Stochastic background of gravitational waves from cosmological sources

Chiara Caprini

IPhT CEA-Saclay
OUTLINE

• overview of GW from the early universe: basic properties
• upper bounds and sensitivities to GW stochastic backgrounds
• examples of sources operating in the early universe:
  - inflation
  - particle production during inflation
  - preheating
  - cosmic strings
  - scalar field self-ordering
  - first order phase transitions in the early universe (EWPT, QCDPT)
Primordial GW: “fossil radiation”

- as the universe expands, particles can get out of thermal equilibrium

- the weakest the interaction, the earlier in the history of the universe the particles decouple

\[
\frac{\Gamma(T)}{H(T)} < 1
\]

rate of the interaction
maintaining thermal equilibrium

\[
\Gamma = n\sigma v
\]

rate of expansion of the universe

- they propagate freely after decoupling, without interaction

- particles that decouple at temperature \( T_{\text{dec}} \) carry direct information about the universe at that temperature
GW: unique probe of the very early universe

a few such decoupling events in the universe history
GW: unique probe of the very early universe

photons decouple: CMB

\[ T_{\text{dec}} = 0.3 \text{ eV} \]

- confirm big bang theory
- temperature fluctuations: seeds for structure formation
- informations on the physics generating them (inflation)
- informations on the content of the universe, the curvature
- ...

today: \[ T_0 \approx 2 \cdot 10^{-4} \text{ eV} \]
neutrinos decouple:

\[ T_{\text{dec}} = 0.3 \text{ eV} \]

photons decouple: CMB

\[ T_{\text{dec}} = 1 \text{ MeV} \]

neutrinos decouple:

\[ \nu \text{ background} \]

- indirect evidence: CMB, structure formation, BBN
- masses, species...
GW: unique probe of the very early universe

for gravitons in thermal equilibrium, the decoupling temperature would be

$$T_{\text{dec}} = 10^{19} \text{ GeV}$$

photons decouple: CMB

$$T_{\text{dec}} = 1 \text{ MeV}$$

neutrinos decouple:

$$\nu \text{ background}$$

$$T_{\text{dec}} = 0.3 \text{ eV}$$
GW: unique probe of the very early universe

because of the weakness of the gravitational interaction, the universe is transparent to primordial GW

GW from any generation process in the early universe carry direct information on the process itself

detection of the GW “fossil radiation”: big step forward in our knowledge of the very early universe
GW from cosmological sources

tensor perturbations of FRW metric:

\[ ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j] \]

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij} \]

**source:** \( \Pi_{ij} \) tensor anisotropic stress

- **fluid:** \( \Pi_{ij} \sim \gamma^2 (\rho + p) v_i v_j \)
- **electromagnetic field:** \( \Pi_{ij} \sim \frac{(E^2 + B^2)}{3} - E^i E^j - B^i B^j \)
- **scalar field:** \( \Pi_{ij} \sim \partial_i \phi \partial_j \phi \)
GW from cosmological sources

source: amplification of vacuum fluctuations during inflation

Inflation:
phase of accelerated expansion of the background
possibility of particle creation

\[ \frac{a''}{a} = a^2 H^2 > 0 \]

\[ v_{\pm}(t) = M_{Pl} a(t) h_{\pm}(t) \]

✓ canonically normalised free field
✓ quantisation
✓ homogeneous wave equation
✓ harmonic oscillator with time dependent frequency
✓ quantum field: zero point fluctuations

\[ v''_{\pm}(t) + (k^2 - a^2 H^2)v_{\pm}(t) = 0 \]
GW from cosmological sources

source: amplification of vacuum fluctuations during inflation

\[ v''_{\pm}(t) + (k^2 - a^2 H^2)v_{\pm}(t) = 0 \]

\[ k \gg aH \quad \text{sub-hubble modes} \]
\[ k \ll aH \quad \text{super-hubble modes} \]

\[ \omega^2(t) = k^2 \]
\[ \omega^2(t) = -a^2 H^2 \]

free field in vacuum
zero occupation number

source: a spectrum of gravitons has been generated by the fast expansion of the background and the stretching of the modes outside the horizon

\[ n_k = 0 \]
\[ n_k \gg 1 \]
**stochastic background of GW**

- **sources from the early universe:**
  stochastic background of GW, statistically homogenous, isotropic and Gaussian

\[ \langle \dot{h}_{ij}(k)\dot{h}_{ij}^{*}(q) \rangle = (2\pi)^3 \delta(k - q)|\dot{h}(k)|^2 \]

- statistical homogeneity and isotropy
- power spectrum

- causal source: many independent horizon volumes visible today

- inflation: intrinsic, quantum fluctuations that become classical (stochastic) outside the horizon
stochastic background of GW

• sources from the early universe:
  stochastic background of GW, statistically homogenous, isotropic and Gaussian

\[ \langle \hat{h}_{ij}(k)\hat{h}^*_{ij}(q) \rangle = (2\pi)^3 \delta(k - q)|\dot{h}(k)|^2 \]

statistical homogeneity and isotropy

power spectrum

GW energy density:

\[ \Omega_{GW} = \frac{\rho_{GW}}{\rho_c} = \frac{\langle \dot{h}_{ij}\dot{h}_{ij} \rangle}{32\pi G \rho_c} = \int \frac{df}{f} \frac{d\Omega_{GW}}{d\ln f} \]

frequency today (redshifted by expansion)

\[ f = \frac{k}{2\pi} \frac{a(t)}{a_0} \]
characteristic frequency for causal sources

causal (not inflation) source of GW cannot operate beyond the cosmological horizon:

\[ k_* \leq H_* \]
Characteristic frequency for causal sources

causal (not inflation) source of GW cannot operate beyond the cosmological horizon:

$$k_* \leq H_*$$

standard thermal history

$$H = \frac{\dot{a}}{a} \quad a = \frac{T_0}{T}$$

characteristic frequency today

$$f_c = \frac{k_* a_*}{2\pi a_0} \leq 1.6 \cdot 10^{-4} \text{ Hz} \quad \frac{T_*}{1 \text{ TeV}}$$

temperature (energy density) of the universe at the source time
Characteristic frequency for causal sources

$T (\text{GeV})$

$h^2 \Omega_{GW}$

- PTA
- eLISA
- ET
- adv LIGO

QCDPT
EWPT

$\text{f (Hz)}$
Characteristic frequency for causal sources

T (GeV)

$\Omega_{\text{GW}}$ vs. $f$ (Hz)

PTA

eLISA

QCDPT

EWPT

physics beyond the standard model

$10^{-9}$ $10^{-7}$ $10^{-5}$ $0.001$ $0.1$ $10$ $1000$

$10^{-13}$ $10^{-11}$ $10^{-9}$ $10^{-7}$ $10^{-5}$ $0.001$ $0.1$ $10$ $1000$

$10^2$ $10^4$ $10^6$ $10^8$ $10^{10}$

$0.001$ $0.1$ $10$ $1000$

$10^{-1}$ $10^0$ $10^1$

$10^{-7}$ $10^{-5}$ $10^{-3}$ $10^{-1}$

$10^{-9}$ $10^{-7}$ $10^{-5}$ $0.001$ $0.1$ $10$ $1000$

$10^{-13}$ $10^{-11}$ $10^{-9}$ $10^{-7}$ $10^{-5}$ $0.001$ $0.1$ $10$ $1000$

$10^2$ $10^4$ $10^6$ $10^8$ $10^{10}$

$0.001$ $0.1$ $10$ $1000$
Inflation

Cosmological dark age (reheating, baryogenesis, phase transitions...)

BBN

CMB anisotropies

GW

GW

light elements

\[ h_{ij} \]

\[ E_{\text{Planck}} \quad 10^{16} \text{GeV} \quad \text{SUSY} \quad \text{EW} \quad \text{QCD} \quad \text{MeV} \quad \text{eV} \]

\[ (\text{BICEP}_2) \]

\[ \gamma \]

CMB

LIGO

eLISA

Pulsars
Limits on a stochastic background

- **Nucleosynthesis and CMB**: measure of the relativistic energy density in the universe
  \[
  h^2 \Omega_{GW} \lesssim 7.8 \cdot 10^{-6} \quad h^2 \Omega_{GW} < 6.9 \cdot 10^{-6} \\
  f > 10^{-10} \text{ Hz} \quad f > 10^{-16} \text{ Hz} \quad \text{Smith et al, astro-ph/0603144}
  \]

- **LIGO science run 2005-2007**
  \[
  h^2 \Omega_{GW} \lesssim 6.9 \cdot 10^{-6} \quad 41 \text{ Hz} < f < 169 \text{ Hz} \quad \text{Abbott et al, 0910.5772}
  \]

- **PPTA 2013**
  \[
  h^2 \Omega_{GW} < 1.3 \cdot 10^{-9} \quad f \approx 2.8 \text{ nHz} \quad \text{Shannon et al 1310.4569}
  \]

- **COBE, WMAP, Planck** measured TEMPERATURE fluctuations in CMB
  \[
  h^2 \Omega_{GW} < 7 \cdot 10^{-11} \left( \frac{H_0}{f} \right)^2 \quad 10^{-18} \text{ Hz} < f < 10^{-16} \text{ Hz}
  \]
Limits on a stochastic background

Before BICEP2

COBE WMAP Planck

LIGO

BBN

CMB

PTA

$10^{-18}$ $10^{-14}$ $10^{-10}$ $10^{-6}$ $0.01$ $100$

$f$ (Hz)

$10^{-8}$ $10^{-5}$ $10^{-2}$ $1$ $10^2$ $10^4$ $10^6$ $10^8$ $10^{10}$

$T$ (GeV)

$h^2 \Omega_{GW}$

$10^{-15}$ $10^{-13}$ $10^{-11}$ $10^{-9}$ $10^{-7}$ $10^{-5}$ $10^{-3}$ $10^{-1}$
Limits on a stochastic background

$T$ (GeV)

$10^{-8}$ $10^{-5}$ $10^{-2}$ $1$ $10^2$ $10^4$ $10^6$ $10^8$ $10^{10}$

$\hbar^2 \Omega_{GW}$

$10^{-8}$ $10^{-5}$ $10^{-2}$ $1$ $10^2$ $10^4$ $10^6$ $10^8$ $10^{10}$

$10^{-18}$ $10^{-14}$ $10^{-10}$ $10^{-6}$ $0.01$ $100$

(hop) detection of GW from inflation by BICEP2: POLARISATION

COBE WMAP Planck

BBN

PTA

LIGO
GW influence CMB photons and leave an imprint in CMB anisotropies

- **temperature**: limit by COBE, WMAP, Planck
  \[ \frac{\delta T}{T} = - \int_{t_{\text{dec}}}^{t_0} \dot{h}_{ij} n^i n^j dt \]

- **polarisation**: BB spectrum measured by BICEP2
  generated at photon decoupling time, from Thomson scattering of electrons by a quadrupole temperature anisotropy in the photons

distortion of an homogeneous photon patch by GW:
imprint quadrupole anisotropy in the photon distribution
GW influence CMB photons and leave an imprint in CMB anisotropies

- **polarisation**: BB spectrum measured by BICEP2 generated at photon decoupling time, from Thomson scattering of electrons by a quadrupole temperature anisotropy in the photons

if the incident radiation is not isotropic, the scattered light at the end of decoupling (CMB) is polarised
GW influence CMB photons and leave an imprint in CMB anisotropies

- **polarisation**: BB spectrum measured by BICEP2 generated at photon decoupling time, from Thomson scattering of electrons by a quadrupole temperature anisotropy in the photons

  polarisation patterns (independent on the reference frame)

  generated by **scalar** and **tensor** perturbations

  generated only by **tensor** perturbations (or foregrounds)

  E mode     B mode
B polarisation power spectra from BICEP last release
GW background from inflation

inflation amplifies both vacuum fluctuations of graviton (tensor mode) and of the scalar field driving inflation (scalar mode)

amplitude of scalar perturbations power spectrum

$$S \propto \frac{1}{\epsilon} \frac{H^2}{M_P} = \frac{1}{\epsilon} \frac{V}{M_P^4}$$

measured by CMB temperature

amplitude of tensor perturbations power spectrum

$$T \propto \frac{H^2}{M_P} = \frac{V}{M_P^4}$$

measured by CMB B polarisation

from the measurement of $T/S$ one can infer the energy scale of inflation

$$V^{1/4} \simeq 2.25 \times 10^{16} \text{ GeV} \left(\frac{r}{0.2}\right)^{1/4}$$

high scale inflation
GW background from inflation

\[ V^{1/4} \approx 2.26 \cdot 10^{16} \text{GeV} \]

\[ k > H_{\text{inf}} \]
smooth transition
no particle creation

\[ H_0 < k < H_{\text{eq}} \]
enter the horizon
in the MD era

\[ H_{\text{eq}} < k < H_{\text{inf}} \]
abrupt transition
particle creation

\[ \frac{d \Omega_{GW}}{d \ln k} \approx 10^{-5} \left( \frac{H_{\text{inf}}}{m_{\text{Pl}}} \right)^2 \left( \frac{k}{k_{\text{eq}}} \right)^{n_T} \]

Grishchuk 1974, Starobinsky 1979, Abbott and Harari 1986, ...
BUT... there are other possible sources of GW in the early universe promising for detection with future interferometers or PTA mechanisms that produce a non-zero tensor anisotropic stress

\[ \ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij} \]
Possible GW sources in the early universe

- inflation
- particle production during inflation
- fluid stiffer than radiation after inflation
- preheating after inflation
- phase transitions at the end or during inflation
- cosmic (super)strings
- first order phase transitions
- non-perturbative decay of SUSY flat directions
- unstable domain walls
- primordial black holes
- scalar field self-ordering
- ...

GW background from particle production during inflation

- production of particles in the time dependent background due to the evolution of the inflaton field

- observable signal: gauge fields coupled to pseudoscalar inflaton in linear inflationary potential

\[
\frac{1}{4} \phi F_{\mu\nu} \tilde{F}_{\mu\nu}
\]

\(N_{\text{CMB}} = 60\) (p=1)

\(\xi_{\text{CMB}} = 2.66\)

\(\xi_{\text{CMB}} = 2.33\)

\(\xi = 0\)

Cook and Sorbo 2012
Barnaby et al 2012
GW background from preheating

- reheating: the energy density driving inflation is converted in radiation and matter

- preheating: possible first stage of this conversion, the inflaton decays in an explosive and highly inhomogeneous way

\[ R_* \approx \frac{4 \cdot 10^{10} \text{ Hz}}{f_*^{1/4} R_* \rho_p^{1/4}} \]

**GW background from cosmic strings**

- one dimensional topological defects formed during symmetry breaking phase transitions or in the context of string theory at the end of brane inflation

- form a **cosmological network** which reach a scaling regime: it looks statistically the same at any time, the only relevant scale is the Hubble length

- decay by GW emission: long strings intercommute and form smaller loops which oscillate relativistically and emit GW

GW background from cosmic strings

- parameters: tension, reconnection probability (loop size: probably solved)
- spectral shape extended in frequency because of continuous production
GW background from scalar field self-ordering

- after the spontaneous breaking of a global symmetry, N component scalar field gets different vev in different causal patches

- horizon grows, the field re-orders but anisotropic stress is present at the boundaries and sources GW

- spectral shape extended in frequency because of continuous production

![Graph showing spectral shapes and frequency ranges](image-url)
GW background from first order phase transitions

universe expands and temperature decreases: PTs, if first order lead to GW

potential barrier separates true and false vacua

quantum tunneling across the barrier: nucleation of bubbles of true vacuum

source: $\Pi_{ij}$ tensor anisotropic stress

- collisions of bubble walls
- magnetohydrodynamic turbulence in the primordial fluid
GW background from first order phase transitions: EWPT

\[ T_* \simeq 100 \text{ GeV} \quad \rightarrow \quad \text{eLISA frequency band} \]

\[ V(H) = -\frac{\mu^2}{2}H^2 + \frac{\lambda}{4}H^4 + \frac{1}{8M^2}H^6 \]

SM + dimension six operator, \( \eta=0.2 \)

Huber and Konstandin 2008

Binétruy et al. 2012

increasing strength
GW background from first order phase transitions: EWPT

Holographic Phase Transition

Randall and Servant 2007
Konstandin et al 2010

$T_\ast \approx 10^4$ GeV
$T_\ast \approx 100$ GeV

$eLISA$

$T_\ast \approx 10^4$ GeV
$T_\ast \approx 100$ GeV

Binétruy et al 2012
GW background from first order phase transitions: QCDPT

QCDPT: if lepton asymmetry is large

\[ T_* = 100 \text{ MeV} \]

Schwarz and Stuke 2009

Current NANOGrav sensitivity

PTA 2020

LISA

CC et al 2010
Conclusions

• we have little information about the physics and the processes operating in the very early universe

• due to their small interaction rate, GW can in principle provide us with this information

• the frequency of GW maps the temperature/energy scale in the early universe

• GW by inflation at the energy scale indicated by the recent BICEP2 result are not visible with the next generation interferometers or by PTA

• but it is in principle possible to generate a stochastic background of GW detectable by them: preheating, cosmic strings, PTs...

• GW are a powerful mean to learn about the early universe and high energy physics: detection is extremely difficult but great payoff