Effects of Inhomogeneities on Cosmic Expansion

E. R. Siegel
Department of Physics, University of Florida, P.O. Box 118440, Gainesville, FL 32611, USA
siegel@phys.ufl.edu
J. N. Fry
Department of Physics, University of Florida, P.O. Box 118440, Gainesville, FL 32611, USA
fry@phys.ufl.edu

Abstract

We evaluate the effect of inhomogeneity energy on the expansion rate of the universe. Our method is to expand to Newtonian order in potential and velocity but to take into account fully nonlinear density inhomogeneities. To linear order in density, kinetic and gravitational potential energy contribute to the total energy of the universe with the same scaling with expansion factor as spatial curvature. In the strongly nonlinear regime, growth saturates, and the net effect of the inhomogeneity energy on the expansion rate remains negligible at all times. In particular, inhomogeneity contributions never mimic the effects of dark energy or induce an accelerated expansion.

1. Introduction

The standard Friedmann equation governing the expansion of a flat universe at late times is

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 \equiv \frac{8\pi G}{3}\rho_c = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda), \tag{1}$$

where ρ_Λ is generally added to explain the observation of an apparently accelerated expansion. One alternative to adding Λ, investigated here, is that $\rho_m = \rho_c$, but the binding energy in gravitational inhomogeneities may be responsible for the observed accelerated expansion, as suggested by Kolb et al. [1].

2. Analytical Framework

Contrary to the approach of Kolb et al. [2], we adopt the approach of Seljak and Hui [3] and work in the conformal Newtonian gauge, beginning from the Einstein equation for $G^0_{\ 0}$:

$$3\left(\frac{\dot{a}}{a}\right)^2(1-2\psi) + (2+6\phi)\frac{1}{a^2}\nabla^2\phi + \frac{1}{a^2}(\nabla\phi)^2 = 8\pi G\bar{\rho}(1+\delta)(1+v^2). \tag{2}$$

From equation (2), we derive the modified Friedmann equation below,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\bar{\rho}(1-5W+2K), \tag{3}$$

$$W = \frac{1}{2}\langle(1+\delta)\,\phi\rangle, \qquad K = \frac{1}{2}\langle(1+\delta)\,v^2\rangle. \tag{4}$$

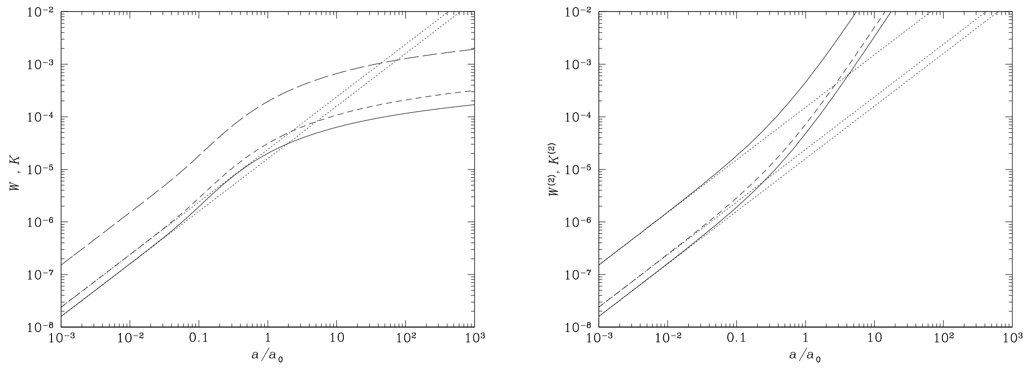

Fig. 1. Inhomogeneity contributions for fully nonlinear (left) and second-order (right) density contrasts. The topmost curve is the total inhomogeneity contribution, the lowest, solid curve is kinetic alone and the dashed, middle curve is potential alone. The straight, dotted lines are for linear-order density contrasts.

Equation (3) is valid to first order in ϕ, v^2, W, and K, but for fully nonlinear density contrasts, $\delta \equiv \delta\rho/\rho \gg 1$.

The Newtonian potential energy (W) can influence cosmic expansion. From the density power spectrum, the dimensionless spectral density, $\Delta^2_W(k)$, is defined by

$$W = -\frac{1}{2}4\pi G\bar{\rho}a^2 \int \frac{d^3k}{(2\pi)^3}\frac{P(k)}{k^2} = -\int \frac{dk}{k}\Delta^2_W(k), \tag{5}$$

Where $P(k)$ is the matter power spectrum.

The kinetic energy (K) can be calculated from the potential energy W through the cosmic energy equation [4],

$$\left(\frac{d}{dt} + \frac{2\dot{a}}{a}\right)K = -\left(\frac{d}{dt} + \frac{\dot{a}}{a}\right)W. \tag{6}$$

By integrating equations (5) and (6), the effect of inhomogeneities on the universe's expansion history can be obtained through equation (3).

3. Results

The maximum contribution to the energy density of the universe arising from inhomogeneities comes on scales of $\mathcal{O}(1\ \mathrm{Mpc})$, with contributions on smaller and larger scales falling as $\sim k^2$. The overall contribution of the potential and kinetic terms is illustrated in Fig. 1 for linear, second order, and fully nonlinear density contrasts. The overall result is that although there is an apparent breakdown in perturbation theory [5], this effect is not physically important, as the fully nonlinear result is well-behaved [6], and is always small compared to the matter contribution.

4. Conclusions

Sub-horizon inhomogeneities, arising mainly from $\mathcal{O}(\mathrm{Mpc})$ scales, have a negligibly small effect on cosmic expansion at all times. In the presence of a true dark energy component, the fractional inhomogeneity energy density falls as $\sim a^{-3}$. Although a perturbative expansion in density contrast appears to break down at late times, use of the fully nonlinear matter power spectrum illustrates that inhomogeneity contributions never induce an accelerated expansion in the universe.

References

[1] Kolb, E. W., Matarrese, S., Notari, A., and Riotto, A. (2005) hep-th/0503117.

[2] Kolb, E. W., Matarrese, S., Notari, A., and Riotto, A. Phys. Rev. D71 (2005) 023524.

[3] Seljak, U., and Hui, L. ASP Conf. Ser. 88: *Clusters, Lensing, and the Future of the Universe*, 88 (1996) 267.

[4] Irvine, W. M. Ph.D. thesis, Harvard University (1961); Layzer, D. Astrophys. J. 138 (1963) 174

[5] Notari, A. (2005) astro-ph/0503715.

[6] Siegel, E. R. and Fry, J. N. (2005) astro-ph/0504421

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5. Title of the Paper

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6. Authors

Author No. 1

- Full Name: Ethan Ryan Siegel
- First Name: Ethan
- Middle Name: Ryan
- Surname: Siegel
- Initialized Name: E. R. Siegel
- Affiliation: University of Florida, Gainesville, FL 32611, USA
- E-Mail: siegel@phys.ufl.edu
- Ship the Proceedings to: P.O. Box 118440, Gainesville, FL 32611-8440, USA

Author No. 2

- Full Name: James Nathan Fry
- First Name: James
- Middle Name: Nathan
- Surname: Fry
- Initialized Name: J. N. Fry
- Affiliation: University of Florida, Gainesville, FL 32611, USA
- E-Mail: fry@phys.ufl.edu
- Ship the Proceedings to: P.O. Box 118440, Gainesville, FL 32611-8440, USA