Precision measurement of planetary gravitomagnetic field and future Chinese PathFinder mission.

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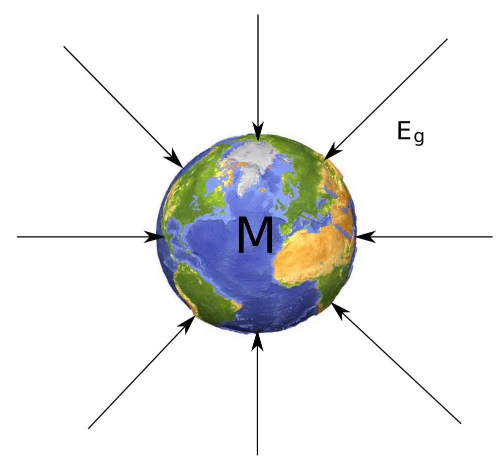
Introduction and Backgrounds

Outstanding tests of GR in the 21st century

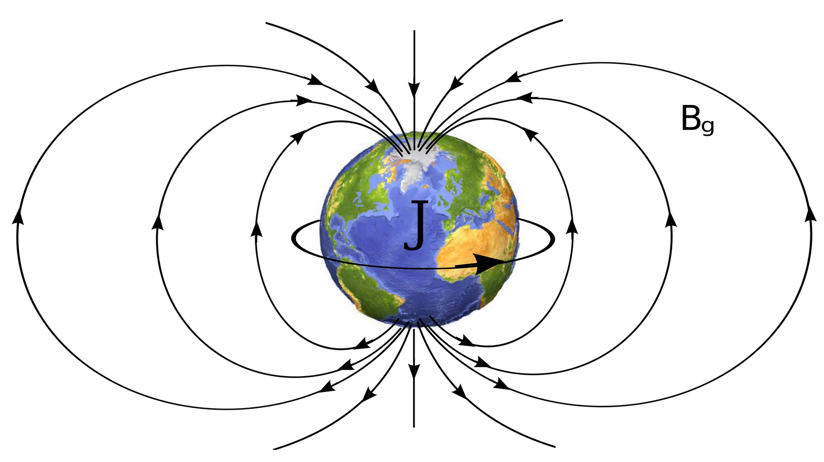
1. Gravitational Waves detection at galactic and cosmological scales.
2. **Gravitomagnetic effect measurements** at planetary and Solar system scales.
   - Poorly tested, remained the major challenge in experimental relativity.
   - Related to fundamental issues such as the origins of inertial, equivalence principle and etc..
   - Applications in future space science such as the determinations of inertial frames, the synchronizations of clocks in space and etc..

Future pathfinder mission for Chinese space-borne gravitational wave antenna.

In the weak field and slow motion limits $\frac{GM}{c^2 r} \sim \frac{v^2}{c^2} \sim \mathcal{O}(\epsilon^2)$, there exists surprisingly rich correspondences between electrodynamics and GR.



$$\nabla \cdot \mathbf{E}_g = -4\pi\rho, \quad \nabla \times \mathbf{E}_g = -\frac{1}{2}\frac{\partial}{\partial t}\mathbf{B}_g.$$



$$\nabla \cdot \frac{1}{2}\mathbf{B}_g = 0, \quad \nabla \times \frac{1}{2}\mathbf{B}_g = \frac{\partial}{\partial t}\mathbf{E}_g - 4\mathbf{j}.$$

The Measurement Principle

- A proof mass m moving in the gravito-electromagnetic field obeys the equation of motion

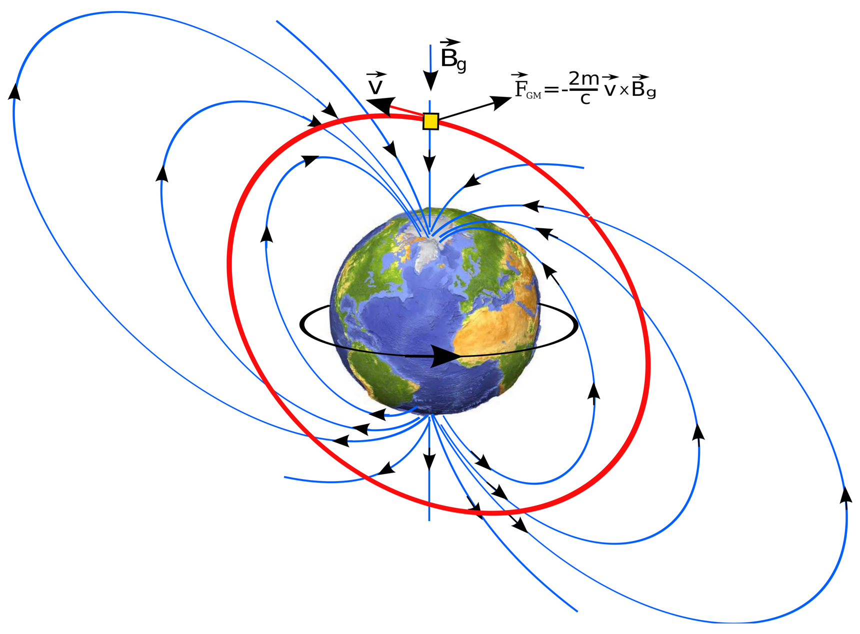
$$m\frac{d^2\mathbf{x}}{dt^2} = \mathbf{F}_{Newton} + \mathbf{F}_{GE} + \mathbf{F}_{GM}$$

$$\mathbf{F}_{GE} = \frac{GmM}{c^2 r^3}[(\frac{4GM}{r} - v^2)\mathbf{x} + 4(\mathbf{x}\cdot\mathbf{v})\mathbf{v}], \quad \mathbf{F}_{GM} = \frac{2Gmv}{c^2} \times [\frac{\mathbf{J}}{r^3} - \frac{3(\mathbf{J}\cdot\mathbf{x})\mathbf{x}}{r^5}].$$

- The gravitomagnetic force $\mathbf{F}_{GM}$, satisfying the Lorentz force formula

$$\mathbf{F}_{GM} = -2m\frac{\mathbf{v}}{c} \times \mathbf{B}_g,$$

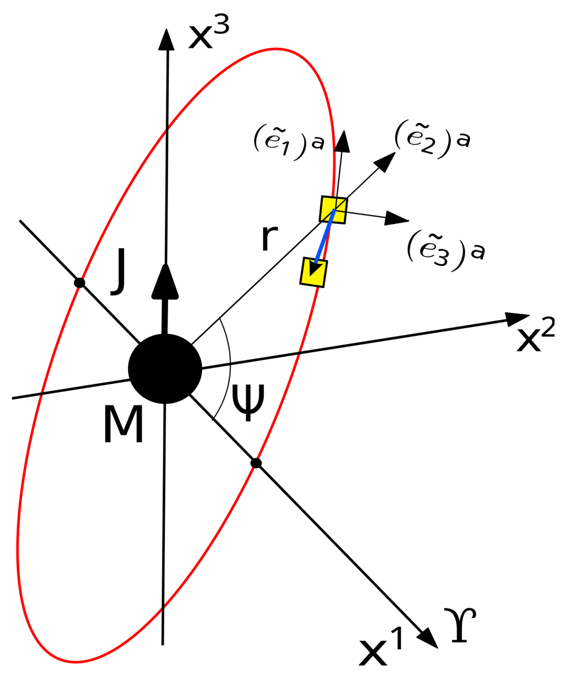
contributes the only component that perpendicular to the instant orbital plane of the proof mass.



- For two closely spaced proof masses following a medium Earth orbit and separated along-track with distance d, their relative motions perpendicular to the initial orbital plane will then encode the very information of the Earth gravitomagnetic field.

The Gravitomagnetic Signal $s_{GM}(t)$

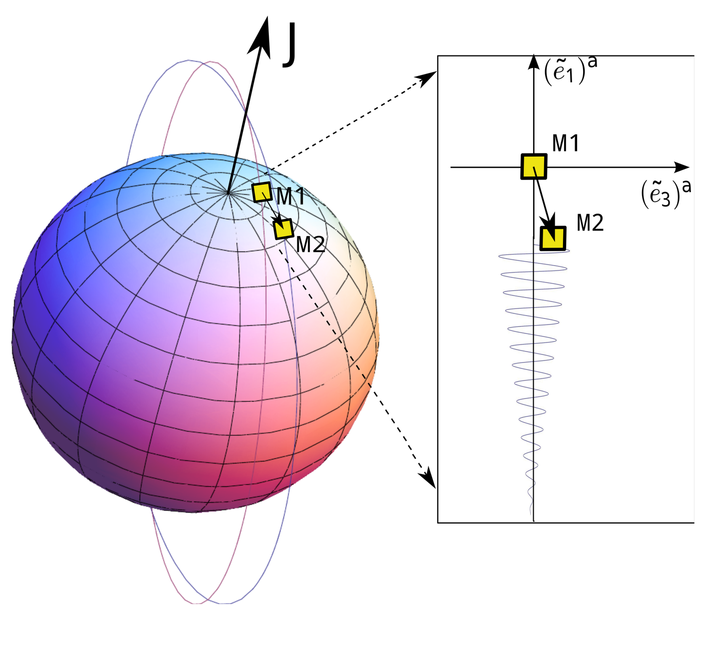
- The local frame is centered at the reference mass and the corresponding tetrad $(\tilde{\mathbf{e}}_\mu)^a$ is attached to the S/C, see the right figure.
- One needs to fix the $(\tilde{\mathbf{e}}_1)^a - (\tilde{\mathbf{e}}_2)^a$ plane (thus the normal $(\tilde{\mathbf{e}}_3)^a$ direction) to parallel to the initial orbit plane by means of attitude controls. This is because that gravitomagnetic effects are of **"Global"** ones, therefore **one needs to compare the locally measured quantities with globally defined ones to read out the corresponding frame-dragging signals.**



The signal to be measured,

$$s_{GM}(t) = \frac{2GJd\sin i\sin(\omega t)}{c^2 r^3}t,$$

is the projection of the relative motion between the two masses along the $(\tilde{\mathbf{e}}_3)^a$ direction. **The magnitude of $s_{GM}(t)$ increases linearly with time.**



$s_{GM}(t)$ Viewed in the Local Frame of the S/C

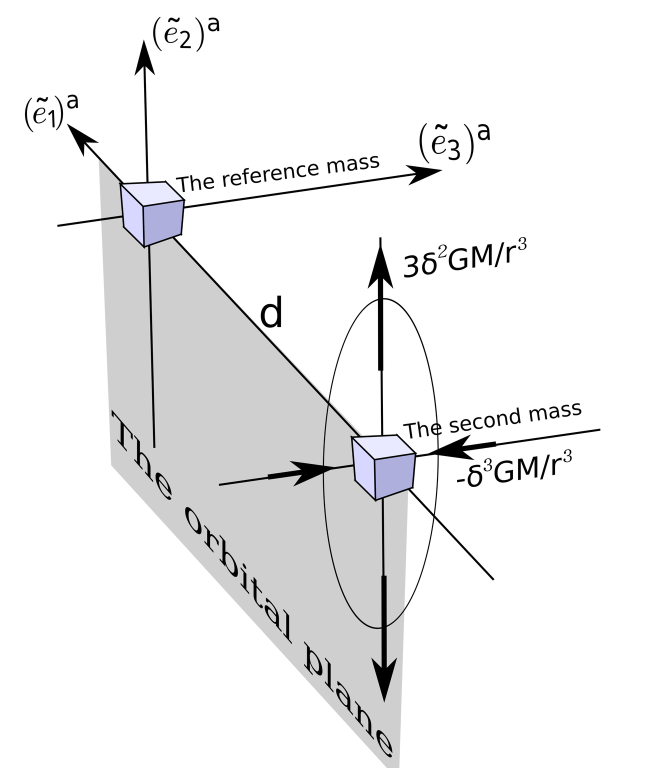
- Locally, the two masses system does not know the precession and other orbital perturbations. The physical observables are the gravitational gradients and the relative motions.
- Along 1PN spherical orbits, the leading Newtonian gradient between the two masses in the S/C local frame has the form $\begin{pmatrix} 0 & 0 & 0 \\ 0 & -\frac{3GM}{r^3} & 0 \\ 0 & 0 & \frac{GM}{r^3} \end{pmatrix}$.

Therefore, along the $(\tilde{\mathbf{e}}_3)^a$ direction in the S/C local frame, the second mass is equivalent to a harmonic oscillator with natural frequency $\omega = \sqrt{GM/r^3}$.

- In the $(\tilde{\mathbf{e}}_3)^a$ direction, the tidal force from the Earth gravitomagnetic field along the 1PN spherical orbit reads

$$\Delta\mathbf{F}^{\perp}_{GM} \sim -4\frac{GmvdJ}{c^2 r^4}\sin i\cos(\omega t),$$

which has the same frequency as the effective oscillator. **Thus, a time accumulating signal is expected from the in-phase action between $\Delta\mathbf{F}^{\perp}_{GM}$ and the recovering Newtonian tidal force.**

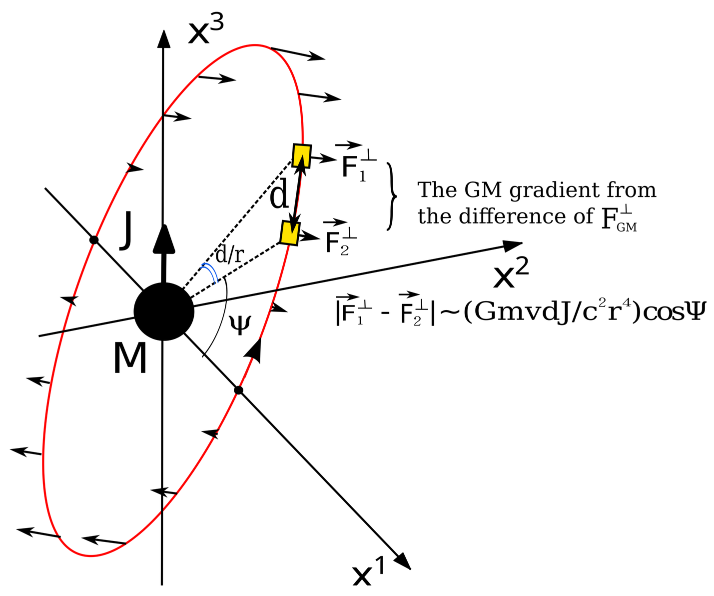


- The evaluation of the geodesic deviation equation along the 1PN spherical orbit gives rise to the relative motions in the local frame

$$\delta^1 = \delta^1_0 + 6(\sin(\omega t) - \omega t)\delta^2_0 + \frac{4\sin(\omega t) - 3\omega t}{\omega}\dot{\delta}^1_0 + \frac{2(\cos(\omega t) - 1)}{\omega}\dot{\delta}^2_0,$$

$$\delta^2 = (4 - 3\cos(\omega t))\delta^2_0 + \frac{2(1 - \cos(\omega t))}{\omega}\dot{\delta}^1_0 + \frac{\sin(\omega t)}{\omega}\dot{\delta}^2_0,$$

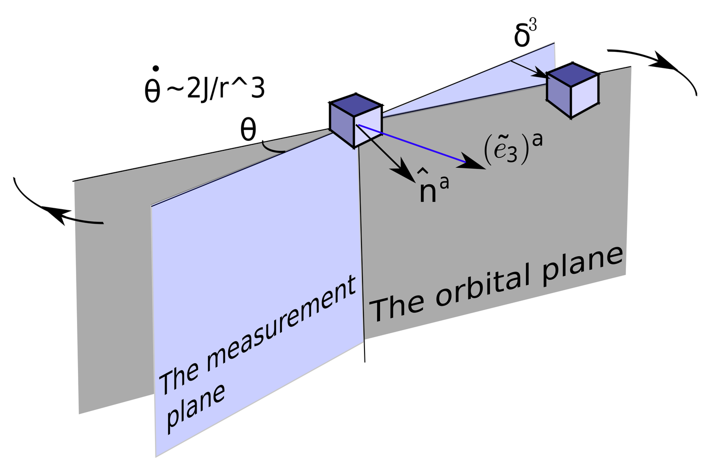
$$\delta^3 = \boxed{\frac{2GJd\sin i\sin(\omega t)}{c^2 r^3}t} + \cos(\omega t)\delta^3_0 + \frac{\sin(\omega t)}{\omega}\dot{\delta}^3_0.$$

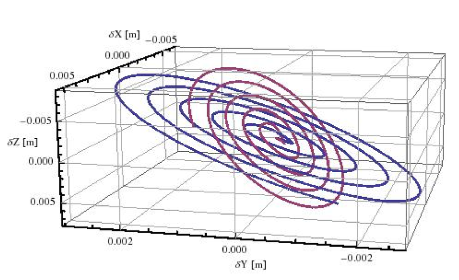


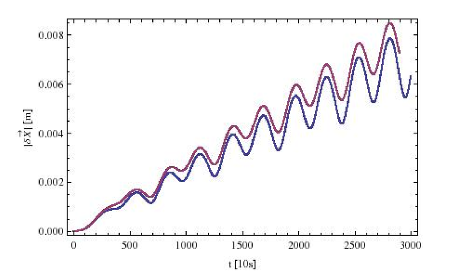
- **The above solutions are approximations for the short time behaviors of the geodesic deviations, which hold true when the deviations are within the 1PN level, that $\delta^i(\tau) \sim d\mathcal{O}(\epsilon^2)$.**

$s_{GM}(t)$ Viewed in the Earth Centered Post-Newtonian System

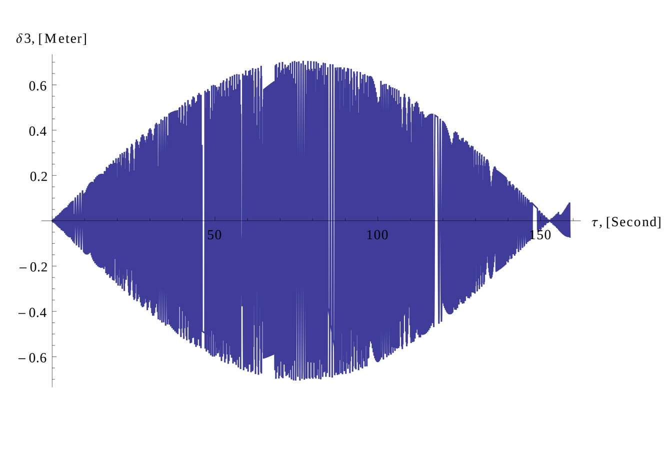
Viewed from the Earth centered coordinates system, the growing magnitude of $s_{GM}(t)$ is produced by the precession of the orbital plane (Lense-Thirring effect) relative to the measurement $(\tilde{\mathbf{e}}_1)^a - (\tilde{\mathbf{e}}_2)^a$ plane that parallel to the initial orbital plane, see the right figure. We also show in the right the growing difference between the precessing orbits and the initial circular orbits with the 59° and 65° inclinations.

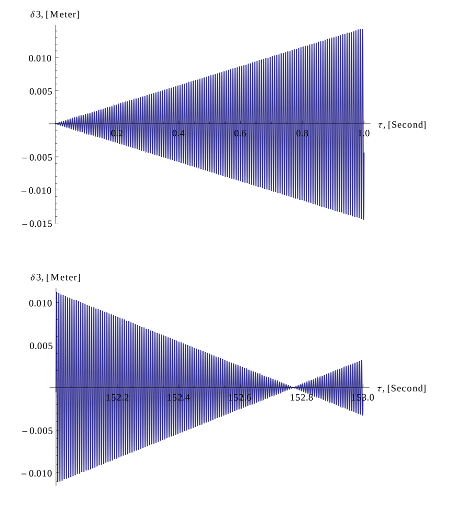




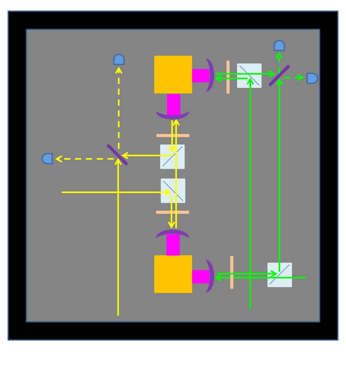


The full solution of the relative motion along the $(\tilde{\mathbf{e}}_3)^a$ direction is modulated by the Lense-Thirring orbital precession with the same period. When the precessing orbit plane, around the J direction, overlaps again with the initial plane, the magnitude of $s_{GM}(t)$ will vanish again. For Earth orbits, it will take the S/C $4 \times 10^7 yrs$ to complete a half round, and therefor the grows of $s_{GM}(t)$ can be take as linearly in time within our mission's lifetime.



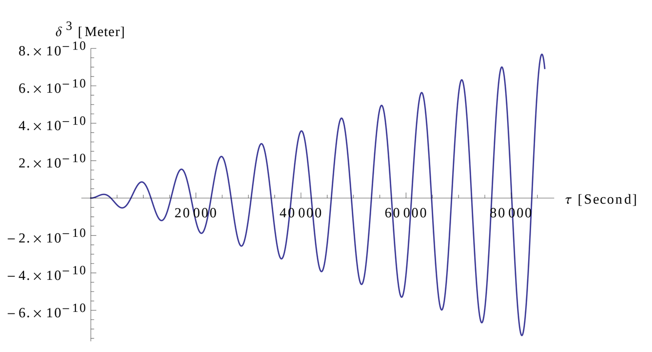


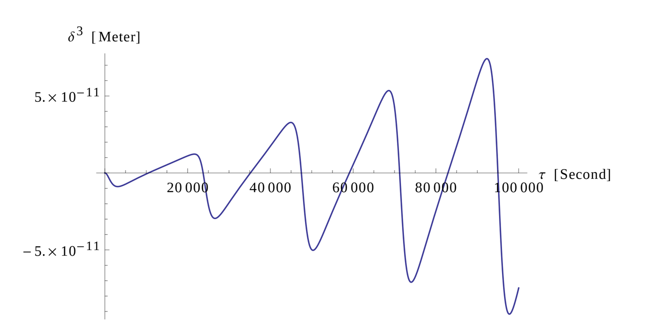
Preliminary Mission Concepts and Scientific Objectives



Mission concepts

- Two proof mass distance **50*cm*** apart.
- On-board laser interferometers as read out system.
- Drag-free system.
- Attitude control.
- Near Polar orbit with altitude **3000*km*** $\sim$ **6000*km***.
- **The gravitomagnetic signal $s_{GM}(t)$ in the cross-track direction will reach a few nanometers in 2 $\sim$ 5*days* operations**
- Make use of the Fermi shifts of the S/C pointing and the orbital eccentricity to add more useful structures in $s_{GM}(t)$, which will benefit in the corresponding data analysis procedure.





Scientific Objectives

- **Direct, precision measurement of Earth's gravitomagnetic field predicted by Einstein to unprecedented accuracy better than 0.1%!**
- Improve the accuracy in the measurement of some post-Newtonian parameters in our solar system, such as α_1 which measures the local Lorentz invariance of gravity theories.
- Track the temporal variation of the Earth gravity field with geoid accuracy better than 1cm and order of earth multiples up to 120.
- New tests and constraints on low energy effective theory related to string theory and quantum gravity, such as Chern-Simons gravity and torsion gravity.

The 10th International LISA Symposium, University of Florida, May 18 - May 23, 2014

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