

Calibration of a Quartz Tuning Fork for Low Temperature Measurements

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Mechanical oscillators are commonly used to measure the properties of cryogenic liquids. Commercial quartz tuning forks have shown promise as viscometers and thermometers for low temperature experiments. These forks are inexpensive, easy to install, and insensitive to magnetic fields. Before a fork can be used, it must be calibrated against a hydrodynamic model. This model is found to be accurate for liquid ^4He at least down to a temperature of 1.5 K. An experiment similar to this one will be incorporated into the University of Florida Advanced Physics Lab 2.

I. Introduction

Studies of the properties and behavior of cryogenic fluids can be done using a mechanical oscillator immersed in the fluid. Through use of a hydrodynamic model [1], changes in the oscillator's amplitude, resonance frequency, and resonance width can be used to infer the temperature, density, or viscosity of the surrounding fluid. However, the oscillator must first be calibrated against a fluid where density and viscosity are known as functions of temperature. In this study, we calibrated a commercial tuning fork against liquid ^4He and compared our data with the hydrodynamic model.

Common mechanical oscillators include wires, spheres, and, more recently, tuning forks. Commercial quartz tuning forks typically measure less than 4 mm in length and are vacuum sealed in a metal canister; these forks are designed to provide reliable timing for watches and

other small devices through a calibrated resonance frequency usually near 2^{15} Hz = 32.768 kHz [1]. To drive oscillations, an alternating voltage is applied across two electrodes plated on the prongs of the fork; the piezoelectric effect converts these mechanical oscillations into electrical current which can be easily measured.

The quartz tuning fork has several advantages and disadvantages over wires and spheres. First, tuning forks are inexpensive and can be installed by simply opening the metal canister and connecting the two provided electrode leads. Second, individual forks can be calibrated through only three parameters from resonance curve measurements in vacuum and in a fluid of known density and viscosity. Finally, forks do not require a magnetic field for operation and are generally insensitive to the presence of such a field [2]. On the other hand, due to their novelty and more complex geometry, far less theoretical information is available on the behavior of such forks in viscous fluids [1]. Also, forks can differ significantly from one model to another and even between samples of the same model; hence, each individual fork must be calibrated before use [1].

To calibrate the fork, we measured current, resonance frequency, and resonance width in liquid ^4He down to a temperature of 1.5 K. Our primary goal was to test the validity of quartz tuning forks as thermometers and viscometers in cryogenic fluids and to identify any potential difficulties with obtaining reliable measurements.

II. Theory

1. Fork in Vacuum

We used the model of Blaauwgeers *et al.* [1] to relate the fork response to the properties of the surrounding fluid. As this model depends on the properties of the fork in vacuum, we first

describe the fork in that domain. The current output by fork is directly proportional to the velocity of the prongs of the fork due to the piezoelectric effect:

$$I(t) = a \cdot \frac{dx(t)}{dt}, \quad (1)$$

where a is the so-called fork constant.

If the amplitude of the oscillations is small then the fork can be described as a harmonic oscillator, which can be treated both as a mechanical oscillator and as an RLC resonance circuit. By comparing these two models and utilizing Eq. (1), we may relate the mechanical and electric behavior of fork:

$$F_0 = \frac{a}{2} U_0, \quad (2)$$

$$R = 2m \Delta\omega / a^2 \quad (3)$$

$$L = 2m / a^2 \quad (4)$$

$$C = a^2 / (2k) \quad (5)$$

where $\Delta\omega$ is the drag coefficient, U_0 is the amplitude of the driving voltage, F_0 is the amplitude of the driving force, and k is the spring constant. The effective mass of one prong of the fork, m , is given in vacuum by [3]

$$m_{vac} = 0.24267 \cdot \rho_q LWT, \quad (6)$$

where $\rho_q = 2659 \text{ kg/m}^3$ is the density of quartz, and the lengths L , W , and D are shown in Figure 1. For our fork [4] $L = 2.809 \text{ mm}$, $W = 0.127 \text{ mm}$, and $D = 0.325 \text{ mm}$, and these dimensions yield $m_{vac} = 7.48 \times 10^{-8} \text{ kg}$. From Eq. (3) we may restate the fork constant as

$$a = \sqrt{\frac{2m \Delta\omega}{R}}. \quad (7)$$

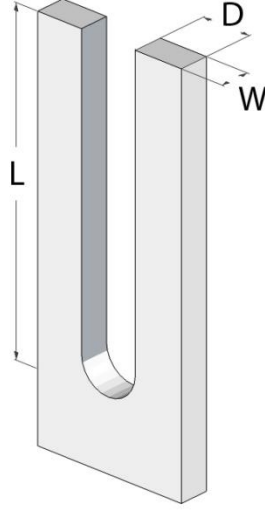


Figure 1. Diagram of a Tuning Fork.

With Eq. (7) the fork constant may be determined after finding R experimentally. For our fork, we obtained $\alpha = 3.4 \times 10^{-6}$ C/m.

As a harmonic oscillator, the response of the fork can be modeled as

$I(t) = I_a \cos(\omega t) + I_b \sin(\omega t)$, where I_a is the absorption component and I_b is the dispersion component. We measured these components separately utilizing a lock-in amplifier.

Both components have well-known theoretical expressions:

$$I_a = \frac{I_0 (\Delta\omega)^2 \omega^2}{(\Delta\omega)^2 \omega^2 + (\omega^2 - \omega_0^2)^2}, \quad (8)$$

$$I_d = \frac{I_0 \Delta\omega \omega (\omega^2 - \omega_0^2)}{(\Delta\omega)^2 \omega^2 + (\omega^2 - \omega_0^2)^2}. \quad (9)$$

We used these expressions to determine I , $\Delta\omega$, and the resonance frequency ω_0 from the fork current response.

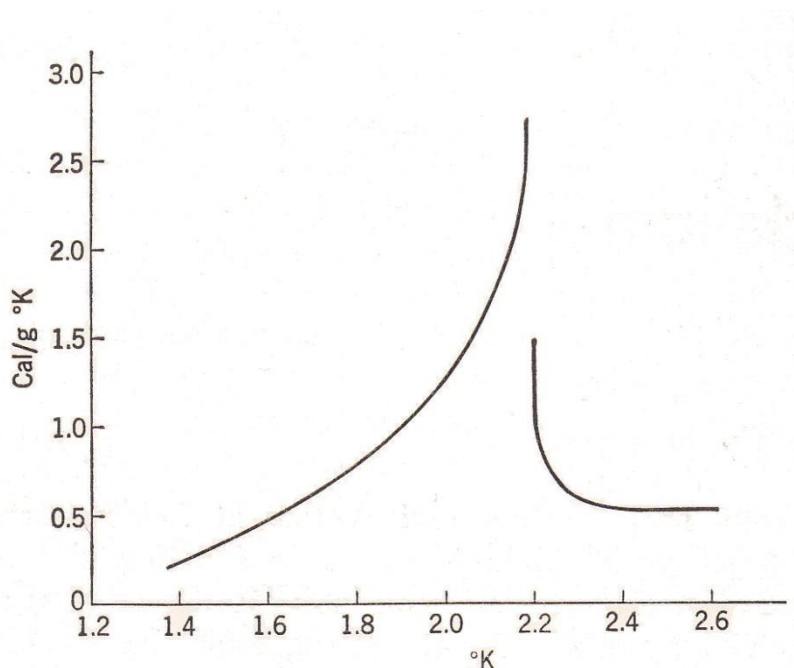


Figure 2. The specific heat capacity of liquid ${}^4\text{He}$ as a function of temperature at saturated vapor pressure. The superfluid transition of ${}^4\text{He}$ is called the λ -point due to the shape of this plot. (Source: C. T. Lane [8]).

2. Superfluid ${}^4\text{He}$

The point at which ${}^4\text{He}$ undergoes the transition to a superfluid, $T_\lambda = 2.1768$ K for saturated vapor pressure, is often called the λ -point due to the shape of heat capacity as a function of temperature around this point, shown in Figure 2. At temperatures below T_λ , the behavior of the liquid ${}^4\text{He}$ can be reasonably described by treating it as a mixture of a normal fluid and a superfluid [5], with the total fluid density now given by $\rho = \rho_n + \rho_s$, where ρ_n is the density of normal fluid and ρ_s is the density of superfluid. The superfluid component is characterized by zero viscosity, zero entropy, and infinite heat conductance. Figures 3, 4, and 5 show the total fluid density, the normal fluid density, and the dynamic viscosity of liquid ${}^4\text{He}$, respectively (using data from [6]). This two-fluid model is necessary to describe the viscosity felt by the tuning fork while immersed in superfluid ${}^4\text{He}$.

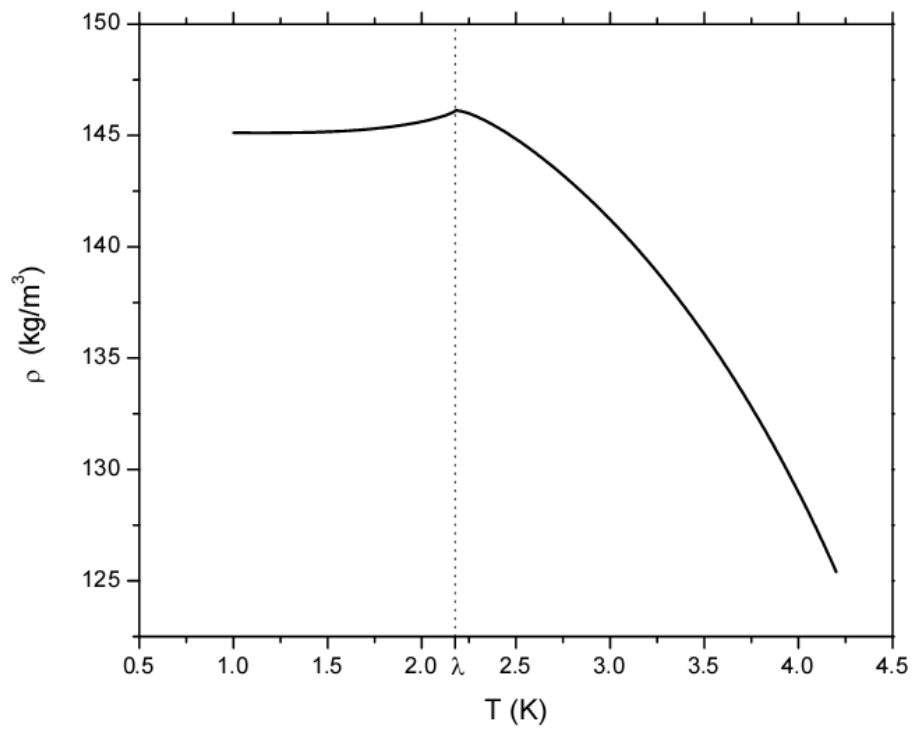


Figure 3. Total fluid density of ${}^4\text{He}$, $\rho = \rho_n + \rho_s$.

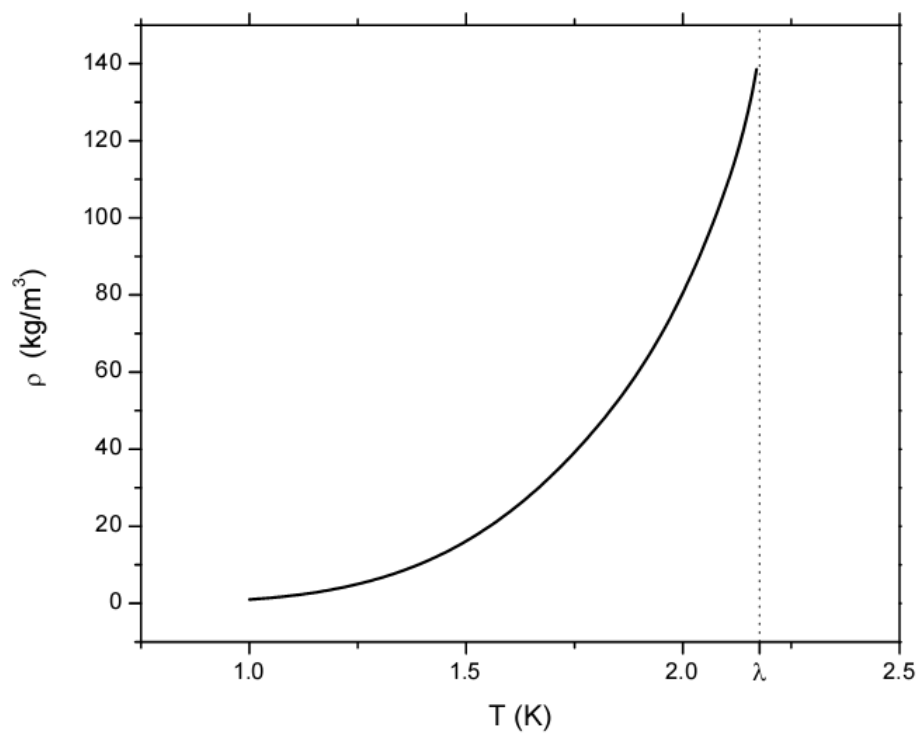


Figure 4. Normal fluid density of ${}^4\text{He}$, ρ_n .

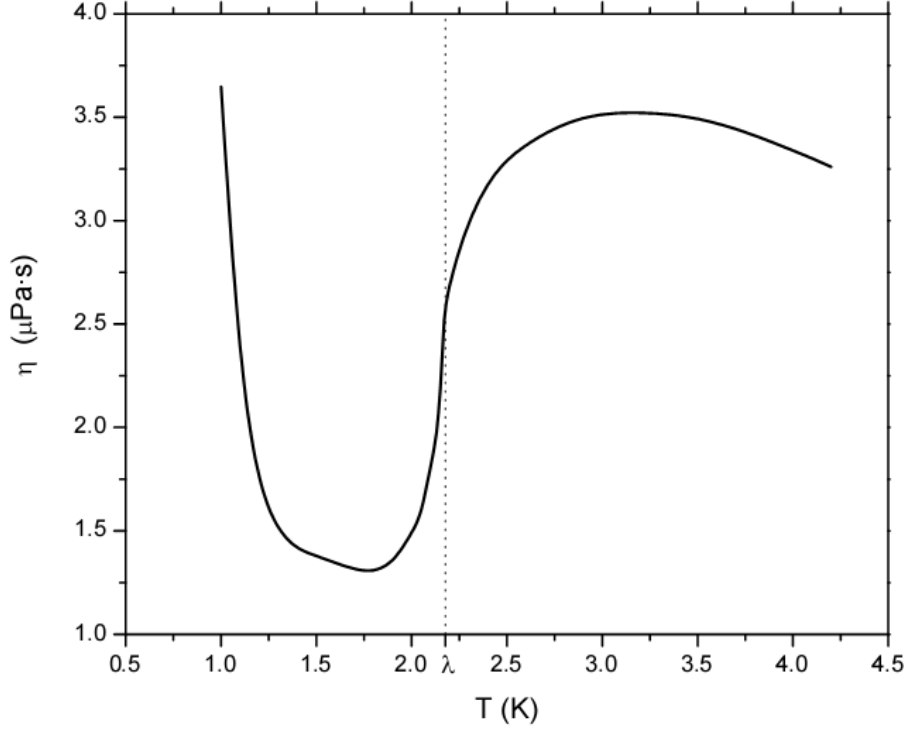


Figure 5. Dynamic viscosity of ${}^4\text{He}$. Past T_λ , the superfluid component is postulated to have no viscosity.

3. Fork in Viscous Fluid

The hydrodynamic model of Blaauwgeers *et al.* [1] adds a drag force, linear in \boldsymbol{v} and $\dot{\boldsymbol{v}}$, to account for the differences between motion in vacuum and motion in a viscous fluid. While this model is simple, it is reasonably accurate as long as several assumptions hold. In classical hydrodynamics [5], the flow near an oscillating body is rotational and governed by the Navier-Stokes equation, whereas the flow at large distances can be treated as an ideal fluid. The length scale of the rotational flow is given by the depth of penetration,

$$\delta = \sqrt{\frac{2\eta}{\rho_n \omega}}, \quad (10)$$

where ω is the frequency of oscillation, η is the dynamic viscosity, and ρ_n is the normal fluid density (excluding superfluid density). We take the characteristic length of our oscillator to be D

and require that $D \gg \delta$ and $D \gg x_0$, where x_0 is the oscillation amplitude. This requirement ensures that the rotational flow is restricted to only a small layer near the surface of the oscillator. From the known properties of liquid He⁴ [6], we estimate δ to be on the order of 0.1 μm for an oscillation frequency of 32 kHz, clearly less than $D = 0.325$ mm. We also note that, at this frequency, a velocity of over 1 m/s would be necessary to violate $D \gg x_0$; we kept the velocity below 1 cm/s for this experiment.

If the preceding conditions are satisfied, then it is reasonably valid to model the oscillator system with the drag force

$$F_D = bv + \tilde{m}\dot{v}, \quad (11)$$

where b represents drag and \tilde{m} represents mass enhancement. The coefficient b is found by solving the potential flow problem around the fork:

$$b = \frac{C}{2} \rho_n V_\delta, \quad (12)$$

where C is a coefficient related to the fork geometry, $S = 2(D + W)L$ is the surface area of a leg of the fork, and $V_\delta = S\delta$ is the volume of rotational flow. The mass enhancement is given by

$$\tilde{m} = \beta\rho V + B\rho_n V_\delta, \quad (13)$$

where both β and B are geometrical coefficients, $V = WDL$ is the volume of a single prong, and ρ is the total fluid density. In Eq. (13), the first term accounts for the fluid displaced by the body, while the second term accounts for fluid moving in comotion with the body.

By adding F_D to the vacuum equation of motion and simplifying, we obtain the following relations:

$$\left(\frac{f_{0vac}}{f_0}\right)^2 = 1 + \frac{1}{\rho_q} \left(\beta \rho + B \frac{S}{V} \sqrt{\frac{\rho n \eta}{\pi f_0}} \right), \quad (14)$$

$$\Delta f = \frac{1}{2m_{vac}} \sqrt{\frac{\rho n \eta f_0}{\pi}} C S \left(\frac{f_0}{f_{0vac}}\right)^2. \quad (15)$$

Note that a term in Eq. (15) dependent on Δf_{vac} has been removed due to its negligible magnitude. Eqs. (14) and (15) can be used to determine β , B , and C if ρ and η are known.

Finally, once these coefficients have been found the tuning fork can be used to measure ρ and η .

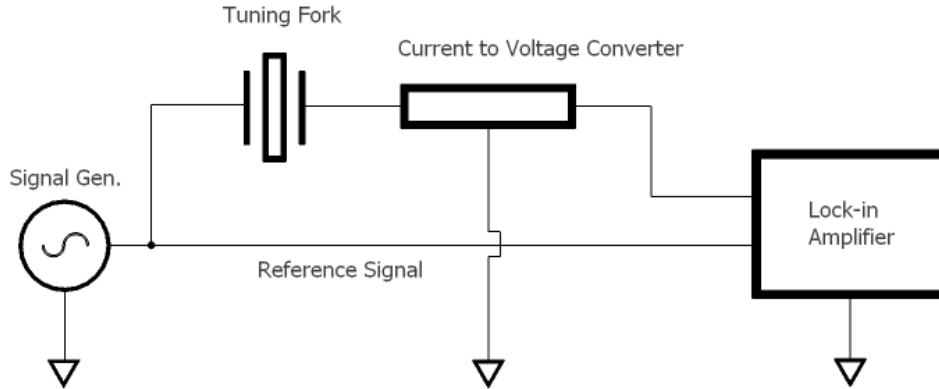


Figure 6. Setup for Tuning Fork measurements. We used the signal generator's SYNC output as a reference signal.

III. Experiment

1. Vacuum

We took measurements using the setup diagrammed in Figure 6; specifically, we utilized a Tegan 2725A function generator and a Stanford Research Systems SR844 lock-in amplifier.

The lock-in amplifier allowed us to detect low amplitude signals in a noisy environment and

separate those signals into components in-phase and out-of-phase with the function generator. To determine the resonance properties of the fork at a given time, we slowly varied the frequency of the function generator and collected the lock-in output as a function of frequency. We fixed the driving voltage at 1.1 mV (except when we measured resistance.) An example pair of resonance curves is shown in Figure 7. These frequency sweeps took from 3 to 7 minutes because the fork oscillations needed at least 5 seconds to stabilize after each change in frequency. Afterwards, the completed curves were fit to Eqs. (8) and (9) to determine I_0 , f_0 , and Δf . This process was automated using LabVIEW to control instrumentation and MATLAB to fit functions with the “levmar” library [7].

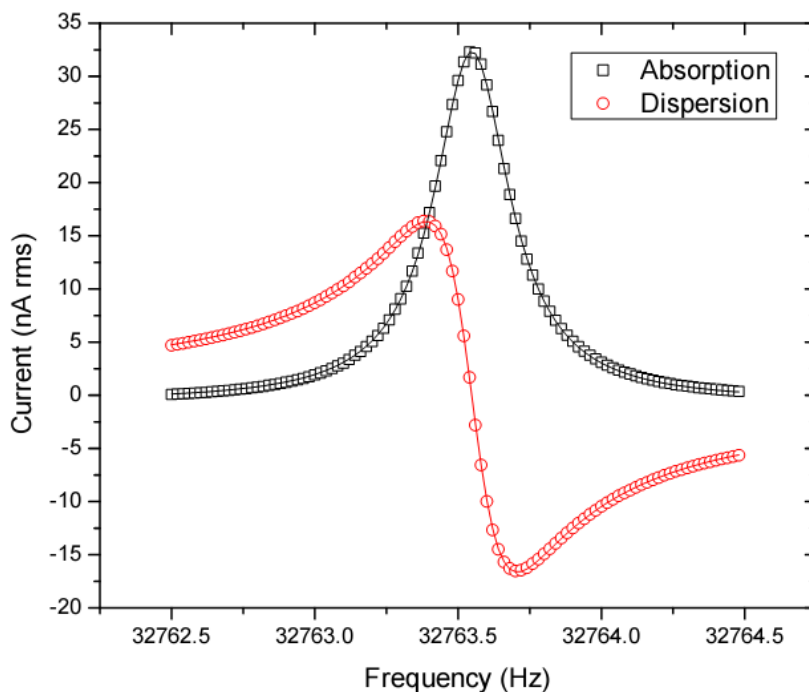


Figure 7. Example of curves generated by the tuning fork in a vacuum cell at room temperature. Here $f_0 = 32763.5$ Hz, $\Delta f = 0.32$ Hz, and $Q = 1.03 \times 10^5$. The solid lines are the fits to Eqs. (8) and (9).

Throughout this experiment we used only a single tuning fork [4]; this fork was opened to the environment by cutting off the top of the canister using a lathe. We first tested this fork in vacuum at room temperature and $\sim 77\text{K}$ to determine f_{0vac} , Δf_{vac} , R and a , given in Table I.

Table I. Vacuum Properties of Tuning Fork

	T = 300 K	T = 77 K
f_{0vac}	32,763.537 Hz	32,725.002 Hz
Δf_{vac}	0.320 Hz	0.511 Hz
R	26 k Ω	40 k Ω
a	3.4×10^{-6} C/m	3.5×10^{-6} C/m

Given that the variance of these parameters had negligible impact on our analysis, we chose to use only those values obtained at room temperature.

2. Liquid ^4He

To test the fork in liquid ^4He , we used something called a “suckstick,” displayed in Figure 8. First, the fork was mounted on the bottom of a long low-temperature probe alongside a precision Cernox thermometry resistor. This probe was then inserted inside a metal tube, the suckstick, with a vacuum jacket and a small capillary at the bottom. The bottom portion of this apparatus is diagramed in Figure 9. To minimize the heat leak when cooling below 4.2 K, the bottom portion of the probe is made out of polycarbonate, a poor conductor of heat. Additionally, the coaxial wires running from the beginning of the polycarbonate portion to the tuning fork use stainless steel conductors, also to minimize heat transfer. The wires connecting to the thermometer, and all wires leading down to the polycarbonate portion, are made from phosphor copper. To accelerate the initial cooldown, we filled the vacuum jacket of the suckstick with

helium gas to promote heat transfer. The suckstick was slowly lowered into a liquid ^4He Dewar until the stick cooled and helium liquid was able to enter through the capillary. Before letting the inner chamber fill with liquid, we evacuated the vacuum jacket in order to maintain lower temperatures in the inner chamber. By pumping out vapor in the inner chamber of the suckstick, we lowered the temperature around the fork to 1.5K.

While the suckstick cooled down relatively fast, it could be used only for one or two days before the capillary became too clogged to allow sufficient liquid ^4He into the chamber. We also noted that the fork measurements are very sensitive to any contaminants that may be present in the helium or the suckstick. Furthermore, we have had to warm the entire suckstick multiple times due to our measurements becoming distorted, once by almost an order of magnitude.

We collected temperature dependent data by controlling the vapor pressure in the inner chamber. Once the thermometer was reading a stable temperature, we ran the frequency sweep program to generate a data point consisting of f_0 , Δf , and T. After collecting sufficient data we fit f_0 and Δf as functions of temperature to Eqs. (14) and (15) to determine the three calibration coefficients. Either due to electrical issues or particles adsorbing to the surface of the fork, we encountered significant problems obtaining consistent series of measurements; therefore, we were forced to settle for a limited number of data points which were taken during a single cooldown.

Additionally, we verified that turbulence was unlikely to affect our measurements by running amplitude tests at the lowest temperature reached by the probe, 1.51 K. Significant changes in resonance only appeared after raising the driving voltage above 300 mV. Because our measurements were taken at 1.1 mV, they are unlikely to have been influenced by turbulent effects.

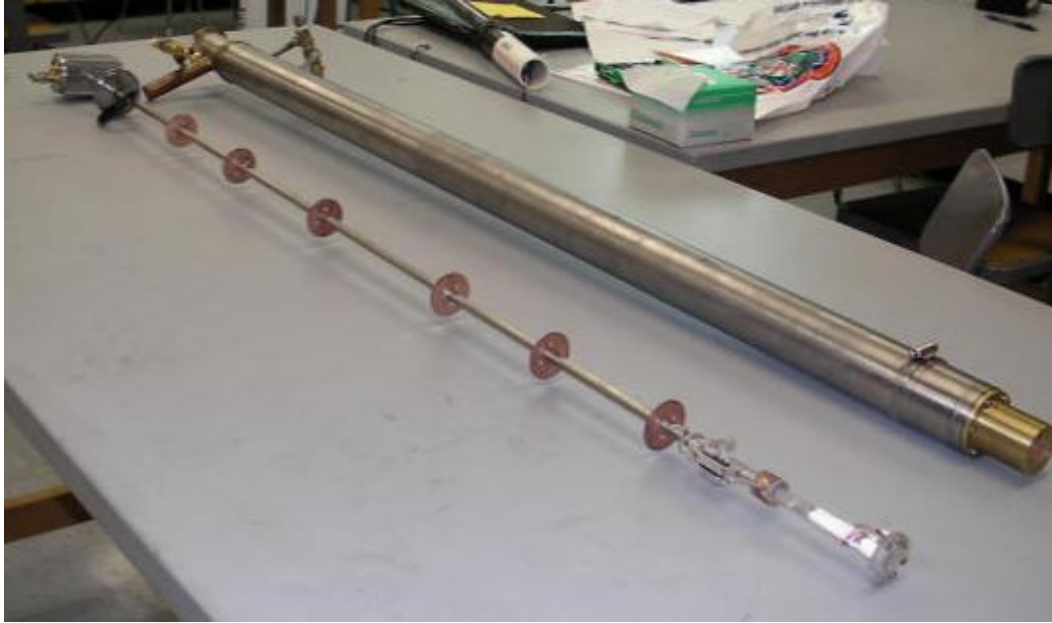


Figure 8. The probe (left) carries the thermometer and tuning fork and is inserted into the suckstick (right).

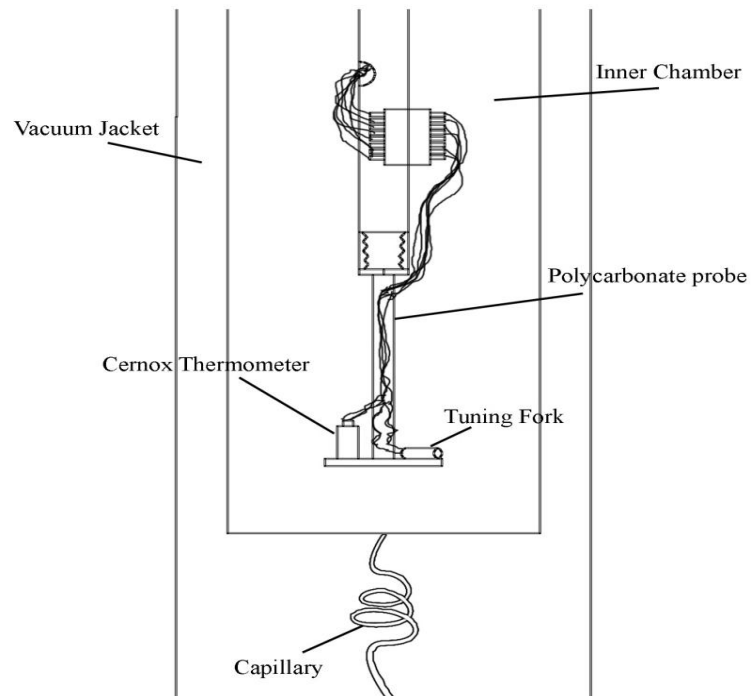


Figure 9. Diagram showing the bottom polycarbonate portion of the probe inside the suckstick. The wire connector located above the polycarbonate portion allows that portion to be unscrewed and removed from the probe. The connector also anchors the wires in order to lessen the effects of contraction due to cooling.

IV. Results and Conclusion

The results of our experiment in liquid ^4He are shown in Figures 10 and 11. Both f_0 and Δf are quite sensitive to the properties of the surrounding fluid; in particular, both show clear changes after the lambda point. Clearly, Eqs. (14) and (15) are reasonably good descriptions of a tuning fork in liquid ^4He . However, it remains to be seen whether or not they will remain accurate below 1K. If this model remains predictive, then the quartz tuning fork could serve as a low temperature thermometer and viscometer that is both easy to install and requires no magnetic field to operate.

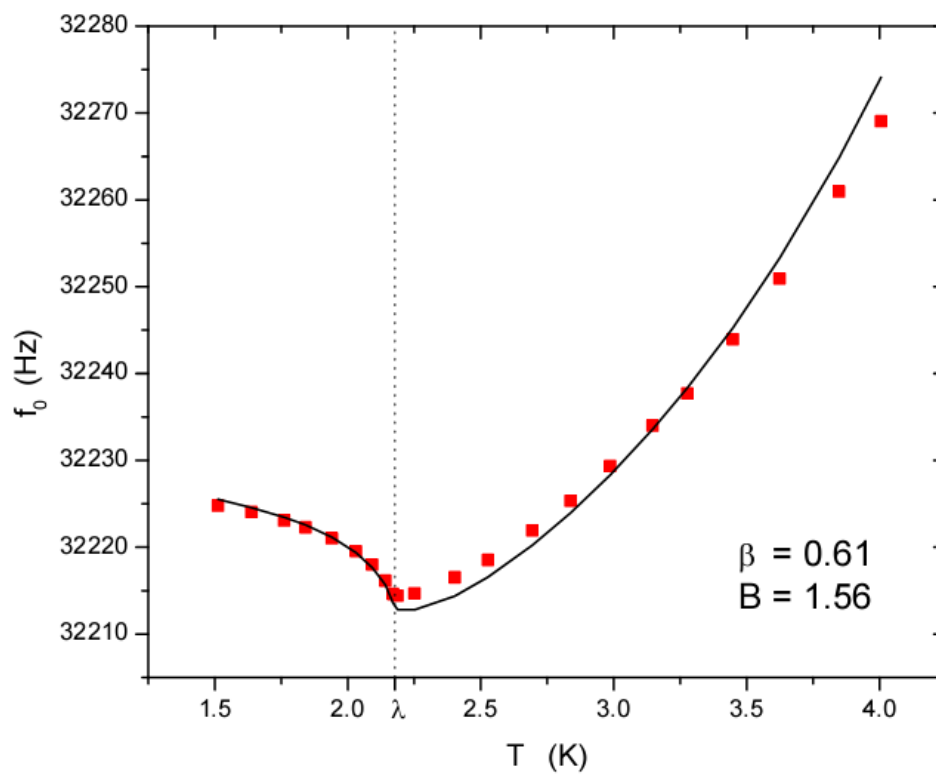


Figure 10. Resonance frequency f_0 as a function of temperature. The black line is the fit to Eq. (14).

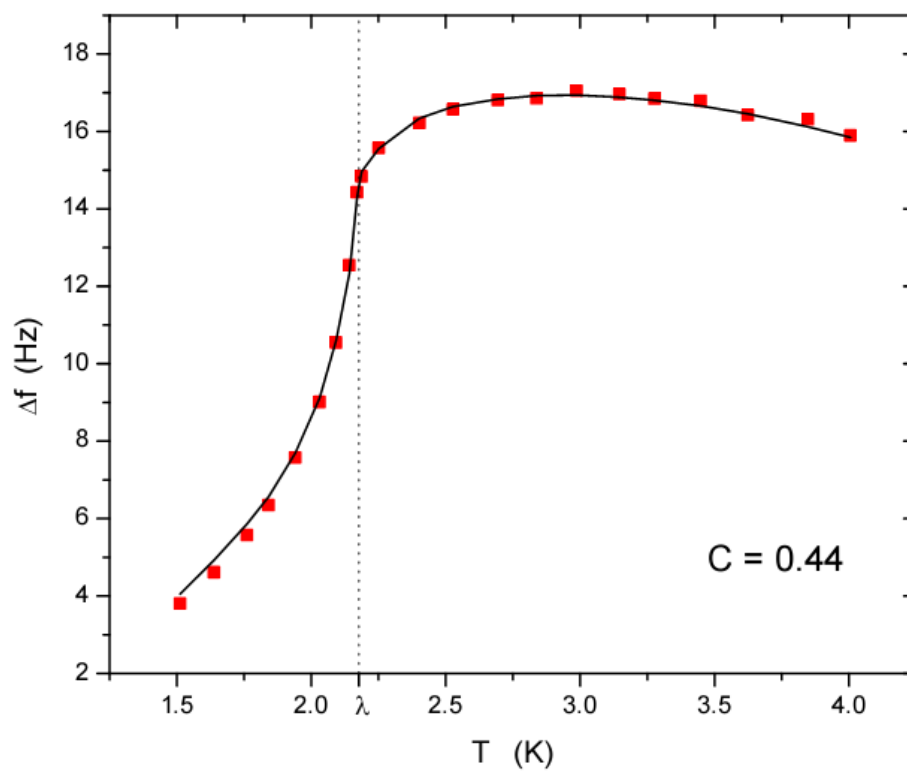


Figure 11. Resonance width Δf as a function of temperature. The black line shows the fit to Eq. (15).

Acknowledgments

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