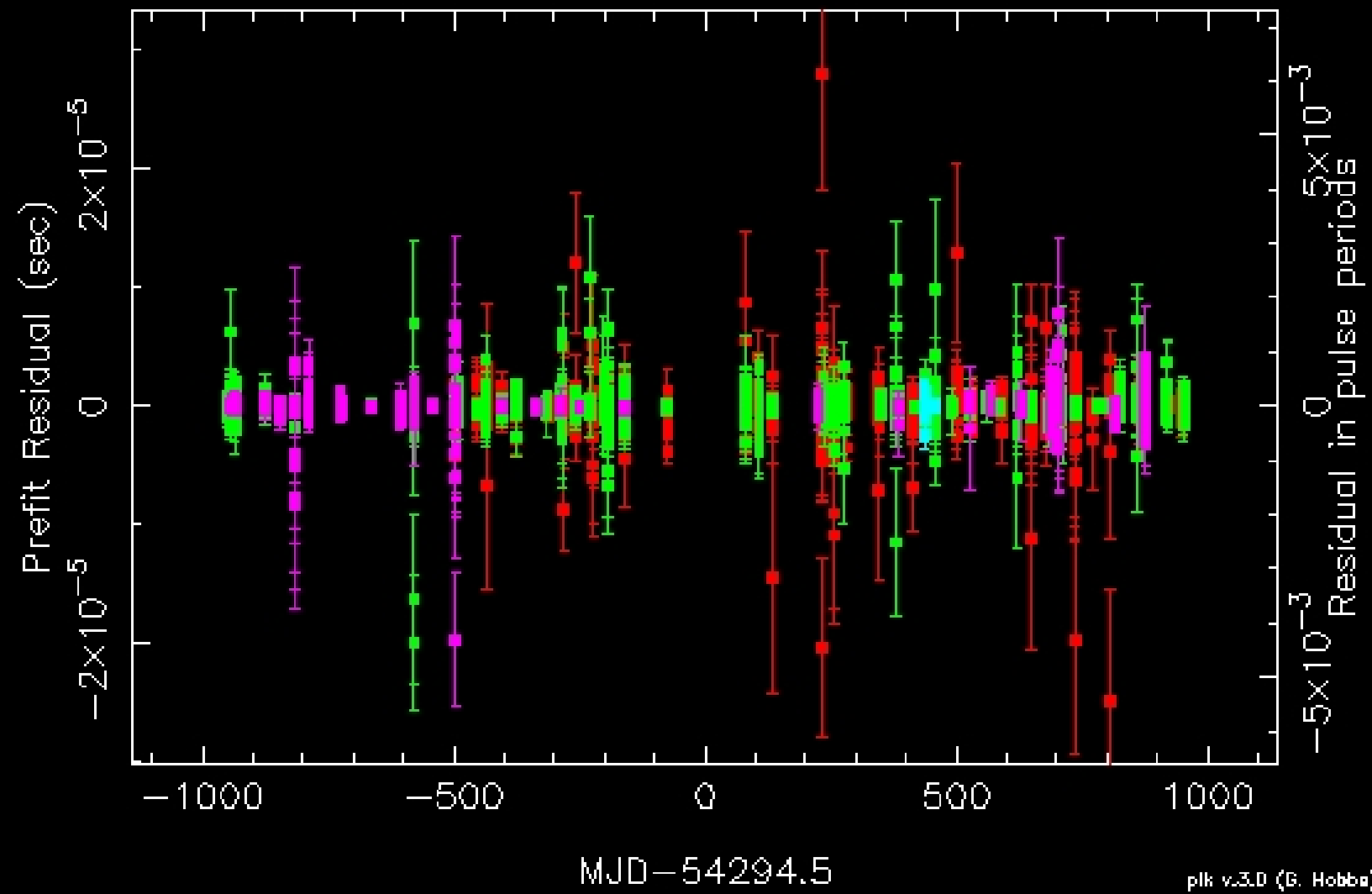
$$\begin{aligned}\Delta t_{S/N} &= 1 \mu s W_{\text{ms}} N_6^{-1/2} \text{SNR}_1^{-1} (\Delta/W)^{1/2} \\ &= 0.71 \mu s (\nu/1.4 \text{ GHz})^{-\alpha_P} P_{\text{ms}}^{-1} W_{\text{ms}}^{3/2} (B N_6)^{-1/2} (S_{\text{sys}}/S_{1400})\end{aligned}$$

# Averaged timing residuals

$$R_I = \frac{\sum_{i=1}^{N_I} R_{Ii} \sigma_{Ii}^{-2}}{\sum_{i=1}^{N_I} \sigma_{Ii}^{-2}}$$

$$\sigma_I^2 = \left( \sum_{i=1}^{N_I} \sigma_{Ii}^{-2} \right)^{-1} \frac{1}{N_I - 1} \sum_{i=1}^{N_I} (R_{Ii} - R_I)^2 \sigma_{Ii}^{-2}$$

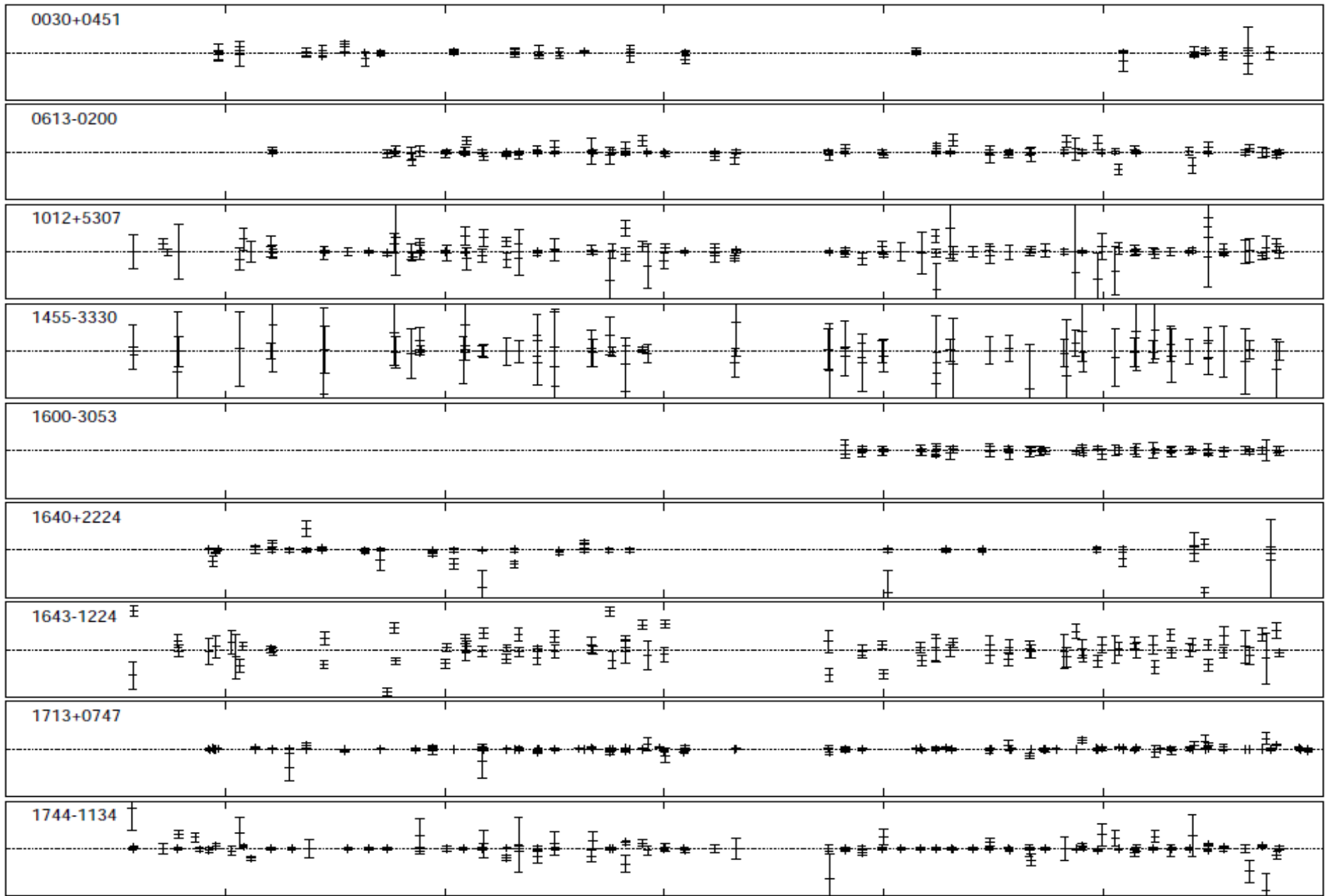




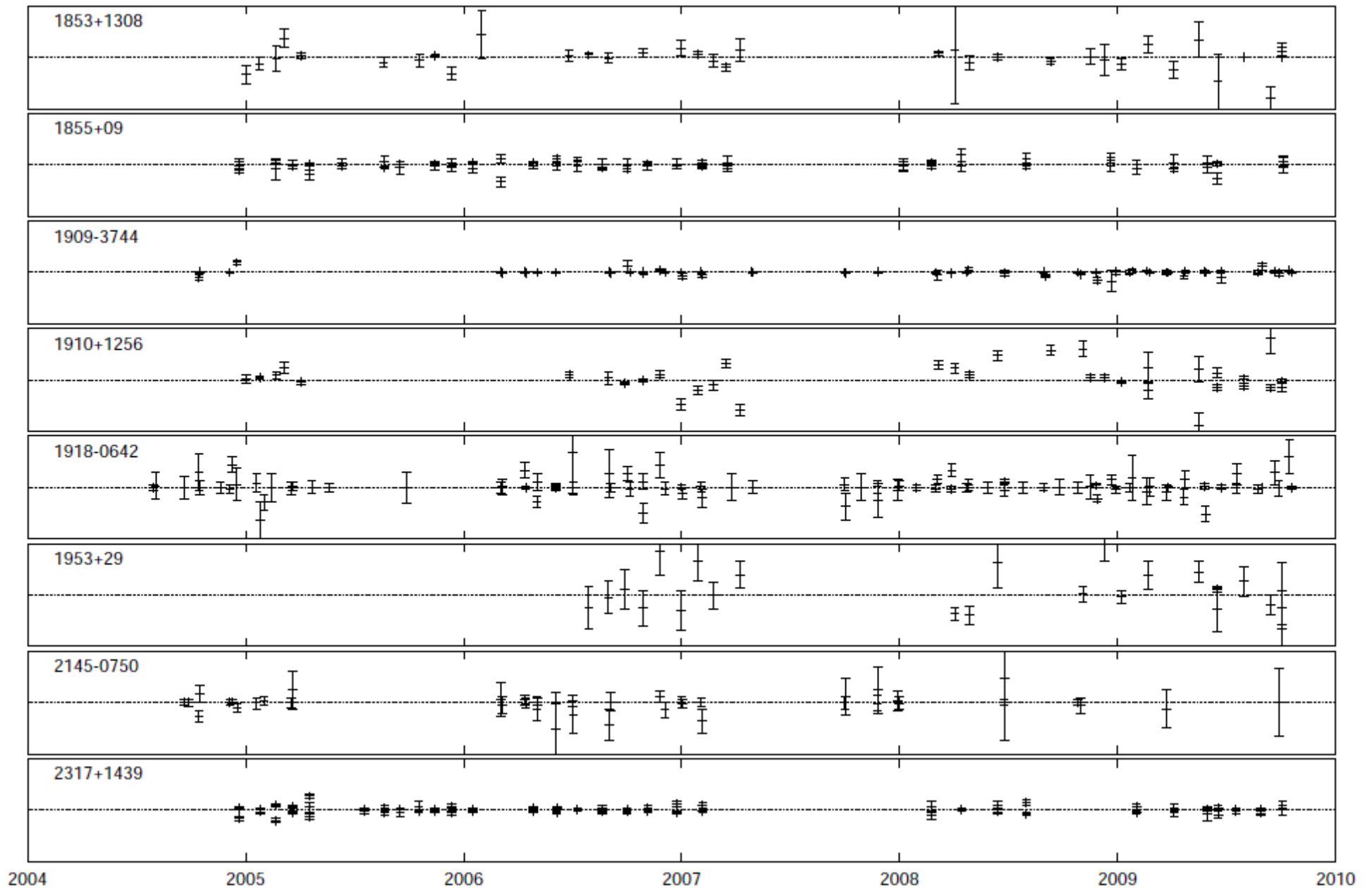
MJD - 54294.5

plk v.3.0 (G. Hobbs)

# Overview of timing residuals 1



# Overview of timing residuals 2



# Selected timing effects

SELECTED TIMING EFFECTS

Term	Type <sup>a</sup>	Mean Part		Stochastic Part		Achromatic or Chromatic <sup>b</sup>	Fluctuation Spectrum		PSR-PSR Correlation <sup>d</sup>	Comments
		Symbol	Value	Symbol	Value		Signature <sup>c</sup>	Shape		
Spin rate	A	$t_{\text{spin}}$	yr	$\Delta t_{\text{spin}}$	$\mu\text{s} - \text{s}$	a	B, R	$f^{-4} - f^{-6}$	U	
Magnetosphere:										
Pulse Shape	A, T	$t_{\text{p}}$	$\mu\text{s} - \text{ms}$	—	—	c	—	—	U	$\nu^{-0.3}$
Pulse Jitter	A, T	—	—	$\Delta t_{\text{J}}$	$< \mu\text{s} - \text{ms}$	c	W, B	see text	U	$\nu^{-0.3}$
Orbital	A	$t_{\text{orb}}$	hr	$\Delta t_{\text{orb}}$	$< \text{ms}$	a	L, R	$f^{-5/3}$	U	
Dispersion	A, T	$t_{\text{DM}}$	$\lesssim \text{s}$	$\Delta t_{\text{DM}}$	$\lesssim 100\mu\text{s}$	C	R	$f^{-5/3}$	U	$\nu^{-2}$
Faraday Rotation	A, T	$t_{\text{RM}}$	$\lesssim \mu\text{s}$	$\Delta t_{\text{RM}}$	$\lesssim \text{ns}$	C	R	$f^{-5/3}$	U	$\nu^{-3}$
Interstellar Turbulence										
Pulse Broadening	A, T	$t_{\text{PBF}}$	$\text{ns} - \text{s}$	$\Delta t_{\text{PBF}}$	$< \text{ns} - \text{ms}$	C	—	complex	U	$\nu^{-4.4}$
DISS	A, T	—	—	$\Delta t_{\delta\text{PBF}}$	$\lesssim \mu\text{s}$	C	W	flat	U	$\nu^{-1.6} - \nu^{-4.4}$
RISS	A, T	$t_{\text{PBF,RISS}}$	$\lesssim \mu\text{s}$	$\Delta t_{\delta\text{PBF,RISS}}$	$\lesssim \mu\text{s}$	C	R	$f^{-7/3}$	U	?
Angle of Arrival	A, T	—	—	$\Delta t_{\text{AOA}}$	$\lesssim \mu\text{s}$	C	R	$f^{-2/3}$	U	$\nu^{-4}$
Angle of Arrival	A, T	—	—	$\Delta t_{\text{AOA,SSBC}}$	$\lesssim \mu\text{s}$	C	R	$f^{-1/3}$	U	$\nu^{-2}$
Multipath averaging	A, T	—	—	$\Delta t_{\text{DM},\nu}$	$\lesssim 0.1\mu\text{s}$	C	R	complex	U	$\nu^{-23/6}$
Astrometric <sup>e</sup>	T	$t_{\text{AST}}$	—	$\Delta t_{\text{AST}}$	—	a	—	—	U	
Newtonian solar perturbations	T	—	—	$\Delta t_{\text{Newt,SSBC}}$	—	a	—	—	C	dipolar
Radiometer Noise	T	—	—	$\Delta t_{\text{S/N}}$	$< \mu\text{s} - \text{ms}$	c→C	W	flat	U	$\nu^0 \rightarrow \nu^{-2.7}$
Polarization	T	—	—	$\Delta t_{\text{pol}}$	—	c	W	flat	U	
Gravitational Lensing	A	$t_{\text{GL}}$	—	$\Delta t_{\text{GL}}$	—	a	—	—	U	Episodic
Cosmic Strings	A	$t_{\text{STR}}$	—	—	—	a	R	$f^{-16/3}$	U	Red noise if multiple events
Gravitational Waves	A	—	—	$\Delta t_{\text{GW}}$	$\lesssim 100 \text{ ns}$	a	R	$f^{-13/3}$	C, U	Two terms

<sup>a</sup>A = astrophysical, T= timing estimation error

<sup>b</sup>a = achromatic, C = strongly chromatic, c = weakly chromatic

<sup>c</sup>Fluctuation spectrum properties: R = red, W = white, B = bandpass, L = lowpass

<sup>d</sup>U = uncorrelated between different pulsar lines of sight, C = correlated

<sup>e</sup>Includes clock errors and Earth spin variations

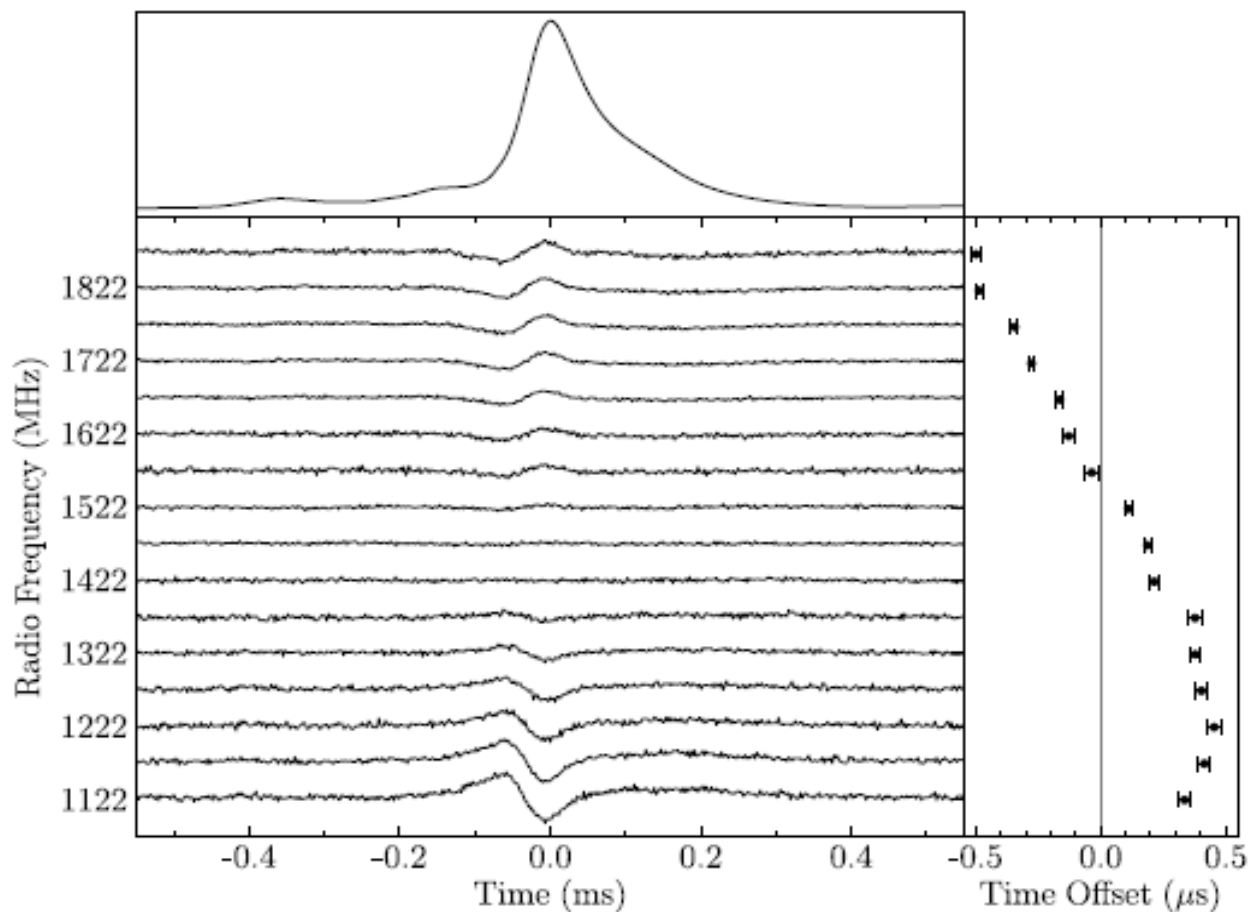


FIG. 4.— Differential profile evolution and offset times vs. frequency for GBT at 1.4 GHz. The top panel shows the broadband template, and each smaller curve (in the same arbitrary units as the top panel) shows the differential profile compared to the broadband. Both a shift and an amplitude change are present. The offset times shown in the right panel are from matched filtering for the different frequencies when the corresponding 50 MHz template is used. Thus, the offset represents the timing errors that would be present if one used the same profile across the band.

