Stochastic background of gravitational waves from cosmological sources

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OUTLINE

- overview of GW from the early universe: basic properties

- upper bounds and sensitivities to GW stochastic backgrounds

- examples of sources operating in the early universe:

- inflation
- particle production during inflation
- preheating
- cosmic strings
- scalar field self-ordering
- first order phase transitions in the early universe (EWPT, QCDPT)

Primordial GW: "fossil radiation"

- as the universe expands, particles can get out of thermal equilibrium

- the weakest the interaction, the earlier in the history of the universe the particles decouple

rate of the interaction maintaining thermal equilibrium

$$\Gamma = n\,\sigma\,v$$

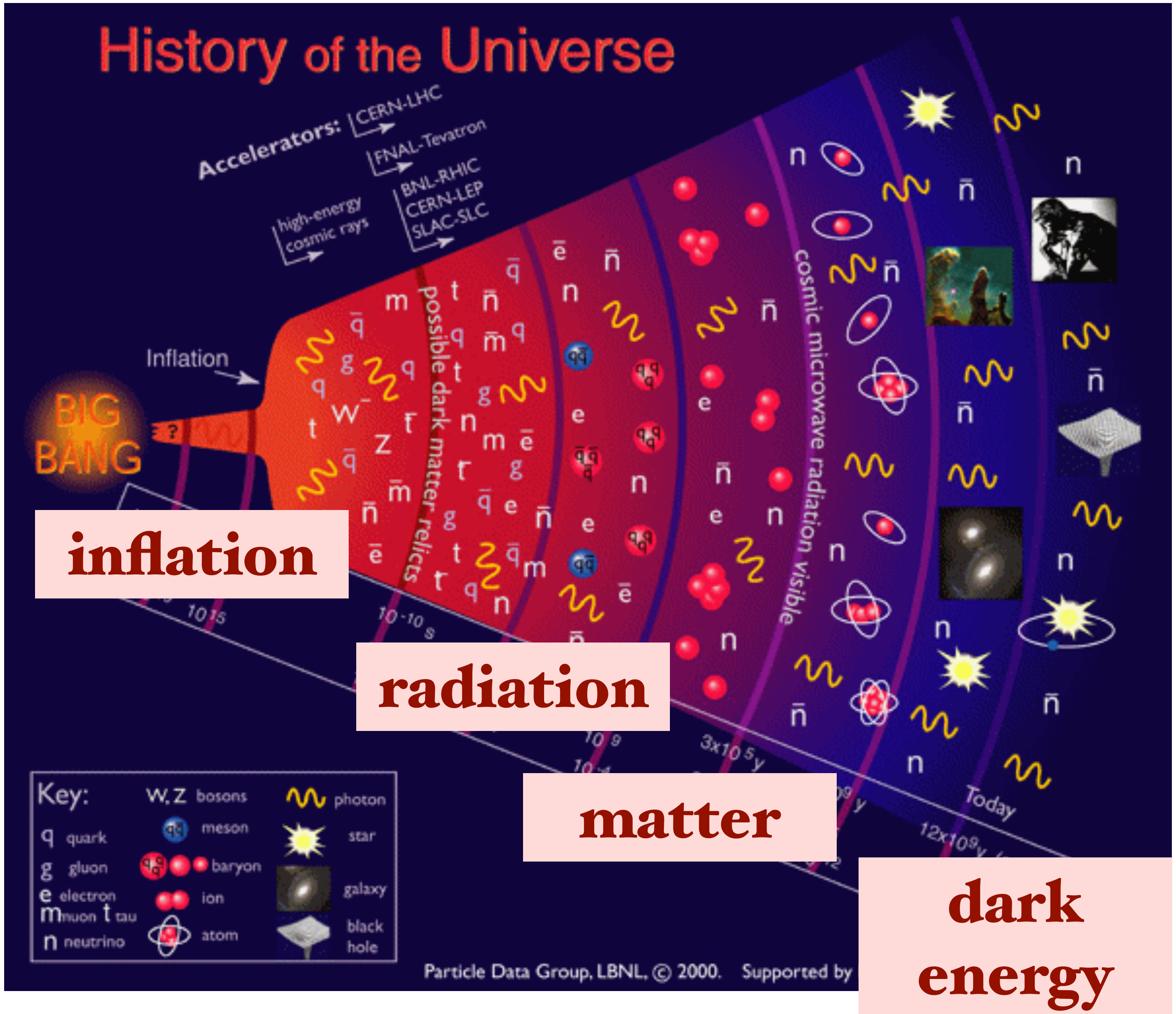
$$\frac{\Gamma(T)}{H(T)} < 1$$

rate of expansion of the universe

- they propagate freely after decoupling, without interaction

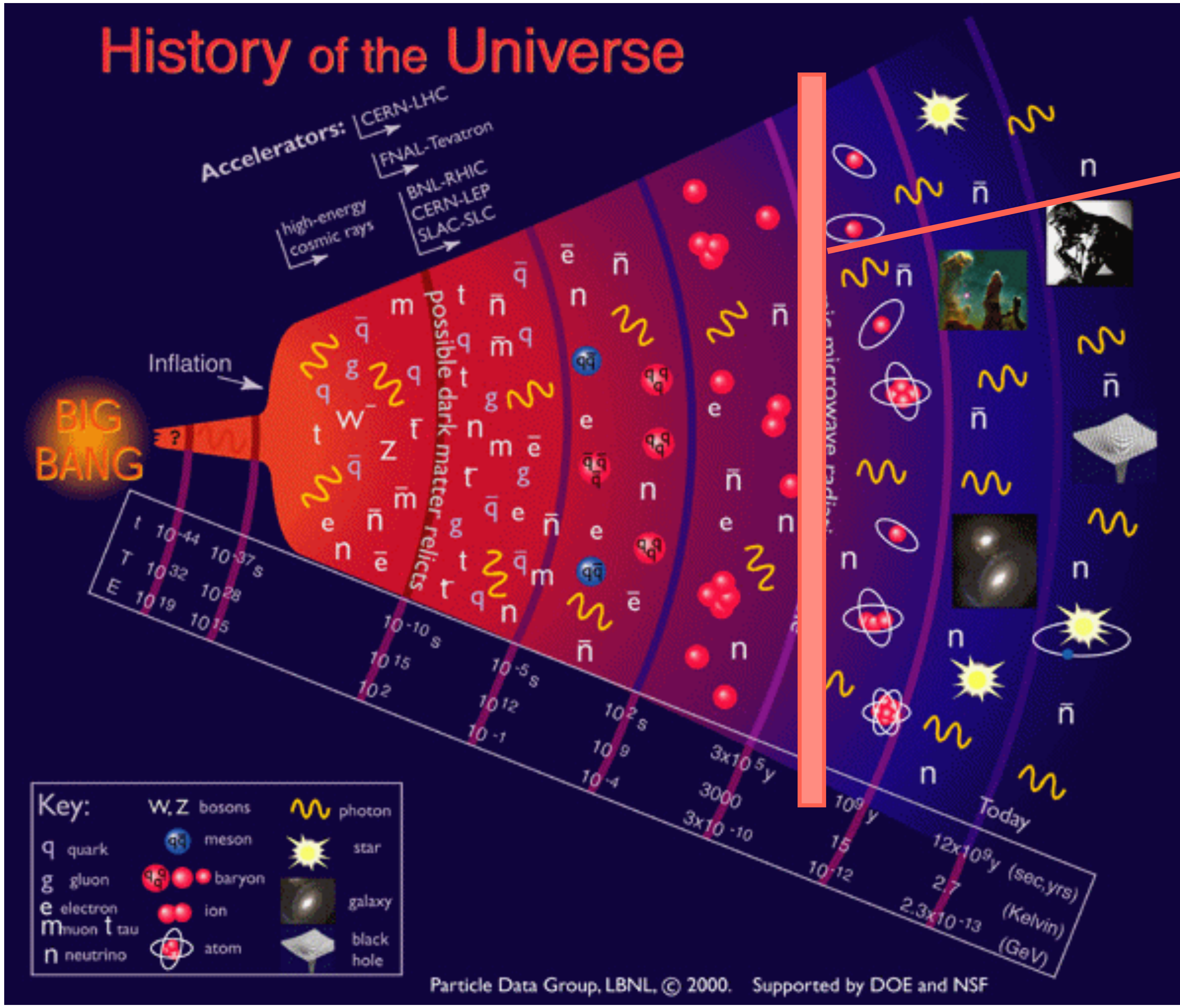
- particles that decouple at temperature $T_{\rm dec}$ carry direct information about the universe at that temperature

GW: unique probe of the very early universe



a few such decoupling events in the universe history

GW: unique probe of the very early universe



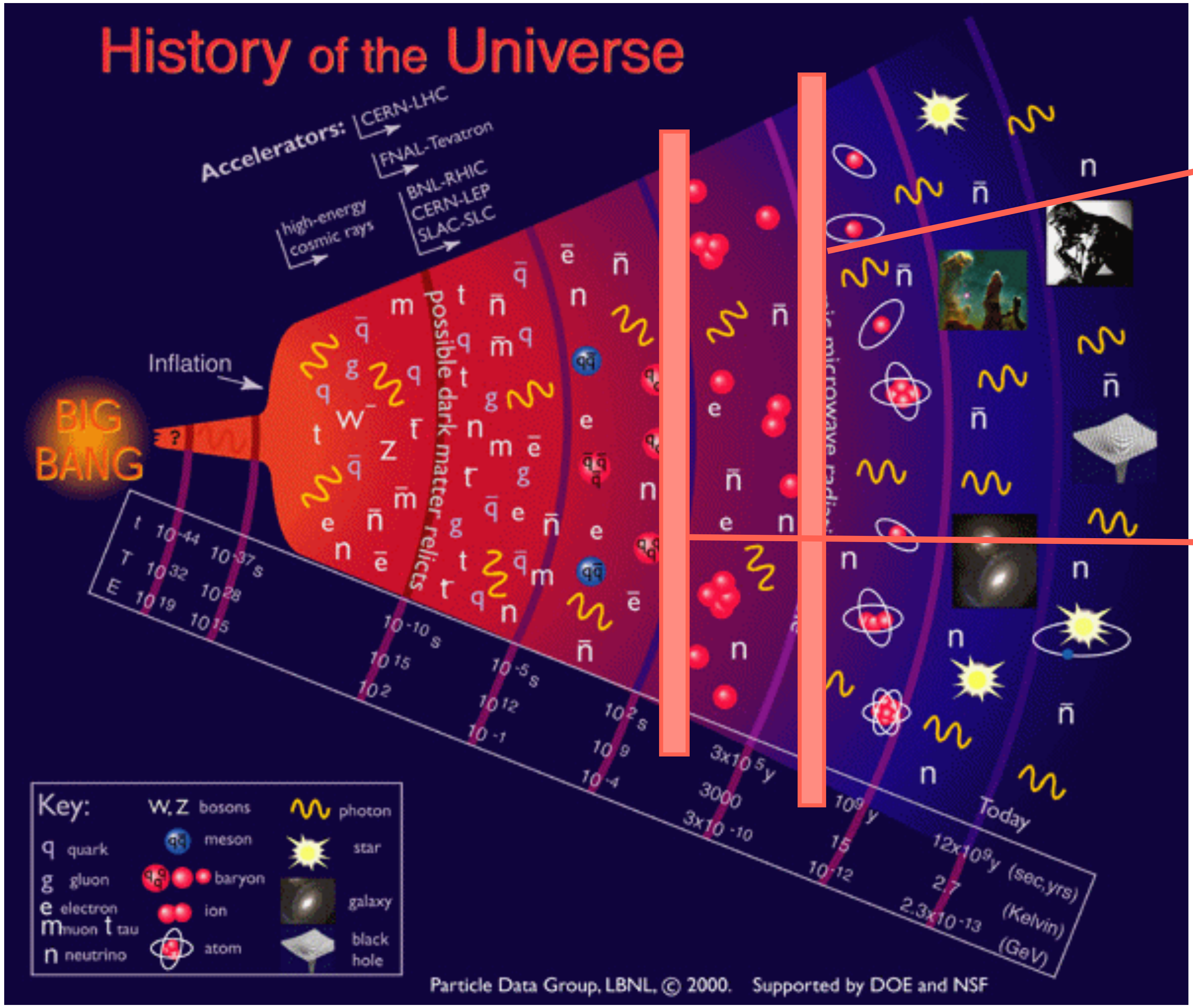
$$T_{\rm dec} = 0.3\,{\rm eV}$$

photons decouple: CMB

- confirm big bang theory
- temperature fluctuations: seeds for structure formation
- informations on the physics generating them (inflation)
- informations on the content of the universe, the curvature
- ...

$$\text{today}: \quad T_0 \simeq 2 \cdot 10^{-4}\,{\rm eV}$$

GW: unique probe of the very early universe



$$T_{\rm dec} = 0.3\,{\rm eV}$$

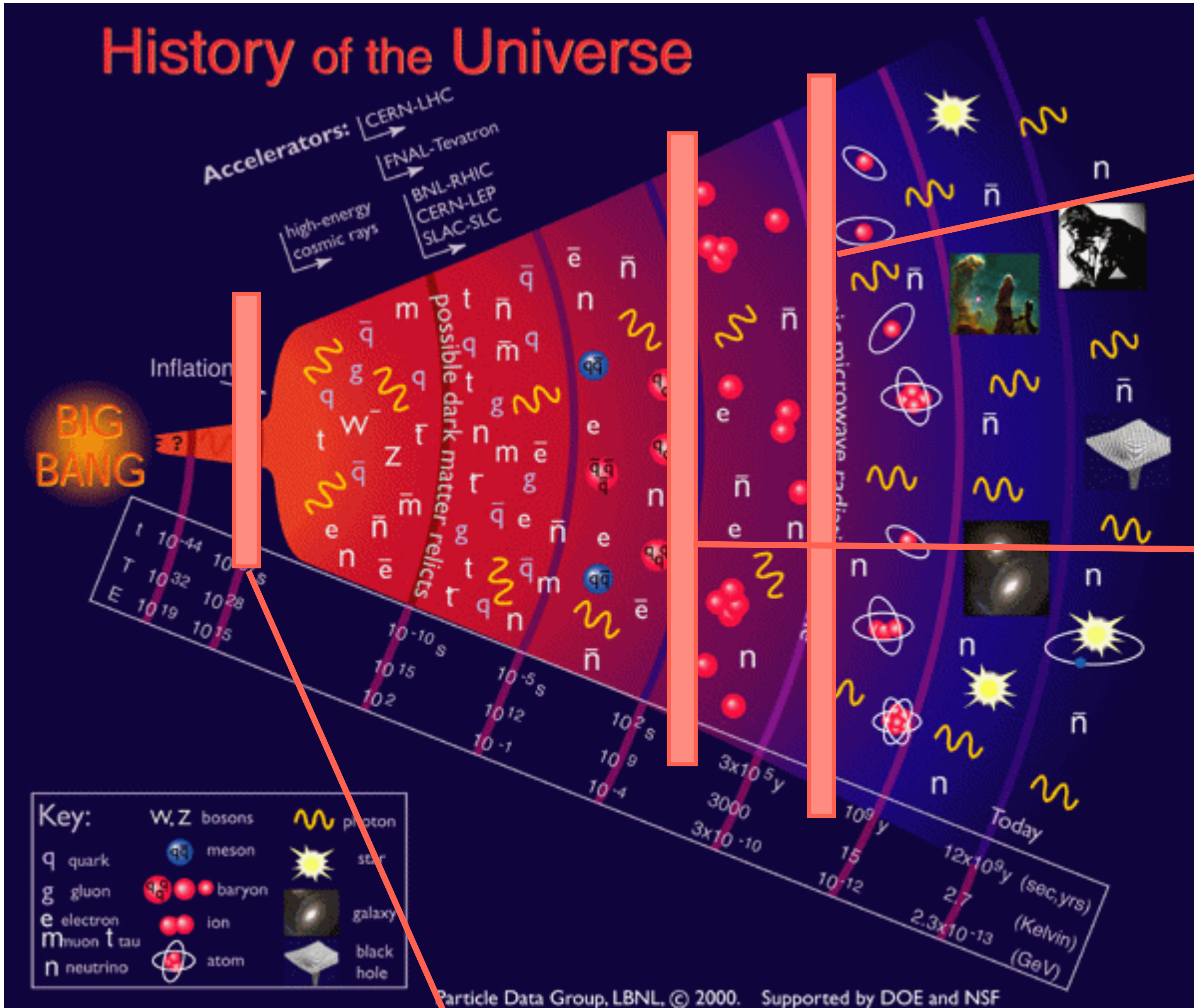
photons decouple: CMB

$$T_{\rm dec} = 1\,{\rm MeV}$$

neutrinos decouple:
ν background

- indirect evidence: CMB, structure formation, BBN
- masses, species...

GW: unique probe of the very early universe



$$T_{\rm dec} = 0.3\,{\rm eV}$$

photons decouple: CMB

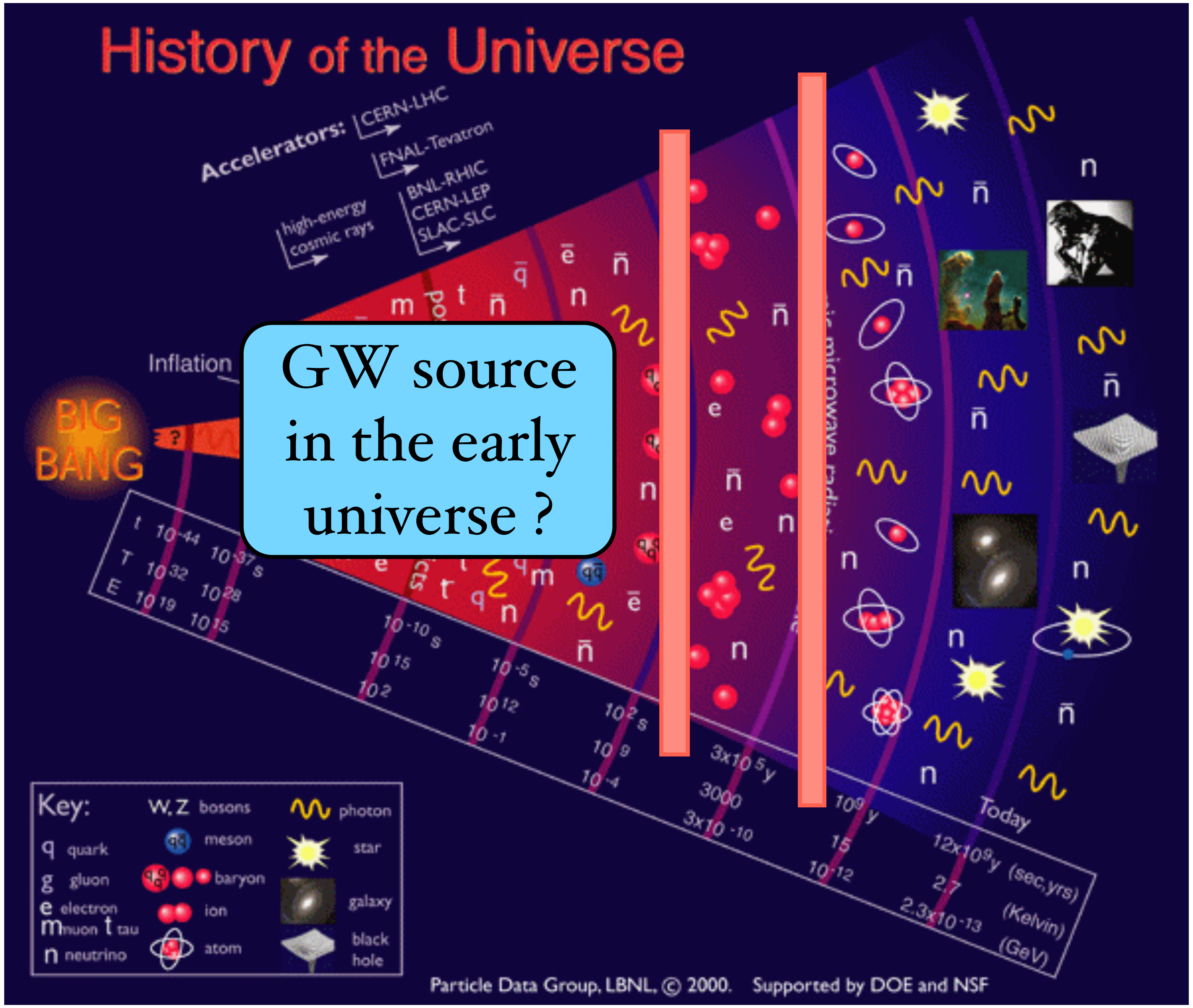
$$T_{\rm dec} = 1\,{\rm MeV}$$

neutrinos decouple:
ν background

for gravitons in thermal equilibrium, the decoupling temperature would be

$$T_{\rm dec} = 10^{19}\,{\rm GeV}$$

GW: unique probe of the very early universe



because of the weakness of the gravitational interaction the universe is transparent to primordial GW

GW from any generation process in the early universe carry direct information on the process itself

detection of the GW “fossil radiation”: big step forward in our knowledge of the very early universe

GW from cosmological sources

tensor perturbations of FRW metric: $\quad (h_i{}^i = h_i{}^j{}_{|j} = 0)$

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\, h_{ij} = 16\pi G\,\Pi_{ij}$$

source: Π_{ij} tensor anisotropic stress

- fluid : $\quad \Pi_{ij} \sim \gamma^2(\rho + p)\, v_i v_j$
- electromagnetic field : $\quad \Pi_{ij} \sim \dfrac{(E^2 + B^2)}{3} - E^i E^j - B^i B^j$
- scalar field : $\quad \Pi_{ij} \sim \partial_i \phi\, \partial_j \phi$

GW from cosmological sources

source: amplification of vacuum fluctuations during inflation

inflation:
phase of accelerated expansion of the background
possibility of particle creation

$$\frac{a''}{a} = a^2 H^2 > 0$$

$$v_\pm(t) = M_{Pl}\, a(t)\, h_\pm(t)$$

- ✓ canonically normalised free field
- ✓ quantisation
- ✓ homogeneous wave equation
- ✓ harmonic oscillator with time dependent frequency
- ✓ quantum field : zero point fluctuations

$$v''_\pm(t) + (k^2 - a^2 H^2) v_\pm(t) = 0$$

GW from cosmological sources

source: amplification of vacuum fluctuations during inflation

$$v''_{\pm}(t) + (k^2 - a^2 H^2) v_{\pm}(t) = 0$$

$k \gg a\,H$ sub - hubble modes

$\omega^2(t) = k^2$

free field in vacuum
zero occupation number

$n_k = 0$

$k \ll a\,H$ super - hubble modes

$\omega^2(t) = -a^2 H^2$

super - horizon modes have
occupation number very large

$n_k \gg 1$

source: a spectrum of gravitons has been generated by the fast expansion of the background and the stretching of the modes outside the horizon

stochastic background of GW

- sources from the early universe:
  stochastic background of GW, statistically homogenous, isotropic and Gaussian

$$\langle \dot{h}_{ij}(\mathbf{k}) \dot{h}^*_{ij}(\mathbf{q}) \rangle = (2\pi)^3 \delta(\mathbf{k}-\mathbf{q}) |\dot{h}(k)|^2$$

statistical homogeneity and isotropy

power spectrum

- causal source: many independent horizon volumes visible today

- inflation: intrinsic, quantum fluctuations that become classical (stochastic) outside the horizon

stochastic background of GW

- sources from the early universe:
  stochastic background of GW, statistically homogenous, isotropic and Gaussian

$$\langle \dot{h}_{ij}(\mathbf{k})\dot{h}^*_{ij}(\mathbf{q})\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{q})|\dot{h}(k)|^2$$

statistical homogeneity and isotropy

power spectrum

GW energy density:

$$\Omega_{\rm GW} = \frac{\rho_{\rm GW}}{\rho_c} = \frac{\langle \dot{h}_{ij}\dot{h}_{ij}\rangle}{32\pi G\,\rho_c} = \int \frac{{\rm d}f}{f}\frac{{\rm d}\Omega_{\rm GW}}{{\rm d}\ln f}$$

frequency today
(redshifted by expansion)

$$f = \frac{k}{2\pi}\frac{a(t)}{a_0}$$

Characteristic frequency for causal sources

causal (not inflation) source of GW cannot operate beyond the cosmological horizon:

$$\boxed{k_* \leq H_*}$$

Characteristic frequency for causal sources

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$$\boxed{k_* \leq H_*}$$

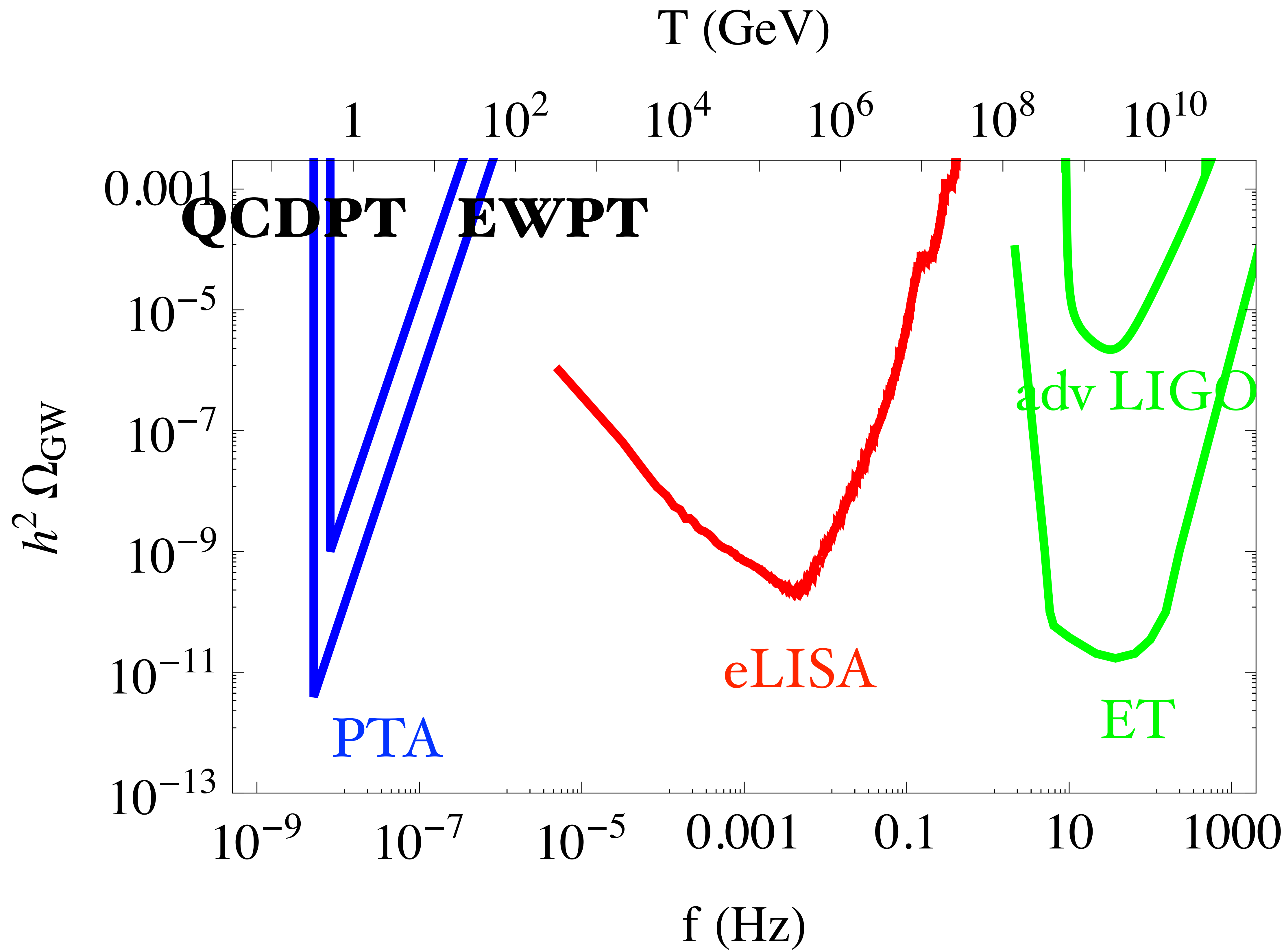
standard thermal history $\quad H = \frac{\dot{a}}{a} \qquad a = \frac{T_0}{T}$

$$f_c = \frac{k_*}{2\pi}\frac{a_*}{a_0} \leq 1.6 \cdot 10^{-4}\,\mathrm{Hz}\,\frac{T_*}{1\,\mathrm{TeV}}$$

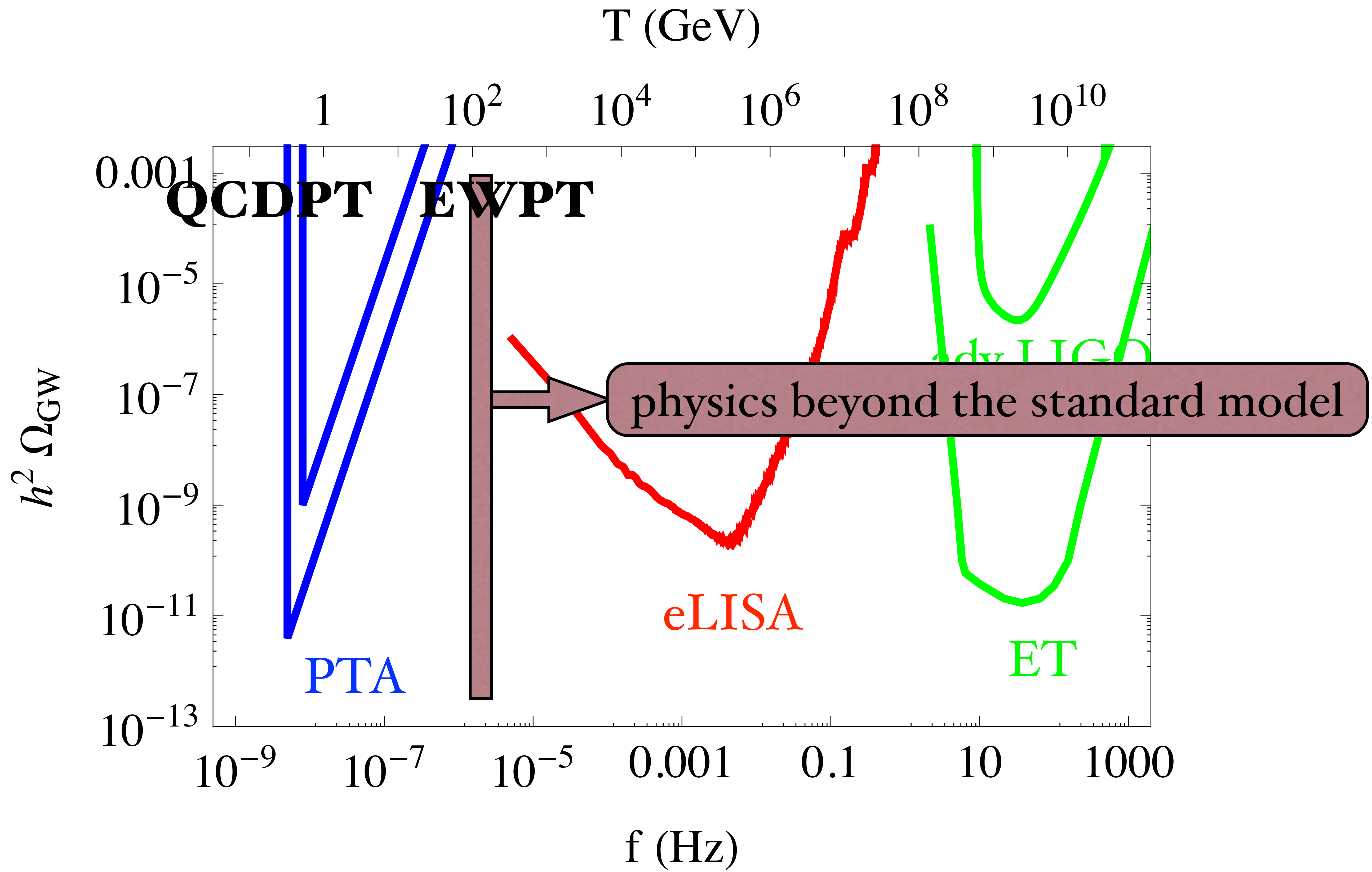
characteristic frequency today

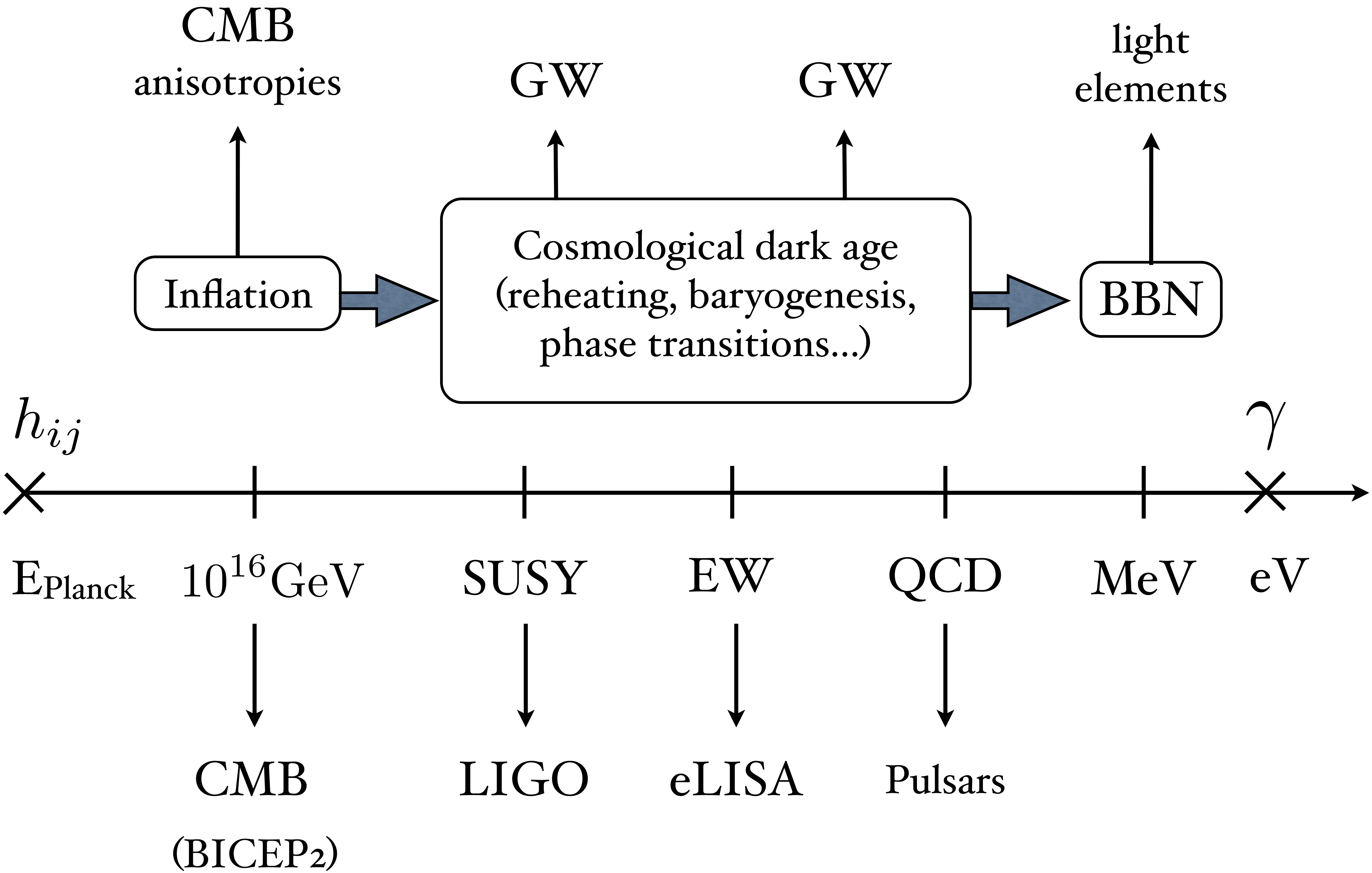
temperature (energy density) of the universe at the source time

Characteristic frequency for causal sources



Characteristic frequency for causal sources




CMB
anisotropies
GW
GW
light
elements
Inflation
Cosmological dark age
(reheating, baryogenesis,
phase transitions...)
BBN
h_{ij}
γ
E_{Planck}
10^{16}GeV
SUSY
EW
QCD
MeV
eV
CMB
(BICEP2)
LIGO
eLISA
Pulsars

Limits on a stochastic background

- Nucleosynthesis and CMB: measure of the relativistic energy density in the universe

$$h^2\Omega_{\rm GW} \lesssim 7.8\cdot 10^{-6} \qquad h^2\Omega_{\rm GW} < 6.9\cdot 10^{-6}$$
$$f > 10^{-10}\,\mathrm{Hz} \qquad f > 10^{-16}\,\mathrm{Hz}$$

Smith et al, astro-ph/0603144

- LIGO science run 2005-2007

$$h^2\Omega_{\rm GW} \lesssim 6.9\cdot 10^{-6} \qquad 41\,\mathrm{Hz} < f < 169\,\mathrm{Hz}$$

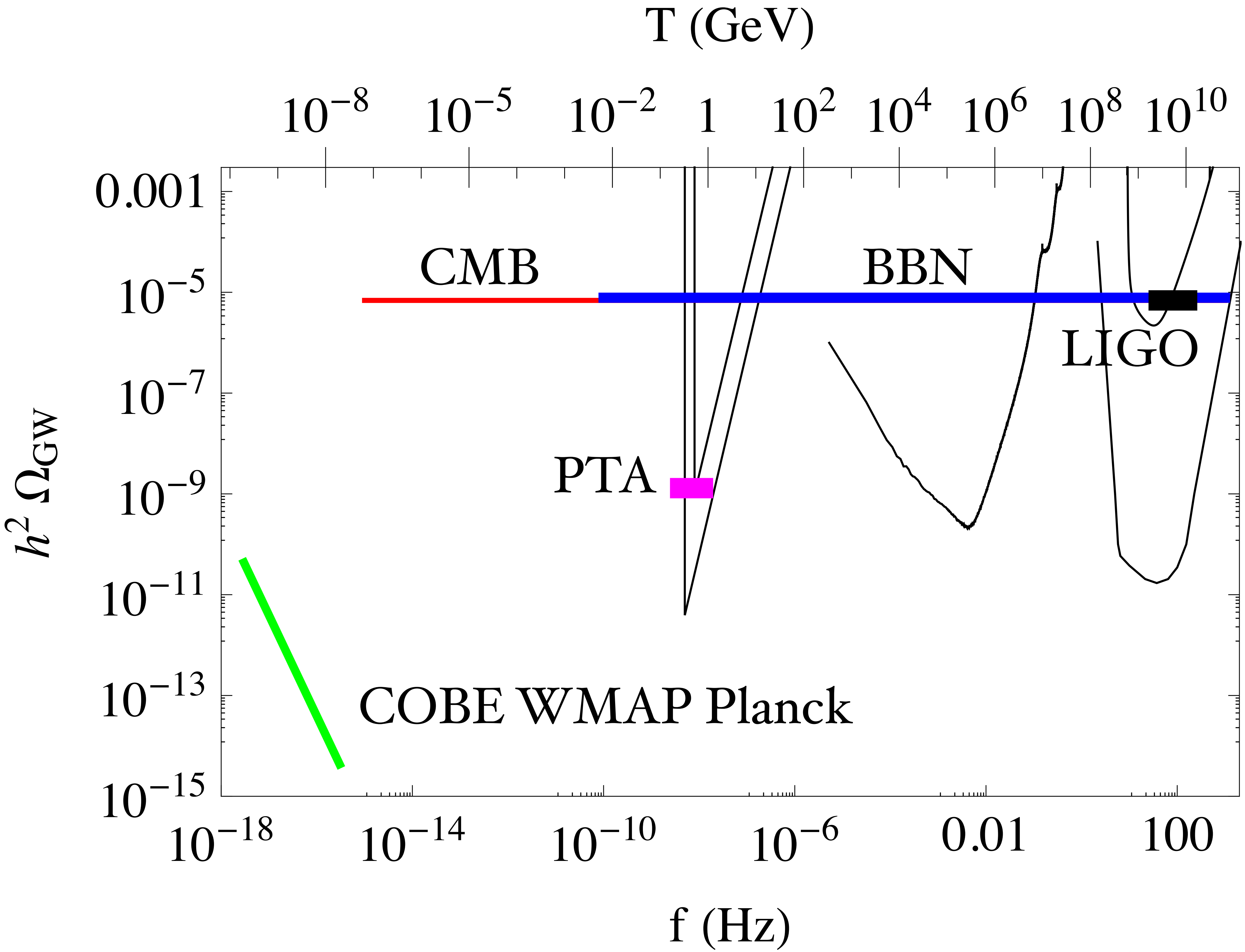
Abbott et al, 0910.5772

- PPTA 2013 $h^2\Omega_{\rm GW} < 1.3\cdot 10^{-9}$ $f \simeq 2.8\,\mathrm{nHz}$ Shannon et al 1310.4569

- COBE, WMAP, Planck measured TEMPERATURE fluctuations in CMB

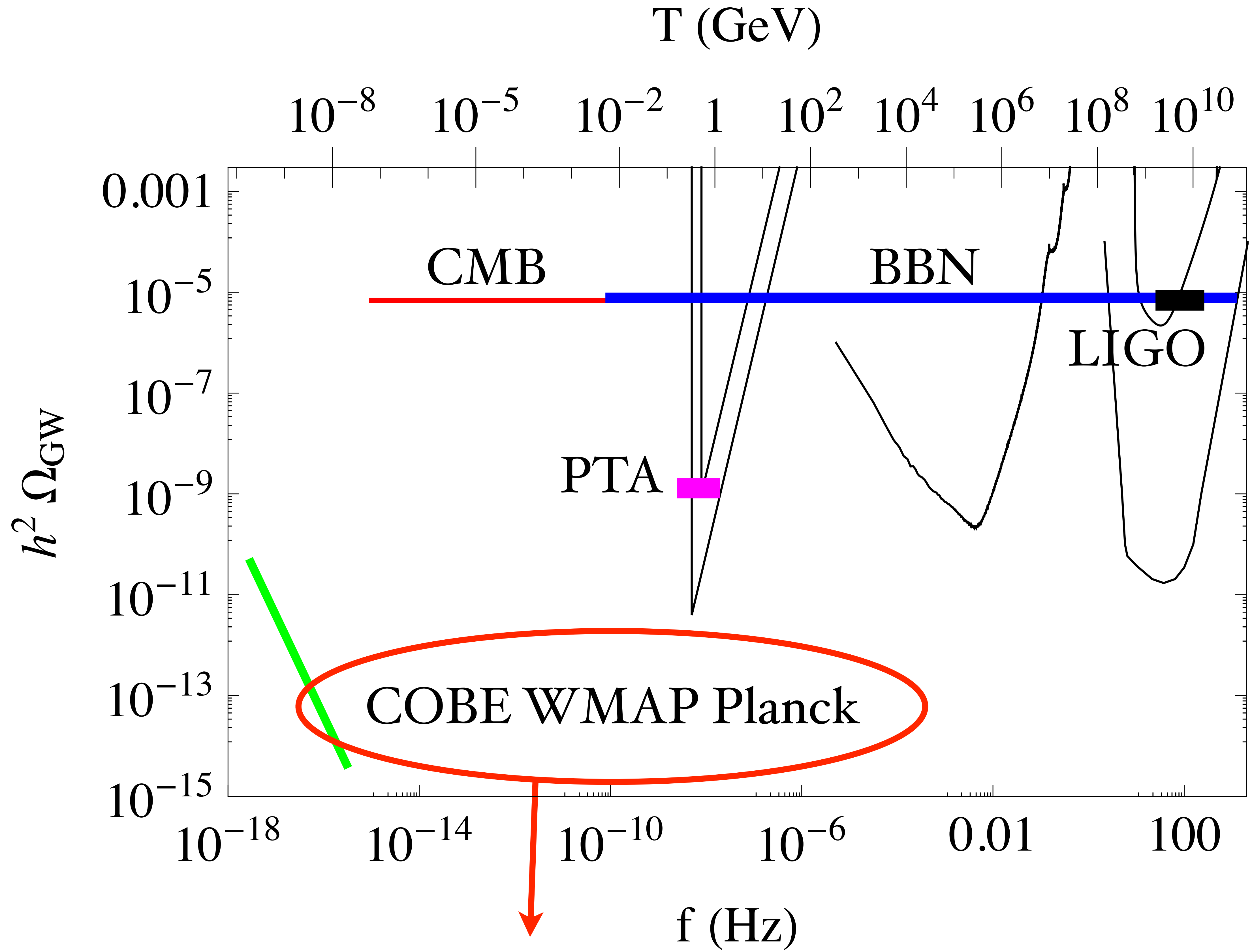
$$h^2\Omega_{\rm GW} < 7\cdot 10^{-11}\left(\frac{H_0}{f}\right)^2 \qquad 10^{-18}\,\mathrm{Hz} < f < 10^{-16}\,\mathrm{Hz}$$

Limits on a stochastic background



before BICEP2

Limits on a stochastic background



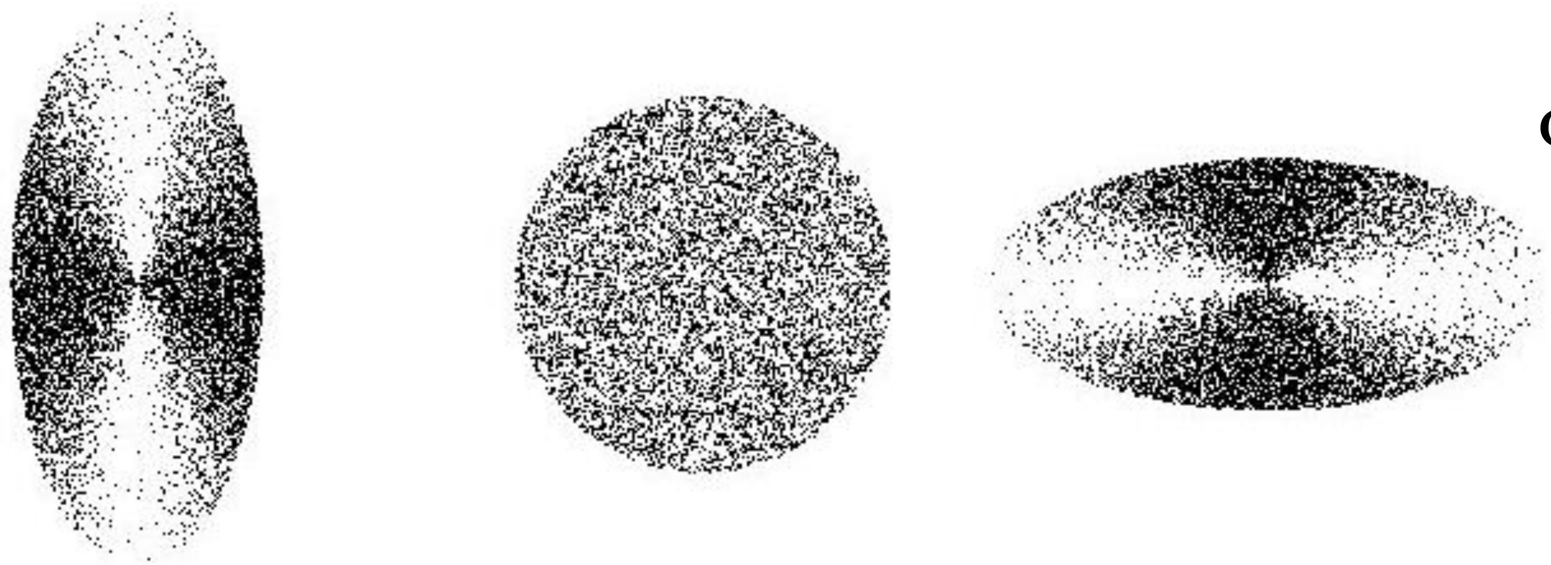
(maybe) detection of GW from inflation by BICEP2 : POLARISATION

GW influence CMB photons and leave an imprint in CMB anisotropies

- temperature : limit by COBE, WMAP, Planck

$$\frac{\delta T}{T} = - \int_{t_{\rm dec}}^{t_0} \dot{h}_{ij}\, n^i n^j \, dt$$

- polarisation: BB spectrum measured by BICEP2
  generated at photon decoupling time, from Thomson scattering of electrons by a quadrupole temperature anisotropy in the photons



distortion of an homogeneous photon patch by GW : imprint quadrupole anisotropy in the photon distribution

GW influence CMB photons and leave an imprint in CMB anisotropies

- polarisation: BB spectrum measured by BICEP2
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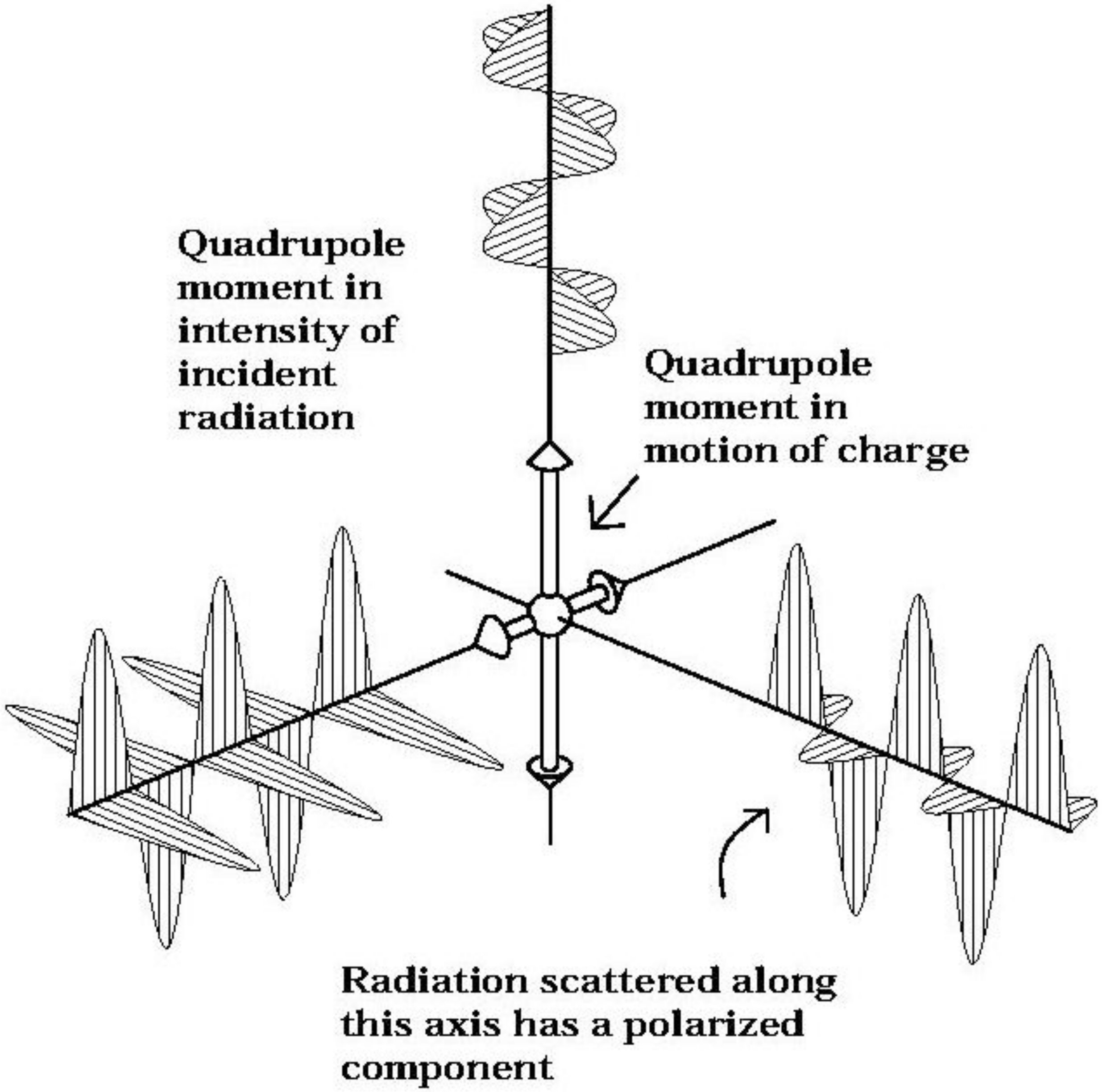


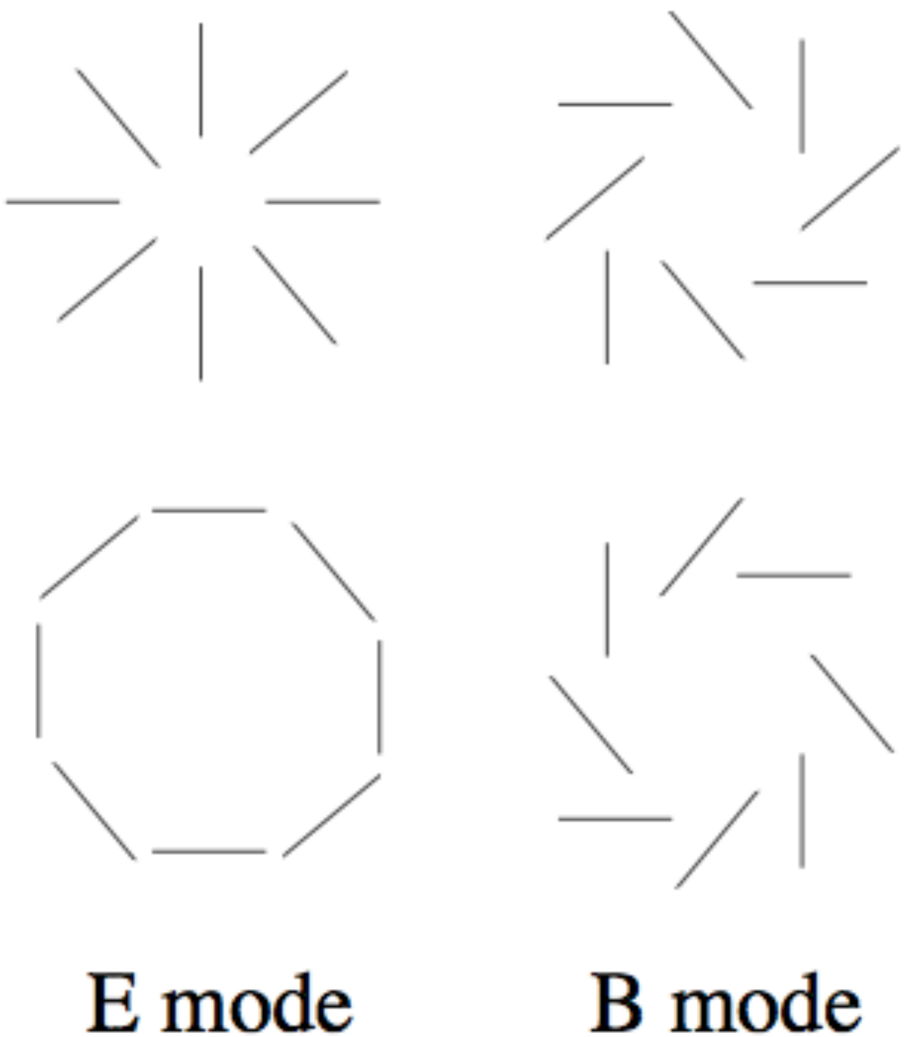
if the incident radiation is not isotropic, the scattered light at the end of decoupling (CMB) is polarised

GW influence CMB photons and leave an imprint in CMB anisotropies

- polarisation: BB spectrum measured by BICEP2
  generated at photon decoupling time, from Thomson scattering of electrons by a quadrupole temperature anisotropy in the photons

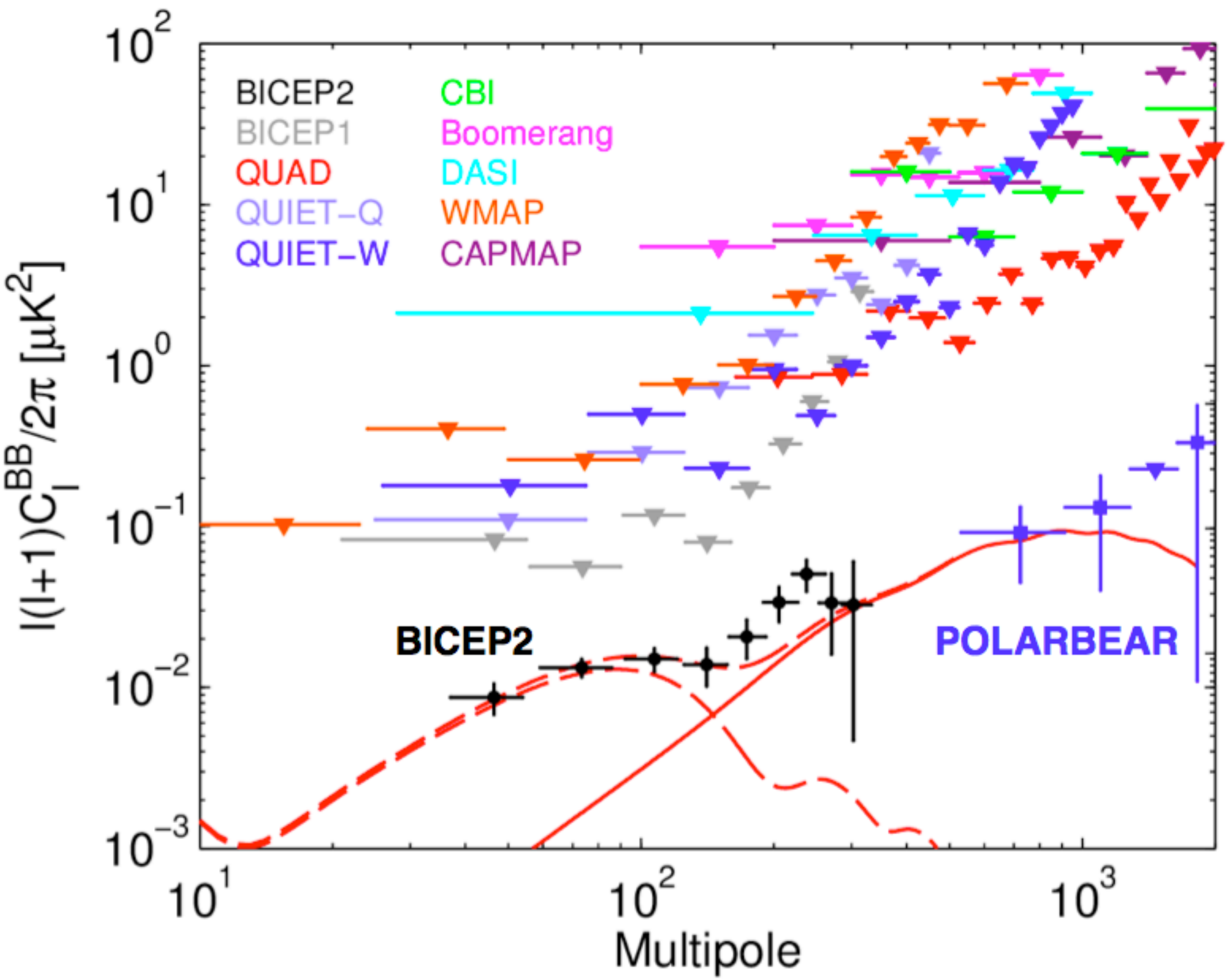
polarisation patterns (independent on the reference frame)

generated by scalar and tensor perturbations



generated only by tensor perturbations (or foregrounds)

B polarisation power spectra from BICEP last release



GW background from inflation

inflation amplifies both vacuum fluctuations of graviton (tensor mode) and of the scalar field driving inflation (scalar mode)

amplitude of scalar perturbations power spectrum

$$S \propto \frac{1}{\epsilon}\frac{H^2}{M_P} = \frac{1}{\epsilon}\frac{V}{M_P^4}$$

measured by CMB temperature

amplitude of tensor perturbations power spectrum

$$T \propto \frac{H^2}{M_P} = \frac{V}{M_P^4}$$

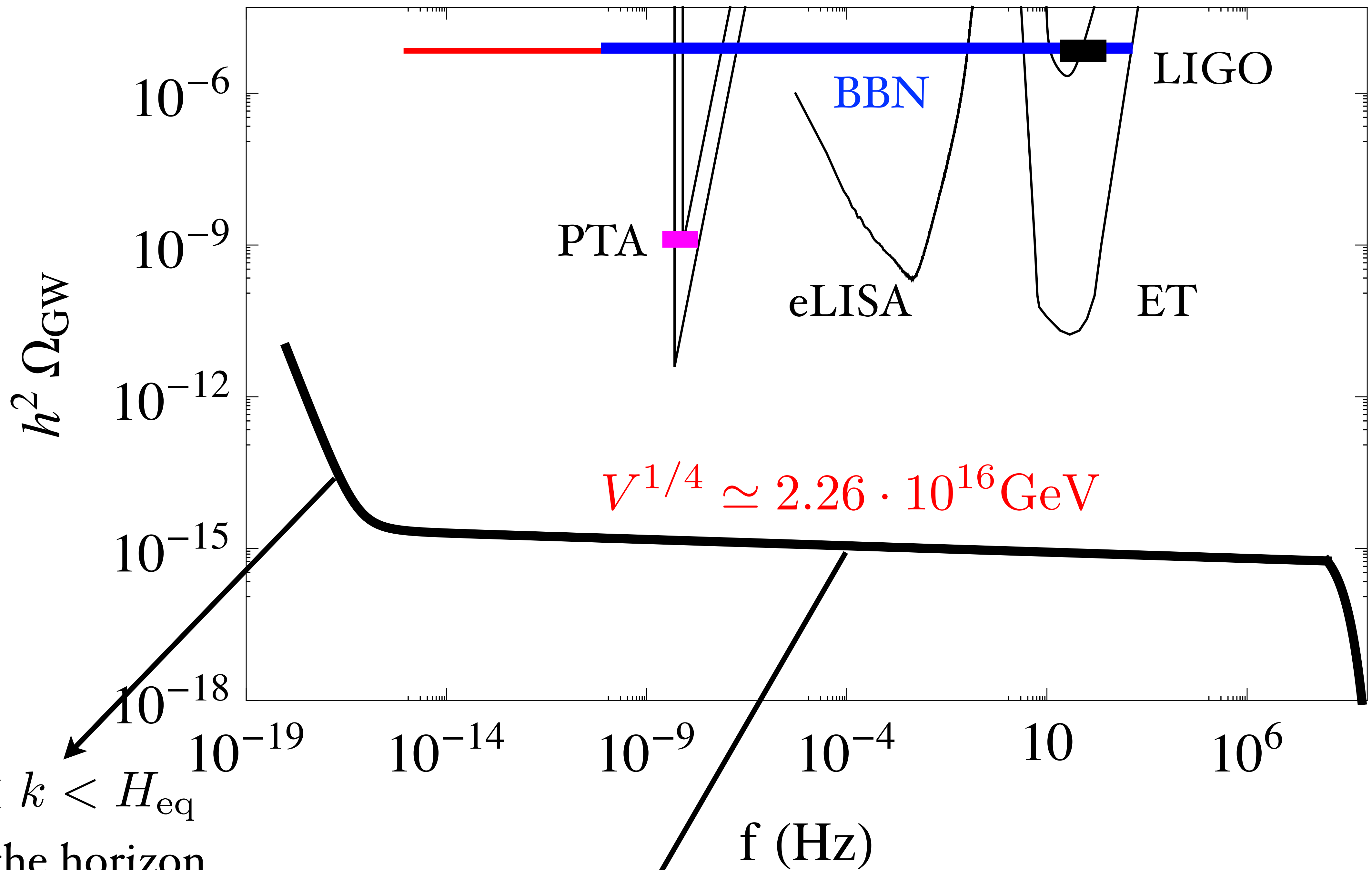
measured by CMB B polarisation

from the measurement of T/S one can infer the energy scale of inflation

$$V^{1/4} \simeq 2.25 \cdot 10^{16}\,\text{GeV}\left(\frac{r}{0.2}\right)^{1/4}$$

high scale inflation

GW background from inflation



$H_0 < k < H_{\rm eq}$
enter the horizon
in the MD era

$k > H_{\rm inf}$
smooth
transition
no particle
creation

$H_{\rm eq} < k < H_{\rm inf}$ abrupt transition particle creation

$$\frac{\mathrm{d}\,\Omega_{\rm GW}}{\mathrm{d}\ln k} \sim 10^{-5}\left(\frac{H_{\rm inf}}{m_{\rm Pl}}\right)^2\left(\frac{k}{k_{\rm eq}}\right)^{n_T}$$

Grishchuk 1974, Starobinsky 1979, Abbott and Harari 1986, ...

BUT... there are other possible sources of GW in the early universe promising for detection with future interferometers or PTA

mechanisms that produce a non-zero tensor anisotropic stress

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}$$

Possible GW sources in the early universe

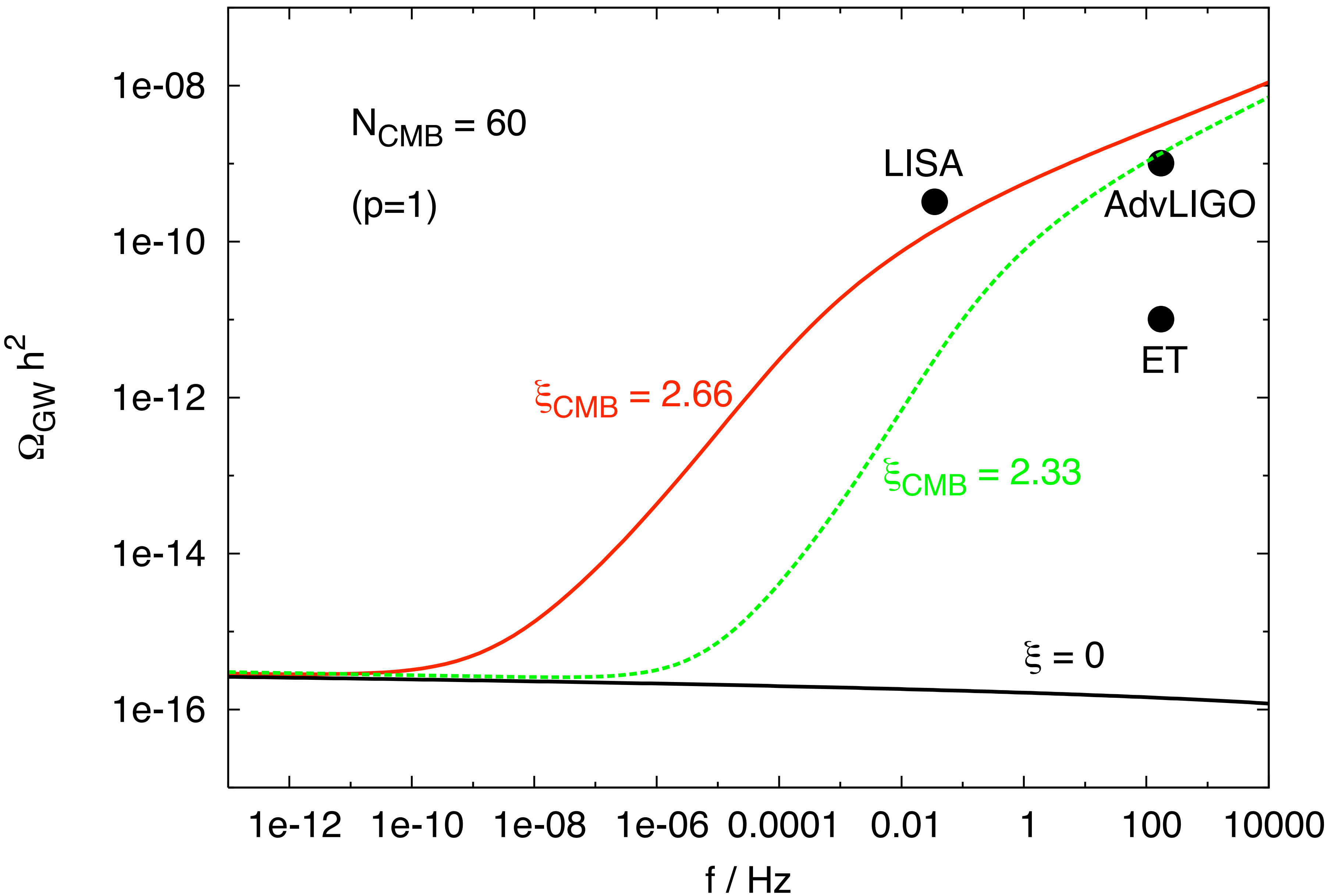
- inflation
  - particle production during inflation
  - fluid stiffer than radiation after inflation
  - preheating after inflation
  - phase transitions at the end or during inflation
- cosmic (super)strings
- first order phase transitions
- non-perturbative decay of SUSY flat directions
- unstable domain walls
- primordial black holes
- scalar field self-ordering
- ...

GW background from particle production during inflation

- production of particles in the time dependent background due to the evolution of the inflaton field

- observable signal : gauge fields coupled to pseudoscalar inflaton in linear inflationary potential

$$\frac{1}{4}\frac{\phi}{f}F_{\mu\nu}\tilde{F}_{\mu\nu}$$



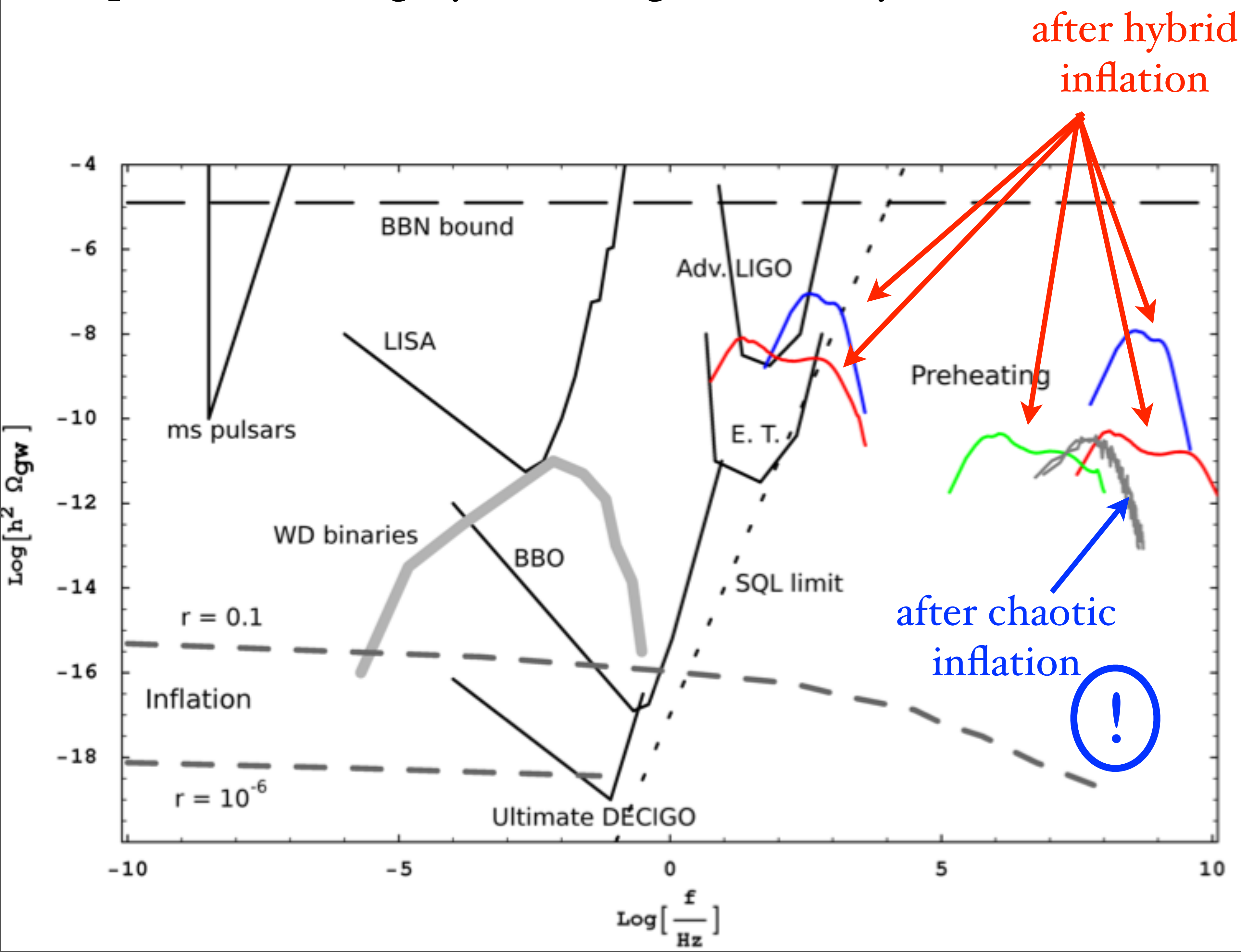
$\xi_{\rm CMB}$

amplitude of the coupling

Cook and Sorbo 2012
Barnaby et al 2012

GW background from preheating

- reheating: the energy density driving inflation is converted in radiation and matter
- preheating: possible first stage of this conversion, the inflaton decays in an explosive and highly inhomogeneous way



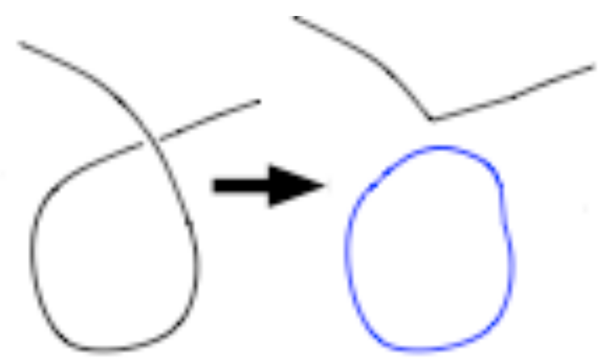
R_* size of the inhomogeneities

$$f_* \sim \frac{4 \cdot 10^{10}\ \mathrm{Hz}}{R_*\, \rho_p^{1/4}}$$

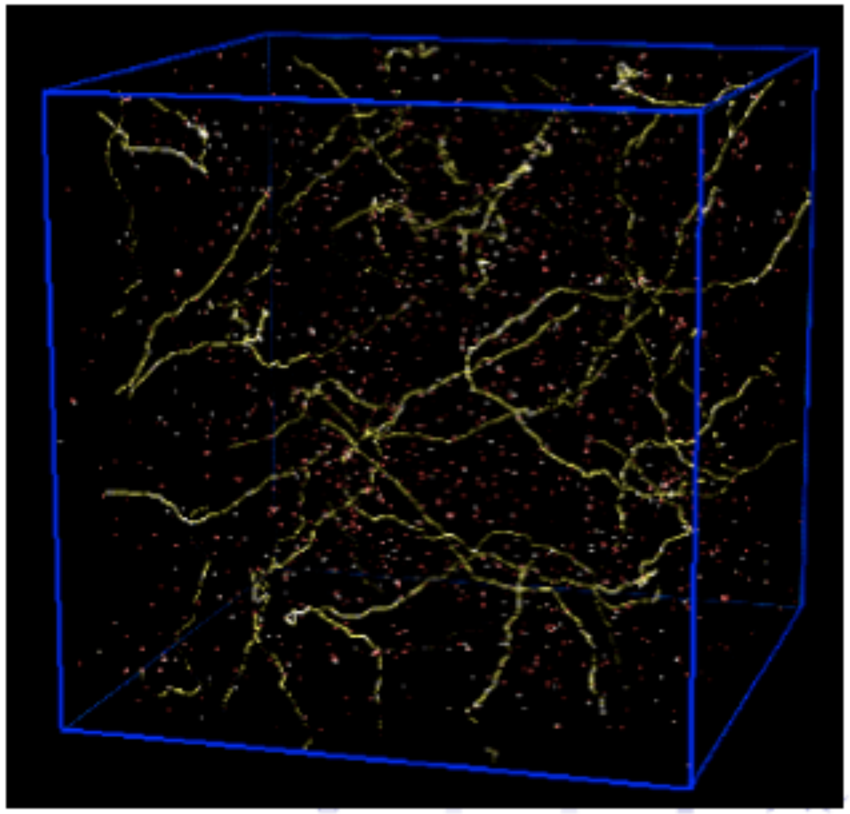
Kofman et al 1994, Khlebnikov and Tkachev 1997, ..., Dufaux et al 2009

GW background from cosmic strings

- one dimensional topological defects formed during symmetry breaking phase transitions or in the context of string theory at the end of brane inflation
- form a cosmological network which reach a scaling regime: it looks statistically the same at any time, the only relevant scale is the Hubble length
- decay by GW emission : long strings intercommute and form smaller loops which oscillate relativistically and emit GW



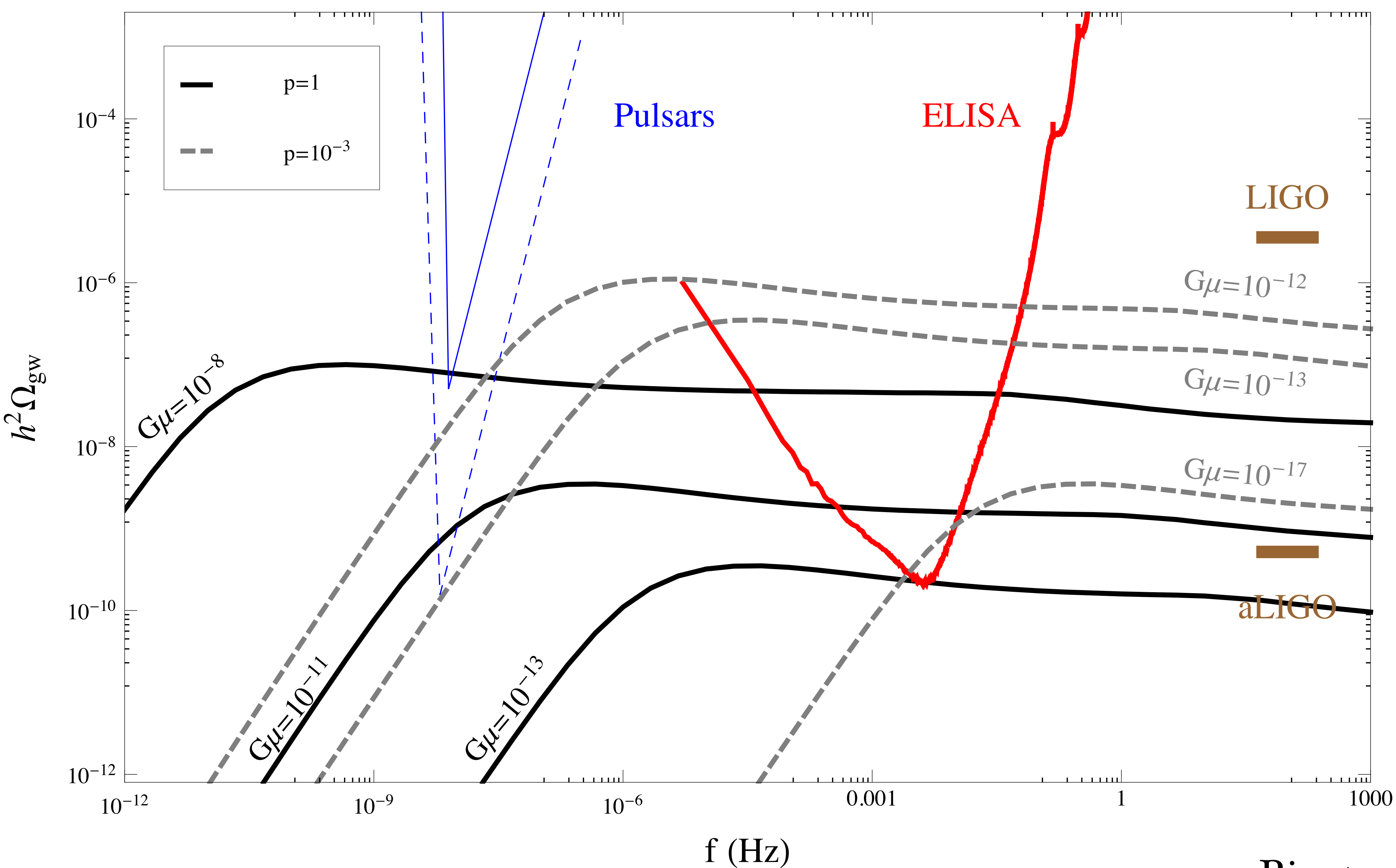
new loops from the long strings continually replace the loops that disappear
continuous GW production



Kibble 1976, Vilenkin 1985, Damour and Vilekin 2000, Sarangi and Tye 2002 ..., Binetruy et al 2012

GW background from cosmic strings

- parameters: tension, reconnection probability (loop size: probably solved)
- spectral shape extended in frequency because of continuous production



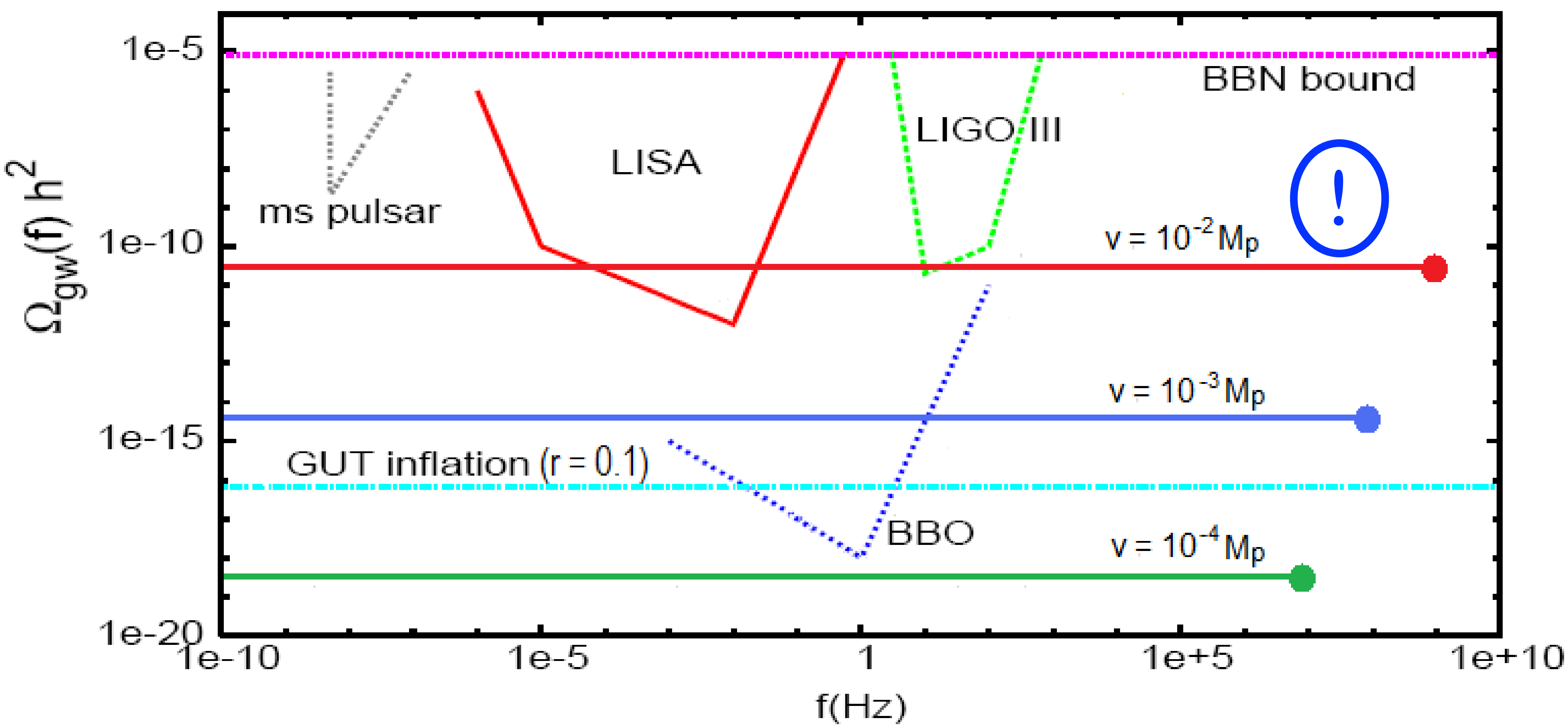
Binetruy et al 2012

GW background from scalar field self-ordering

- after the spontaneous breaking of a global symmetry, N component scalar field gets different vev in different causal patches

- horizon grows, the field re-orders but anisotropic stress is present at the boundaries and sources GW

- spectral shape extended in frequency because of continuous production



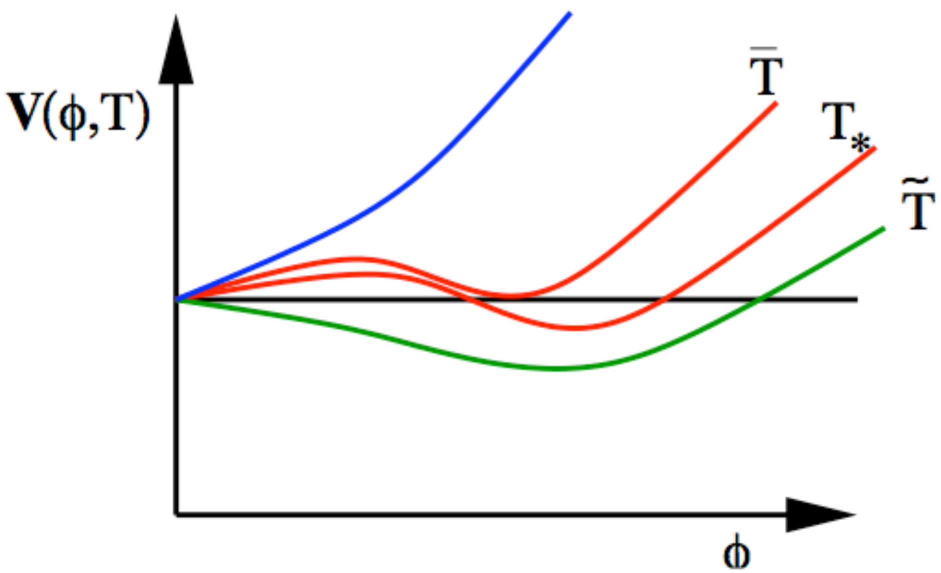
Fenu et al 2009

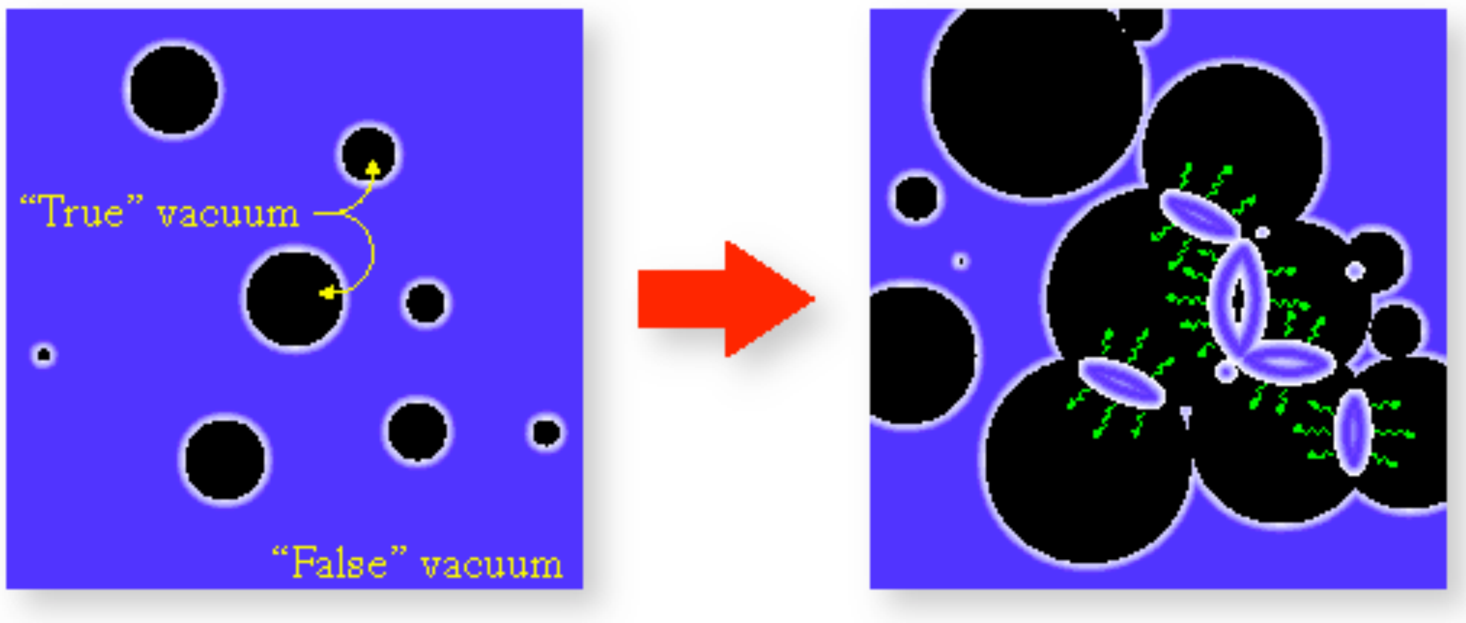
GW background from first order phase transitions

universe expands and temperature decreases : PTs , if first order lead to GW

potential barrier separates true and false vacua

quantum tunneling across the barrier : nucleation of bubbles of true vacuum





source: Π_{ij} tensor
anisotropic stress

- collisions of bubble walls
- magnetohydrodynamic turbulence in the primordial fluid

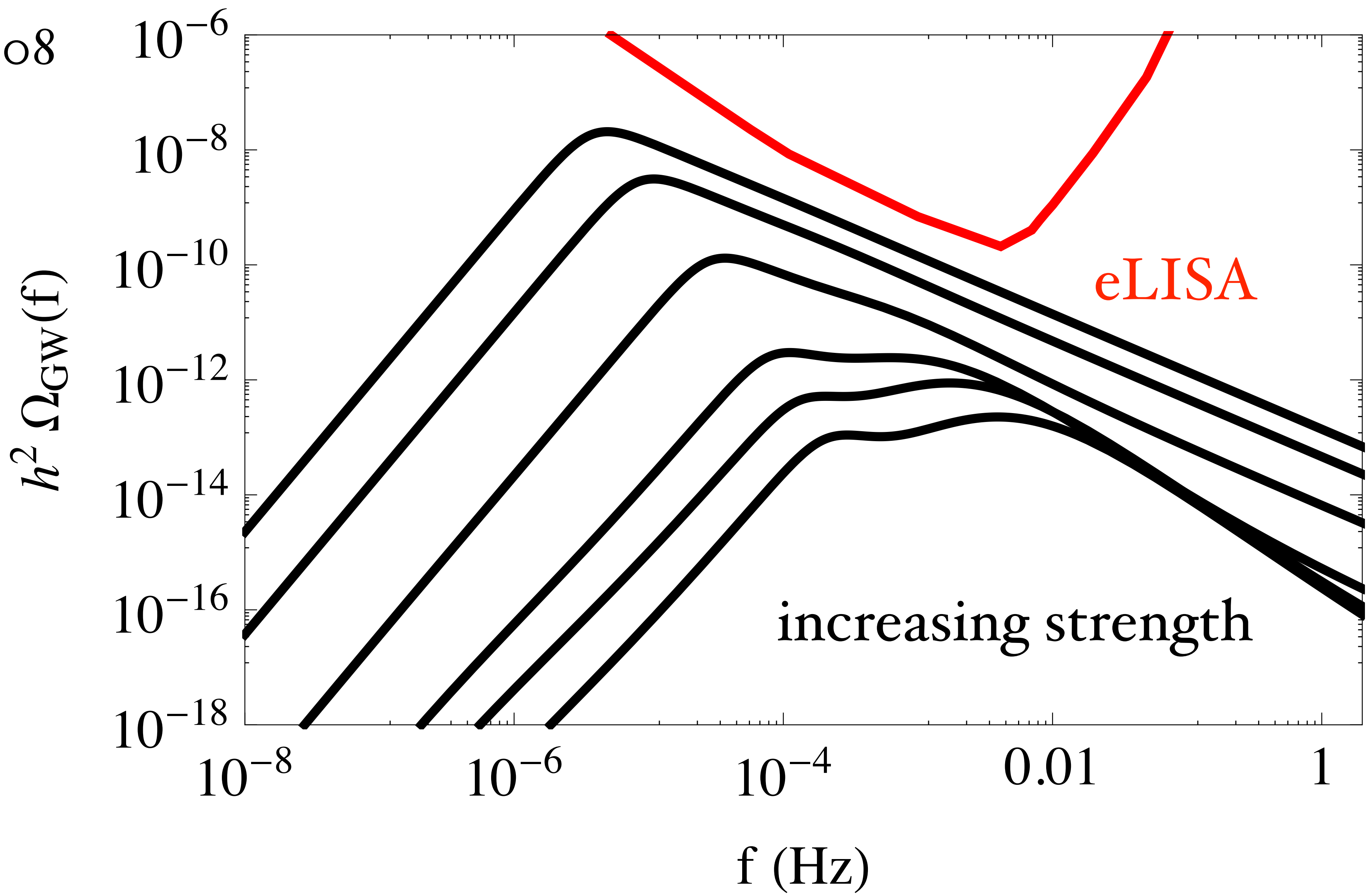
GW background from first order phase transitions : EWPT

$T_* \simeq 100\,\mathrm{GeV}$ $\longrightarrow$ eLISA frequency band

$$V(H) = -\frac{\mu^2}{2}H^2 + \frac{\lambda}{4}H^4 + \frac{1}{8M^2}H^6$$

Huber and Konstandin 2008

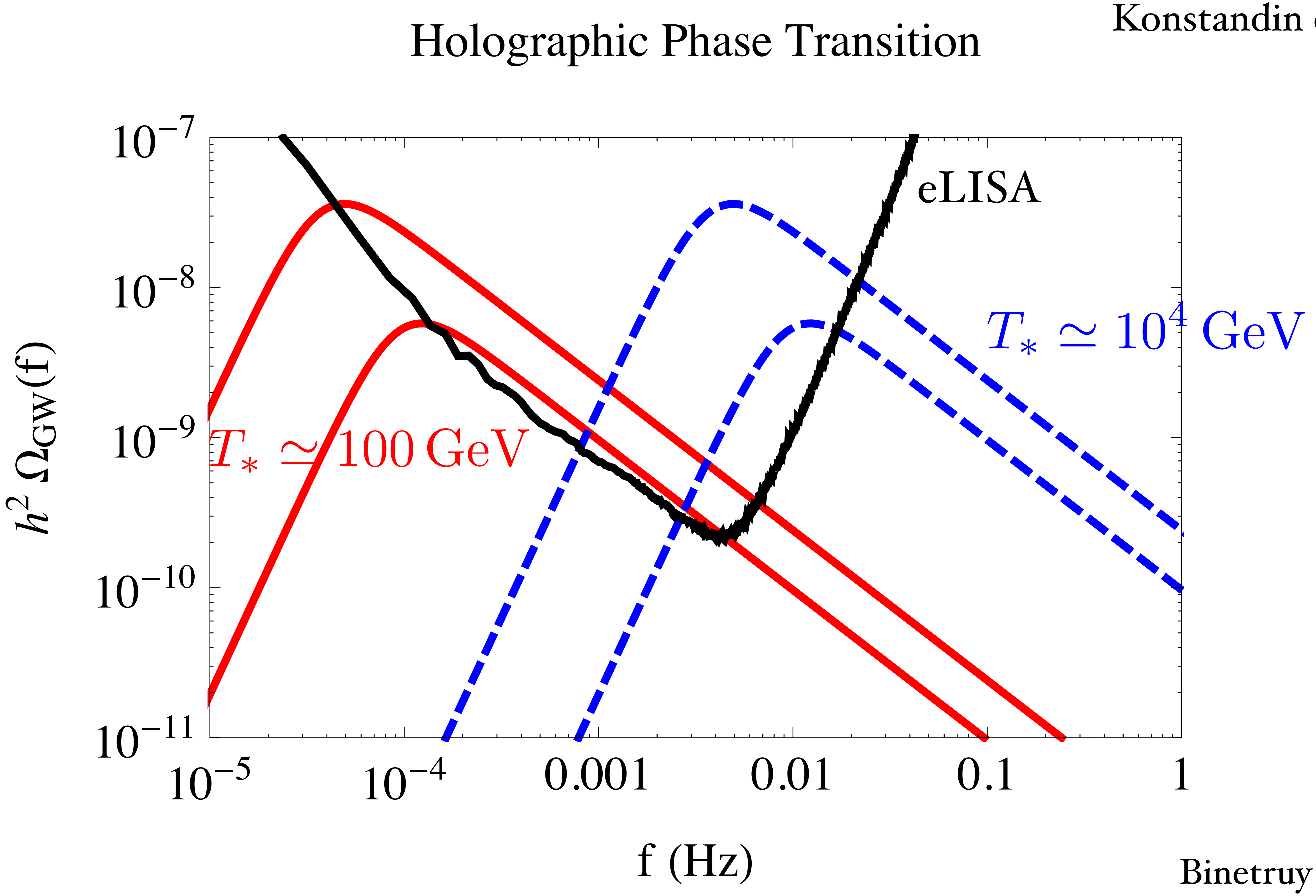
SM + dimension six operator, η=0.2



Binetruy et al 2012

GW background from first order phase transitions : EWPT

Randall and Servant 2007
Konstandin et al 2010

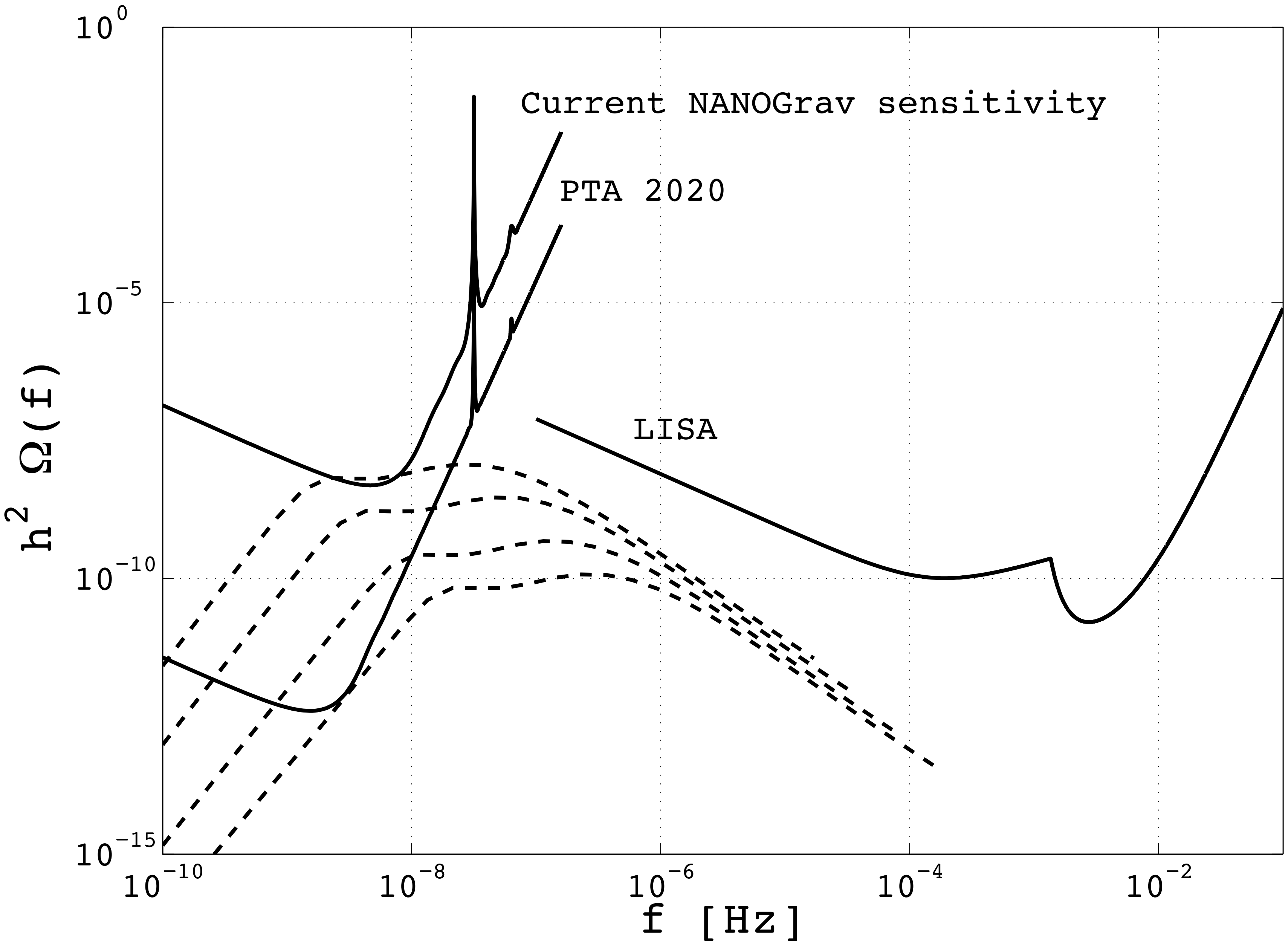


Binetruy et al 2012

GW background from first order phase transitions : QCDPT

QCDPT : if lepton asymmetry is large

Schwarz and Stuke 2009



$$T_* = 100\,\mathrm{MeV}$$

CC et al 2010

Conclusions

- we have little information about the physics and the processes operating in the very early universe
- due to their small interaction rate, GW can in principle provide us with this information
- the frequency of GW maps the temperature/energy scale in the early universe
- GW by inflation at the energy scale indicated by the recent BICEP2 result are not visible with the next generation interferometers or by PTA
- but it is in principle possible to generate a stochastic background of GW detectable by them : preheating, cosmic strings, PTs...
- GW are a powerful mean to learn about the early universe and high energy physics: detection is extremely difficult but great payoff