Developments in Implicit Rotating Source Waveform Models

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Black hole merger waveforms, as predicted by general relativity, are striking for their remarkably simple features. This is advantageous both because it provides clear encoding of system properties on the observable waveforms, and it aids in the development of empirical analytical waveform models which can accurately encode information from costly simulations. Represented in spherical harmonic components aligned with the orbital plane, the waveform phase is strongly circularly polarized through inspiral and ring-down with a smooth transition from orbital frequency to rotational ring-down frequency. We describe recent developments in “Implicit Rotating Source” (IRS) modeling, an empirical approach exploiting these simple waveform characteristics for a compact waveform description. We discuss results parameters of late-time IRS merger description over a broad parameter space, and recent results on extending the model to make analytic contact with PN inspiral waveforms.

Gravitational Waves from Numerical Mergers

NR black hole merger simulations produce waveforms decomposed into (spin-weighted) spherical harmonics: \( \ell \omega_\ell^m \equiv r_{\ell m} \propto \tilde{h}_{\ell m}(t) \).
- We work with strain rate \( h \propto f^{-1} \).
- Each mode has an amplitude and complex phase: \( r_{\ell m} = A_{\ell m} e^{i\phi_{\ell m}} \).
- A handful of modes dominate energy flux most \( (\ell, m) \).
- (2, 2) is sufficient for detection; other modes are important for parameter estimation.

Implicit Rotating Source Picture

We observe: NR waveforms for quasicircular inspiral characterized by simple rotational frequency development:
- Universal view of inspiral-merger-ringdown for quasicircular inspiral.
- Most important IRs model have consistent rotational phase: \( \psi_{\ell m} = 2\pi f \).
- Can apply:
  - To gain insight into detailed waveform features (“fine structure”).
  - In analytic waveform models.
- We have an explicit empirical model for merger-ringdown.
- Exploration: Can we extract any recoil to sensitivity represent earlier plane?

Explicit IRs Phasing Model

[Baker et al. (2006), Kelly et al. (2011)]
- For comparable mass, the most of GR energy.
- Mostly depends on that in a \( a \).
- Rotational frequency model is smoothed “deep function” to fundamental GRM frequency:
  \( \Omega(\ell, m) = \frac{\ell - n}{\ell + n} \Delta_{\ell} + (1 - n) \Delta_{\ell} \)
  \( \Delta_{\ell} = \left( \frac{1}{2} \right)^{\frac{1}{2}} \left( \frac{2 \pi f}{\Omega(\ell, m)} \right) \)
  \( \psi_{\ell m} = 2\pi f \).
- Poor for times earlier than \( < \Omega(\ell, m) \).
  - Non-zero asymptote \( \psi_{\ell m} = 0 \).
  - But turnover stretch the curve toward early time.

Numerical runs for comparison

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<th>( A_{\ell m} )</th>
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New Parametric Model

Simple resolution: Parametrically stretch time.
1. Write time parametrically \( t \rightarrow f(t) \).
2. Rewrite last time with \( f(0) \) and \( f(\infty) \).
3. Stretch time with a term that blow up as \( f \rightarrow 0 \), but this affected for large \( t \).
4. We eliminated one parameter \( b \), added two more: stretch amplitude \( A \) and power \( n \).
5. Note that \( f(t_0 - f(t)) \) at early times (Can engineer early-time phasing).

Does the new model still fit the merger?... Yes!...!

Approach: Apply Bayesian analysis to massive systems (M = 40M\(_{\odot}\), orbital \( \sim 100 \)) where waveforms model a main merger ringdown portion of n, with little distortion.

Can the model fit the late inspiral?... Yes!...!

Approach: Want to add info about late inspiral without strongly biasing the late-time fit.

Equal-mass antialigned (longest waveform): strong constraints (Run4)

Parameter covariance (posterior projections)

Summary

- A small tweak to the IRS phasing model yields a family of analytic functions capable of fitting late-IRs phasing for several cases studied.
- Bayesian techniques are useful for numerical-relativity-empirical studies.
- Future:
  - Try the model on a wider sample of waveforms.
  - Try extending full PN phasing.
  - Fit model parameters from physical parameters.
  - Try to develop a corresponding empirical model.