Developments in Implicit Rotating Source Waveform Models

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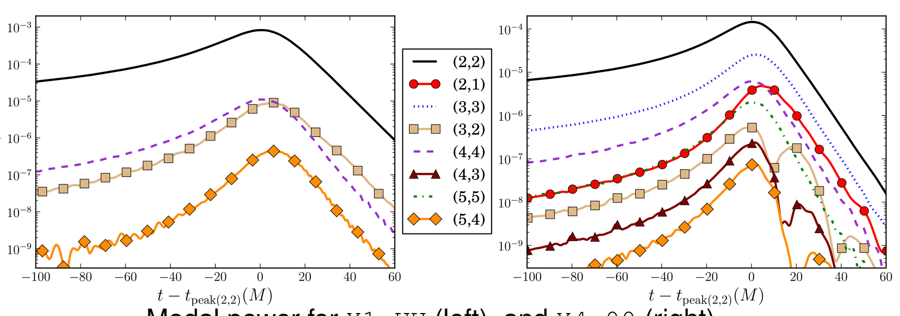
Black hole merger waveforms, as predicted by general relativity, are striking for their remarkably simple features. This is advantageous both because it provides clear encoding of system properties on the observable waveforms, and it aids in the development of empirical analytical waveform models which may accurate encode information from costly simulations. Represented in spherical harmonic components aligned to the orbital plane, the waveform phase is strongly circularly polarized through inspiral merger and ring-down with a smooth transition from orbital frequency to rotational ring-down frequency. We describe recent developments in "Implicit Rotating Source" (IRS) modeling, an empirical approach exploiting these simple waveform characteristics for a compact waveform description. We discuss results providing parameters of late-time IRS merger description over a broad parameter space, and recent results on extending the model to make analytic contact with PN inspiral waveforms.

Gravitational Waves from Numerical Mergers

NR black-hole merger simulations produce waveforms decomposed into (spin-weighted) spherical harmonics: $r\psi_4(t,r,\theta,\phi) = \sum_{\ell m} C_{\ell m}(t,r)\,{}_{-2}Y_\ell^m(\theta,\phi)$.

- We work with *strain-rate* $\dot h = \int \psi_4^* dt$; modal power $\dot E_{\ell m} \propto \dot h_{\ell m}^2$
- Each mode has an amplitude and complex phase: $r\dot h_{\ell m} = A_{\ell m}(t) e^{i\varphi_{\ell m}(t)}$.
- A handful of modes dominate energy flux; mostly $(\ell, \pm\ell)$.
- $(2,\pm 2)$ is sufficient for *detection*; other modes are important for *parameter estimation*.

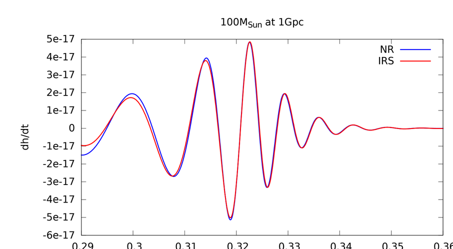


Modal power for `X1_UU` (left), and `X4_00` (right).

Implicit Rotating Source Picture

We observe: NR waveforms for quasicircular insprial characterized by simple rotational frequency development

- Universal view of inspiral–merger–ringdown for quasi-circular inspiral.
- Most important WF modes have consistent *rotational phases* $\Phi_{\ell m} \equiv \varphi_{\ell m}/m$
- Can apply:
  - to gain insight about detailed waveform features ("fine structure")
  - in analytic waveform models
- We have an explict empirical model for merger-ringdown.
- Exploration: **Can we extend this model to sensibly represent earlier phasing?**



Explicit IRS model waveform. ($M = 100M_\odot$, $q = 4$, $S_1 = S_2 = 0$, $D = 1Gpc$)

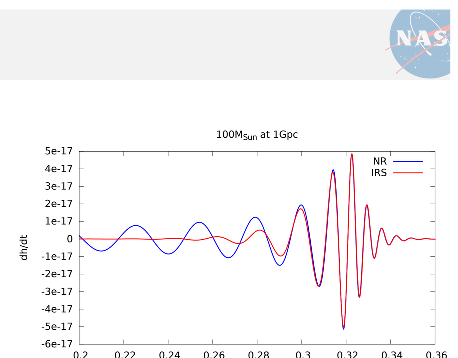
Explicit IRS Phasing Model

[Baker et al. (2008), Kelly et al. (2011)]:

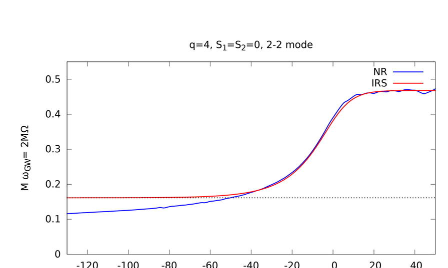
- For comparable mass: fits most of GW energy
- Mainly depends only on final m and a
- Rotational frequency model is a smoothed "step function" to fundamental QNM frequency:

$$\begin{aligned}\Omega(t) &= \Omega_0(\Omega_f, c, \kappa) + \Delta(\Omega_f, c, \kappa)\Theta^\kappa \\ \Omega_0(\Omega_f, c, \kappa) &= \Omega_f\,(1 - \Delta(c,\kappa)) \\ \Delta(c,\kappa) &= \frac{c}{2}\left(\frac{1}{\kappa} + 1\right)^{1+\kappa} \\ \Theta &= (1 + e^{-2x})^{-1} \\ x &= (t - t_0)/b + \log(\kappa)/2\end{aligned}$$

- Poor for times earlier than $\sim 60M$ before merger
- Need to:
  - eliminate non-zero asymptote $\Omega_0 \to 0, \Delta \to 1$
  - but somehow stretch the curve toward early times.



Explicit IRS model waveform goes out of phase toward early times.



Frequency quickly asymptotes to a constant.

Numerical runs for comparison

Aligned-spin BHB configurations "equivalent" to nonspinning 4:1, and 6:1 BBHs.

run label	$q \equiv M_1/M_2$	j_1	j_2	j_f
`run0`	1.0	-0.6365	-0.6365	0.4765
`run1`	2.0	-0.4734	0.0	0.4765
`run2`	3.0	-0.1557	0.0	0.4760
`run3`	4.0	0.0	0.0	0.4748
`run4`	5.0	0.0937	0.0	0.4748
`run5`	2.0	-0.6250	-0.6250	0.3762
`run6`	3.0	-0.3843	0.0	0.3762
`run7`	4.0	-0.1896	0.0	0.3762
`run8`	5.0	-0.0752	0.0	0.3762
`run9`	6.0	0.0	0.0	0.3762

New Parametric Model

Simple resolution: **Parametrically stretch time**.

- 1. Write time parametrically $t \to t(\Theta)$.
  Rewrite last slide with $\Omega_0 \to 0$ and $\Delta -> 1$ as.

$$\begin{aligned}\Omega(t) &= \Omega_f \Theta^\kappa \\ t - t_0 &= \frac{b}{2}\log\frac{\Theta/\kappa}{1-\Theta}\end{aligned}$$

  Note that $\Theta \to 0$ for early-time/low freq. $\Theta \to 1$ for late time/QNM freq.
- 2. Stretch time with a term that blows up as $\Theta -> 0$, but litte affected for large Θ:

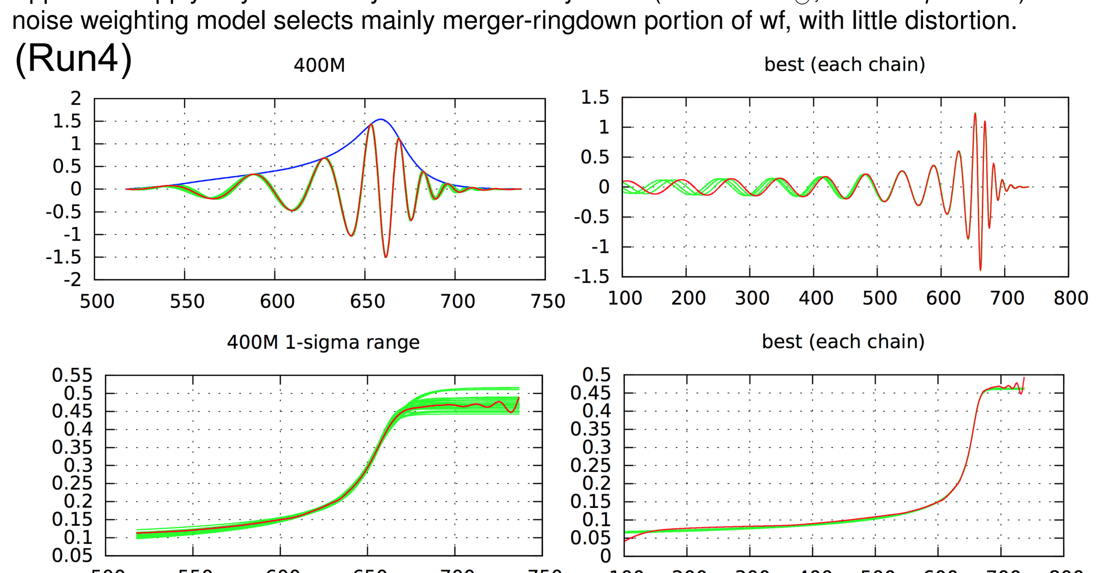
$$t - t_0 = \frac{b}{2}\log\frac{\Theta/\kappa}{1-\Theta} - (\Theta^{-n} - 1)\,A.$$

- We eliminated one parameter c, but added two more: stretch amplitute A and power n.
- Note that $\Omega \sim (t_0 - t)^{-\kappa/n}$ at early times (Can engineer early-time phasing).

Does the new model still fit the merger?... Yes!

Approach: Apply Bayesian analysis to massive systems ($M \sim 400M_\odot$, nominal $\rho = 100$) where noise weighting model selects mainly merger-ringdown portion of wf, with little distortion.

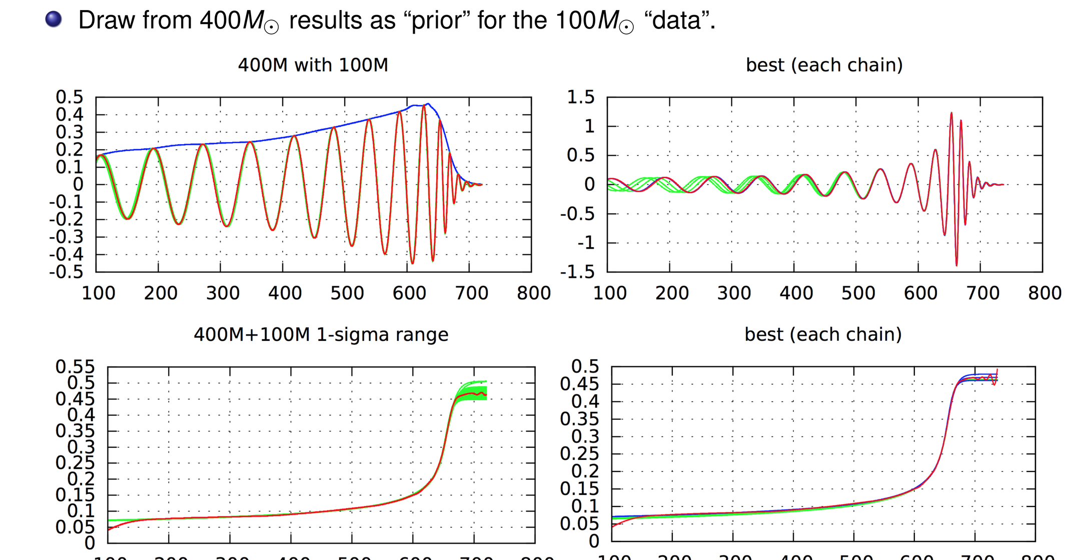
(Run4)



$q = 4$,nonspinning merger. $\dot h_{22}$ (top), ω (bottom).
Left side: Noise-weighted waveforms, with NR (red) plus 30 samples from 1-sigma range (green).
Right side: best (MAP) cases per chain (green).

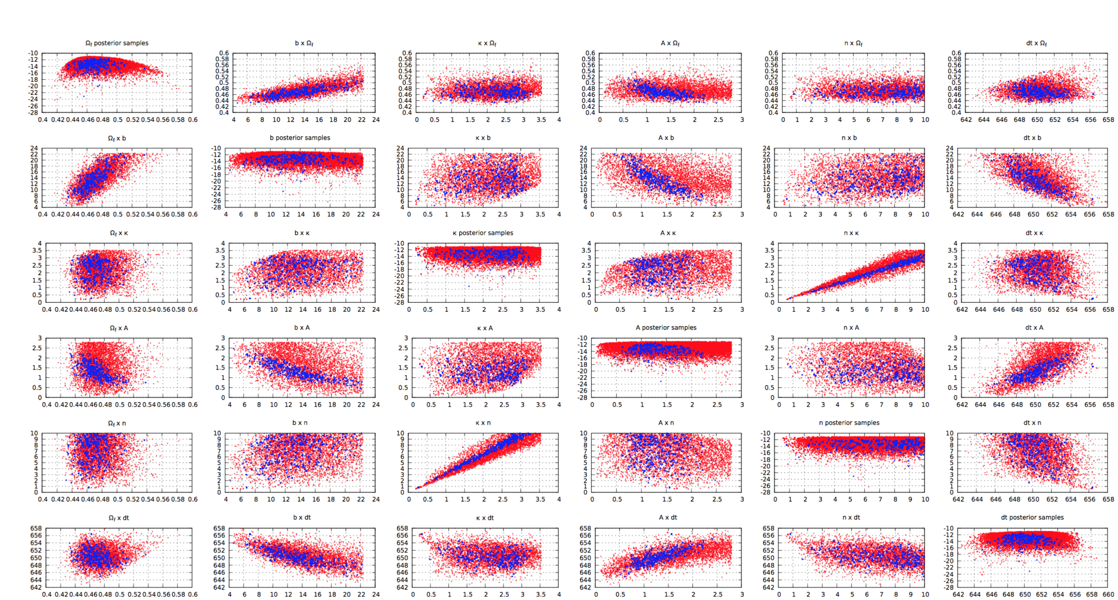
Can the model fit the late inspiral?... Yes!

- Approach: Want to add info about late insprial without strongly biasing the late-time fit.
- Apply Bayesian analysis to massive system together with a smaller, fainter system at lower SNR ($M = 100M_\odot$, $SNR = 16$).
- Draw from $400M_\odot$ results as "prior" for the $100M_\odot$ "data".

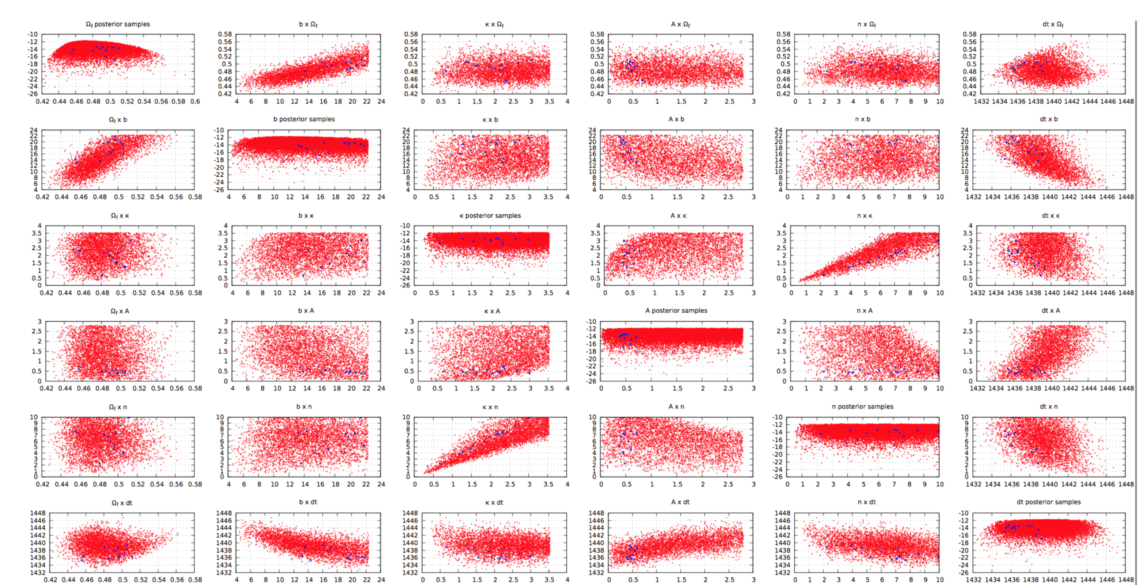


$q = 4$,nonspinning merger. $\dot h_{22}$ (top), ω (bottom).
(Run4)

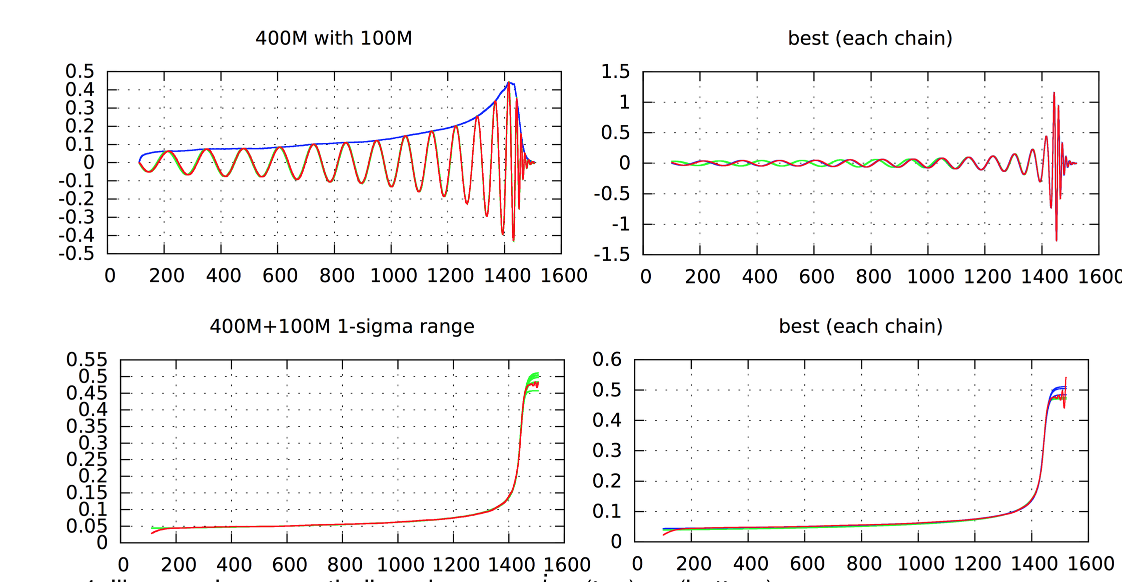
Parameter covariance (posterior projections)



Equal-mass antialigned (longest waveform): stronger constraints (Run0)

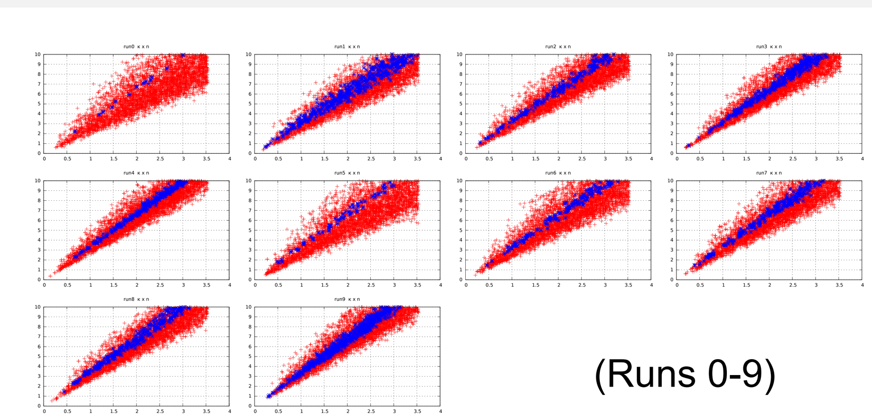


But waveforms still look great! (Run0)



$q = 4$–like equal-mass,anti-aligned merger. $\dot h_{22}$ (top), ω (bottom).
Left side: Noise-weighted (small-mass case) waveforms, with NR (red) plus 30 samples from 1-sigma range (green).
Right side: best (MAP) cases per chain (blue). Green shows best fit from large mass fit only.

A look at the most correlated parameters: n and κ



(Runs 0-9)

n–κ posterior projections for all NR run cases indicate similar correlations, tightened when there is inspiral data.

- Empirically $\kappa/n \approx 0.3$
- Late-time: n and κ compete to shape approach to merger phasing.
- Recall that in early time limit: $\omega \sim (t_0 - t)^{-\kappa/n} \approx (t_0 - t)^{-0.3}$
- Compare: leading PN limit: $\omega \sim (t_0 - t)^{-3/8} \approx (t_0 - t)^{-0.375}$
- *Encouraging toward a PN extention.*

Summary

- **A small tweak to the IRS phasing model yields a family of analytic functions capable of fitting late I-M-R phasing** for several cases studied.
- Bayesian techniques are useful for numerical-relativity-empirical studies.
- Future:
  - Try the model on a wider sample of waveforms.
  - Try enforcing full PN phasing.
  - Fit model parameters from physical parameters.
  - Try to develop a corresponding amplitude model.