Equivalent circuit measurement and complex impedance of calorimeters and bolometers

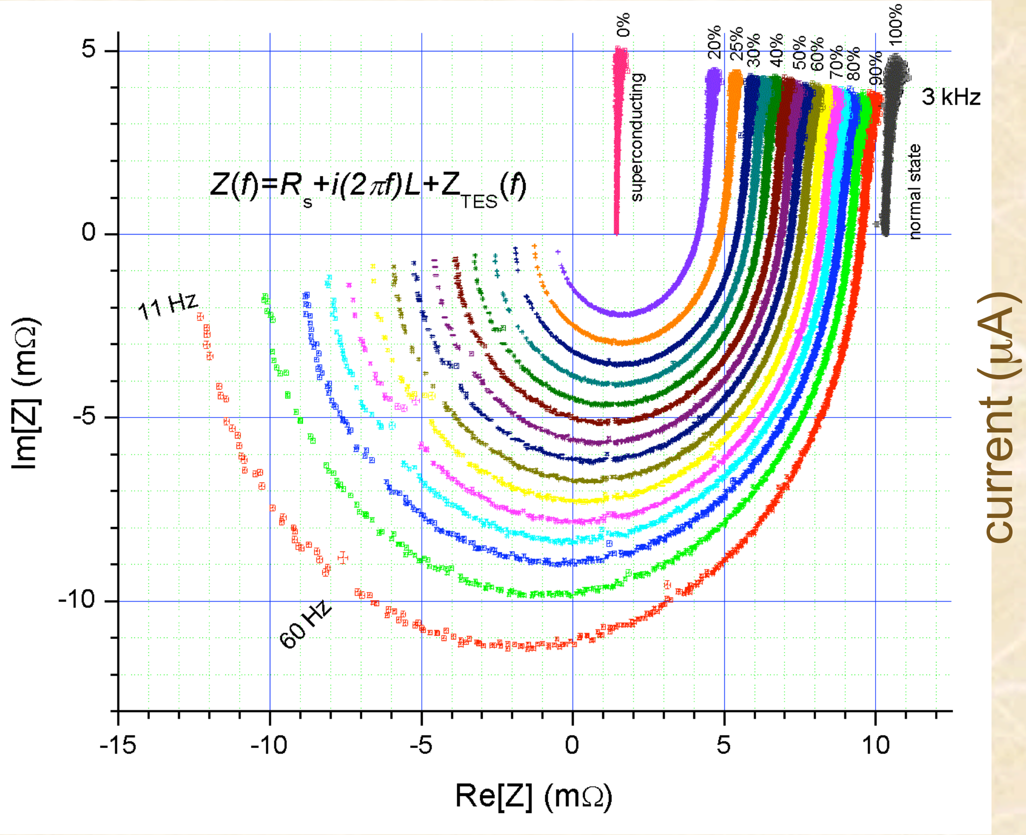
M. A. Lindeman, K. Barger, D. E. Brandl, S. G. Crowder,
L. Rocks, D. McCammon[a)]

University of Wisconsin, Madison

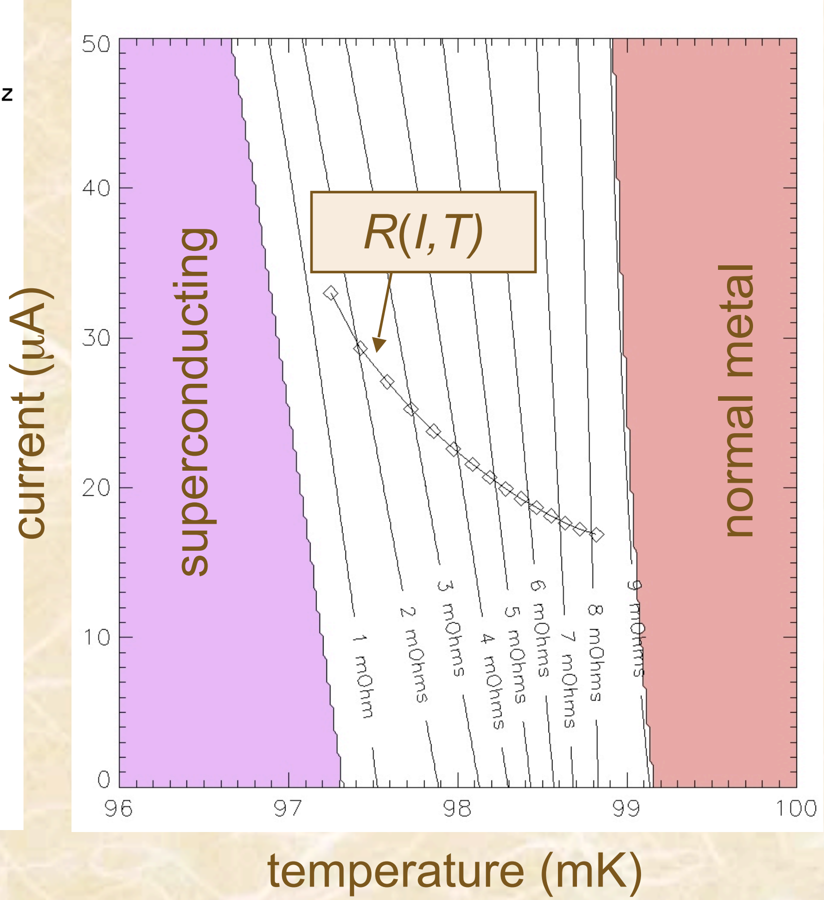
Impedance measurements

THE UNIVERSITY of WISCONSIN MADISON

Impedance of a TES

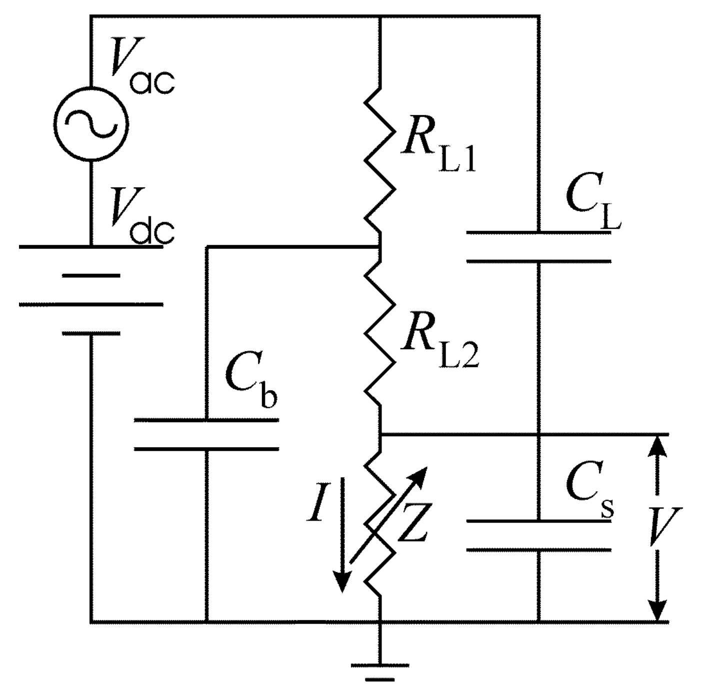


Z used to understand Detector physics

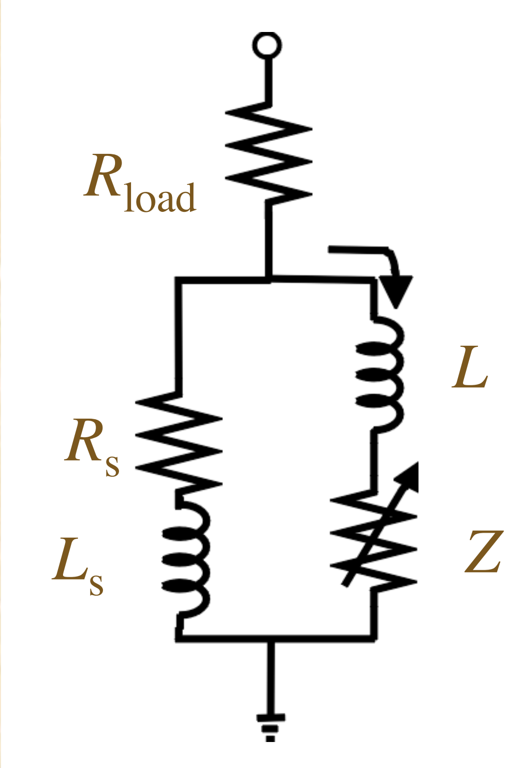


Bias circuit models

Voltage readout
(Si Thermistor)

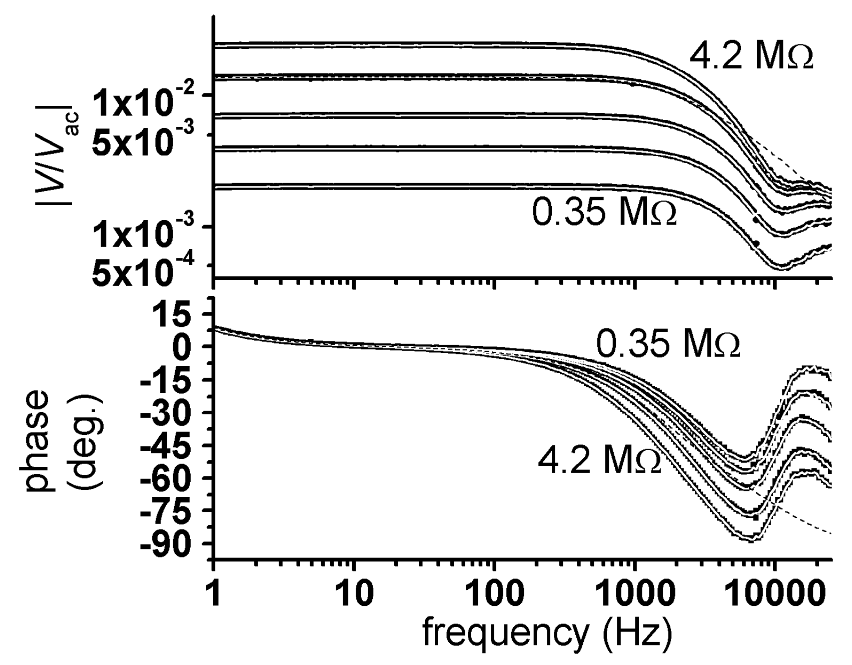


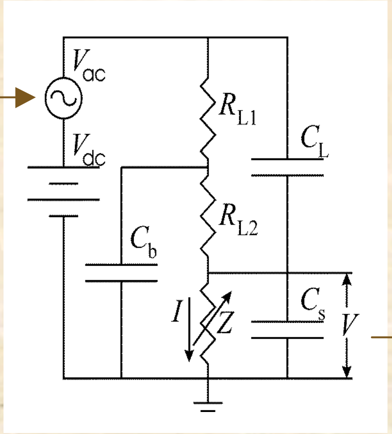
Current Readout
TES



Measurement of transfer functions

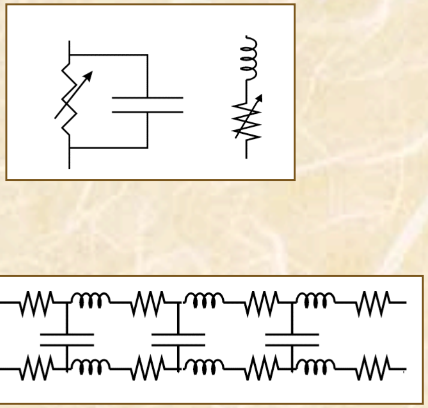
- Measure transfer function
  - Put a signal into the bias
  - Measure response at the detector
  - Ratio of output/input is transfer function
- Use with circuit model to calculate *Z*(*f*)

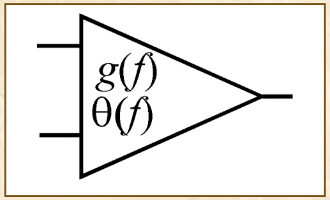




Non-ideal elements

- Parasitic impedance
- Distributed loads
- Gain and phase shifts of amplifier are frequency dependent
- All these things affect measurement of *Z*(*f*)
- May be hard to model every important contribution
- Would like a technique that **easily removes effects of these**

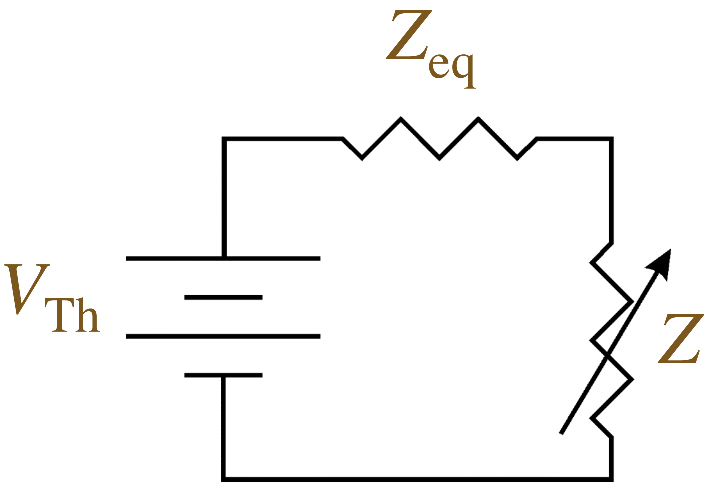




Equivalent circuits

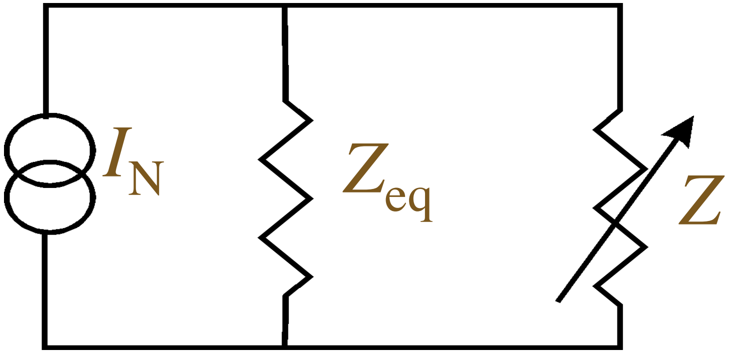
- The bias circuit can be replaced by a Thevenin or Norton equivalent circuit
- Equivalents can be measured independently for each frequency
- Allows you to isolate the impedance Z of thermistor

Thevenin



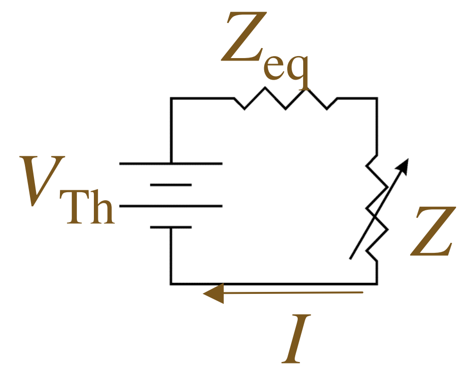
Norton

$I_N = Z_{eq} / V_{Th}$



Finding Thevenin / Norton equivalents

- When Z is real valued:
  - Superconducting TES
  - normal metal TES
  - Unbiased Si thermistor or TES
- If Z is real valued resistance R
  - simple relationships between current and resistance



$$V_{Th} = I(Z_{Th} + Z)$$

Linear equations:

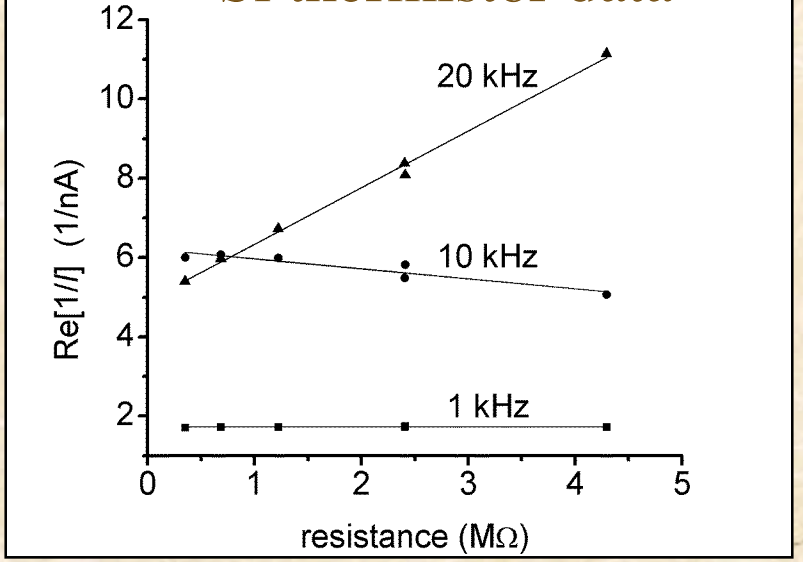
$$\mathrm{Re}[I^{-1}] = \mathrm{Re}[V_{\mathrm{Th}}{}^{-1}]R + \mathrm{Re}[I_{\mathrm{N}}{}^{-1}]$$

$$\mathrm{Im}[I^{-1}] = \mathrm{Im}[V_{\mathrm{Th}}{}^{-1}]R + \mathrm{Im}[I_{\mathrm{N}}{}^{-1}]$$

Linear fits

- Choose *R* of thermistor by setting temperature of the fridge
- (1/I) is a linear function of *R*
  - Slope gives $(1/V_{\mathrm{Th}})$
  - Y-intercept gives $(1/I_{\mathrm{N}})$
- Find resistance R from low frequency measurements
- Plot Re[1/*I*] vs.. *R* and Im[1/*I*] vs.. *R*
- Results for different frequencies are independent

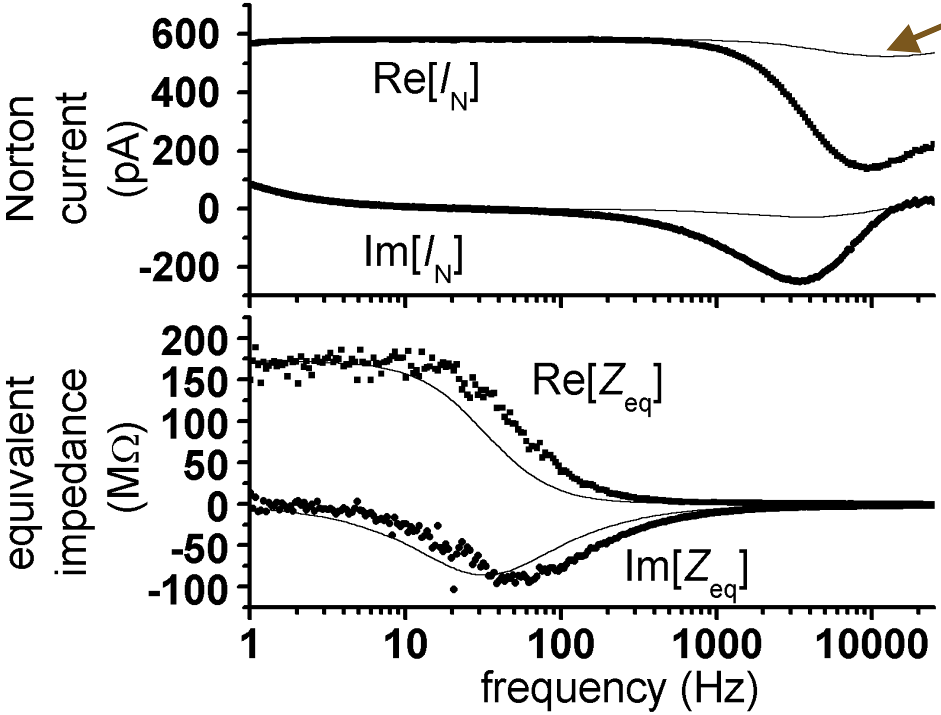
Si thermistor data



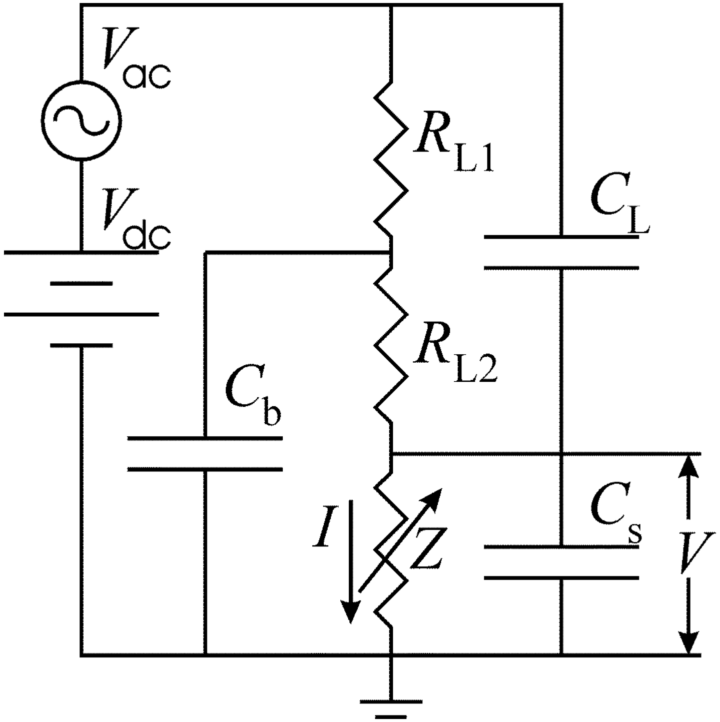
$$\mathrm{Re}\left[\frac{1}{I}\right] = \left(\frac{1}{V_{\mathrm{Th}}}\right) R + \mathrm{Re}\left[\frac{1}{I_{\mathrm{N}}}\right]$$

Measured equivalents

Voltage readout
(Si thermistors)

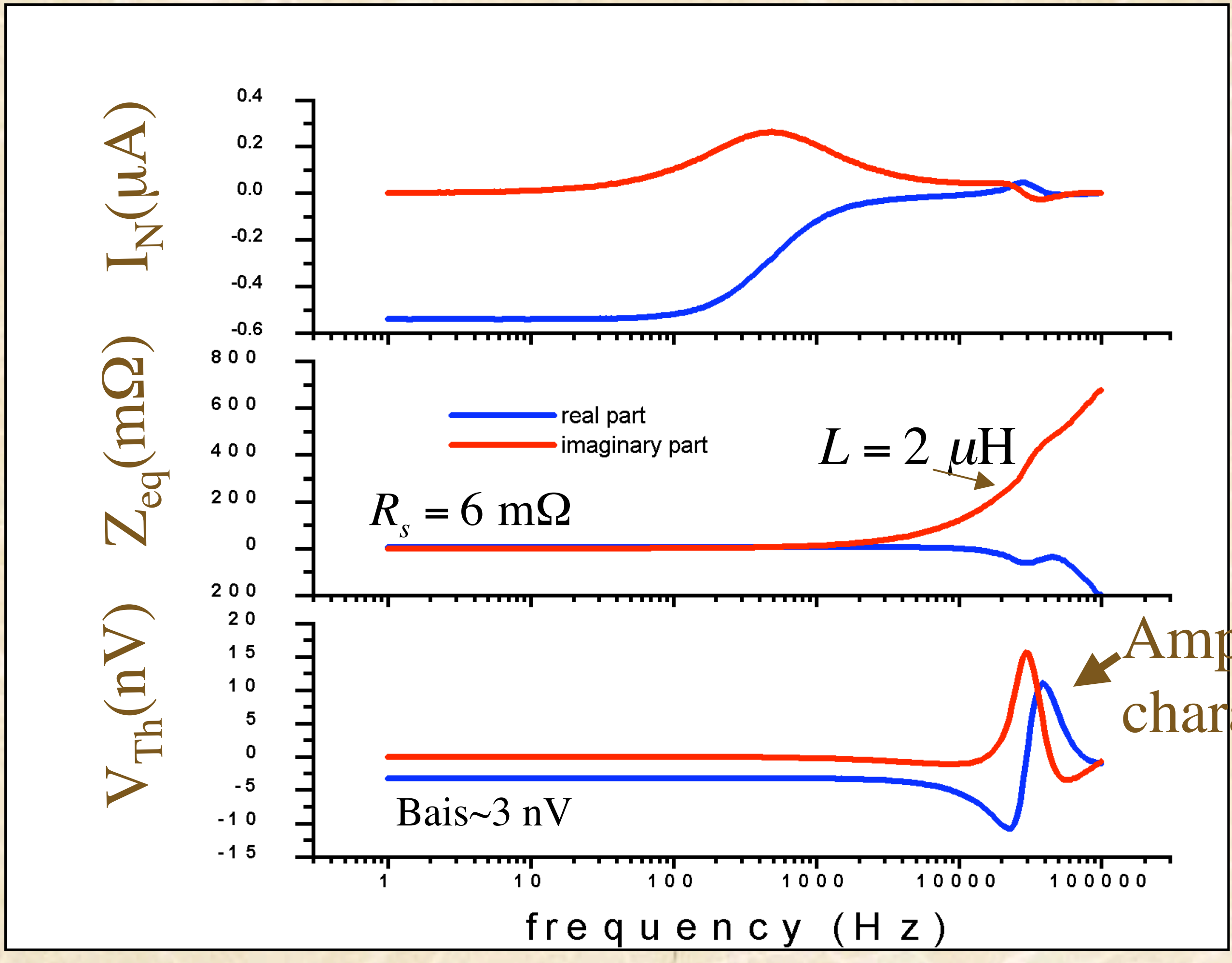


Circuit model not so Good!



Equivalents of a TES readout

Used R=0 and R=R_N to find equivalents



Amplifier characteristics

Thevenin transform

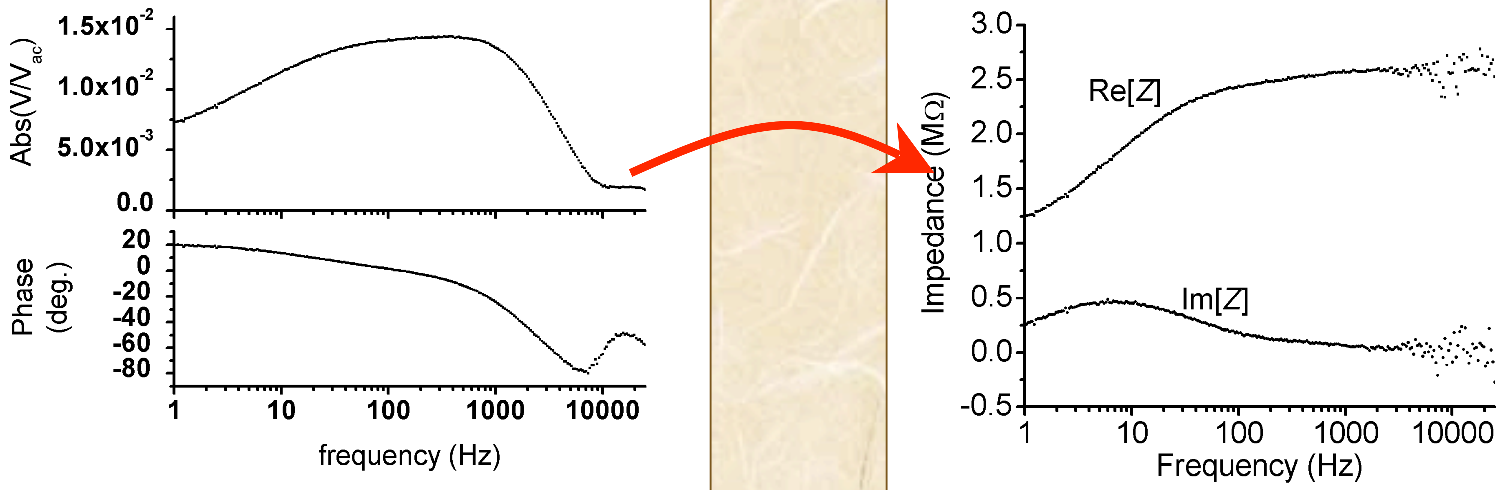
Voltage readout

$$Z(f) = Z_{\mathrm{eq}}(f)\frac{V(f)}{V_{\mathrm{Th}}(f) - V(f)}$$

or

Current readout

$$Z(f) = Z_{\mathrm{eq}}(f)\left(\frac{I_N(f)}{I(f)} - 1\right)$$

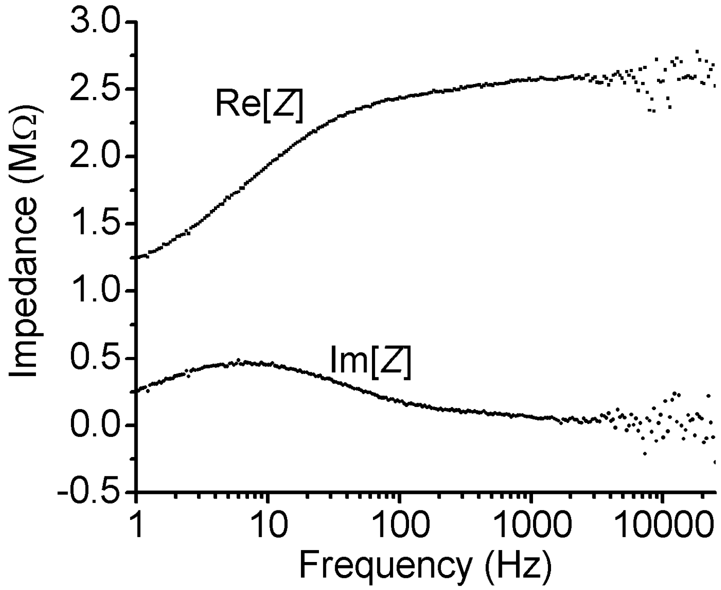


Transfer function

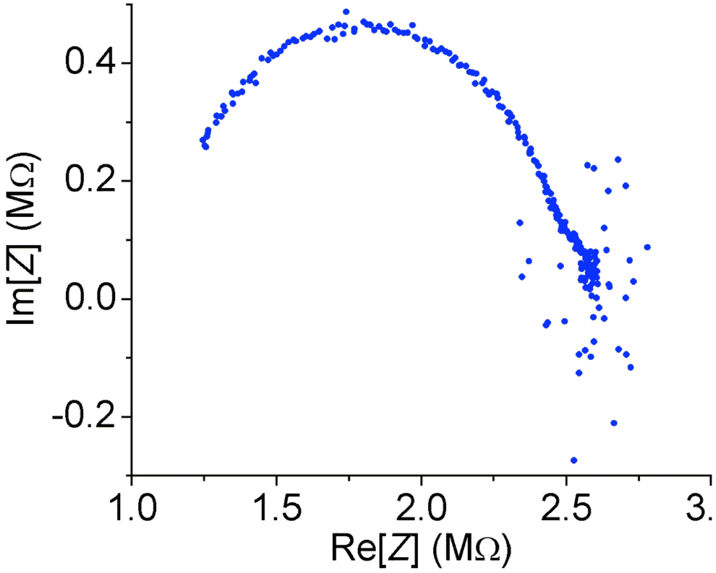
Impedance

Thevenized Si thermistor data

Impedance vs. Frequency

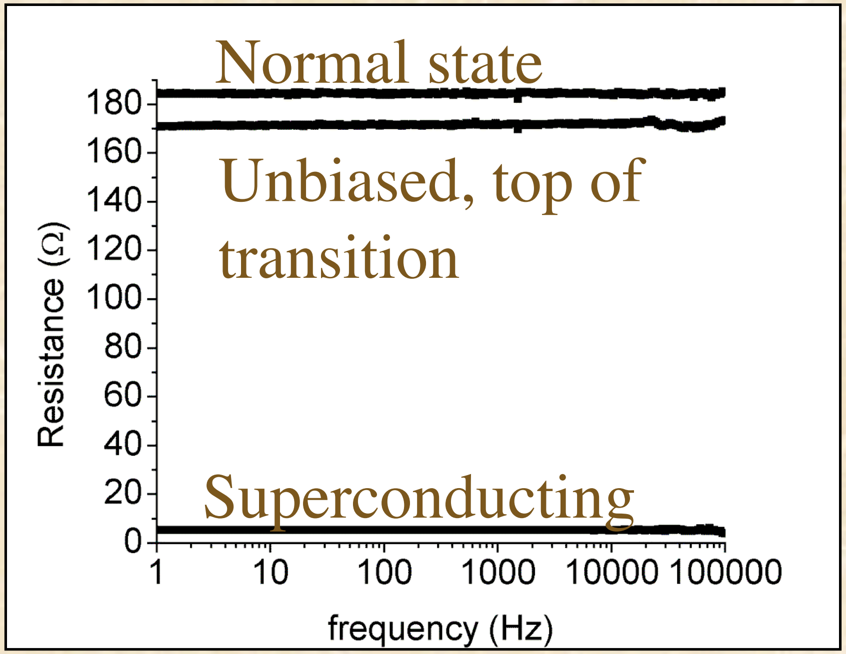


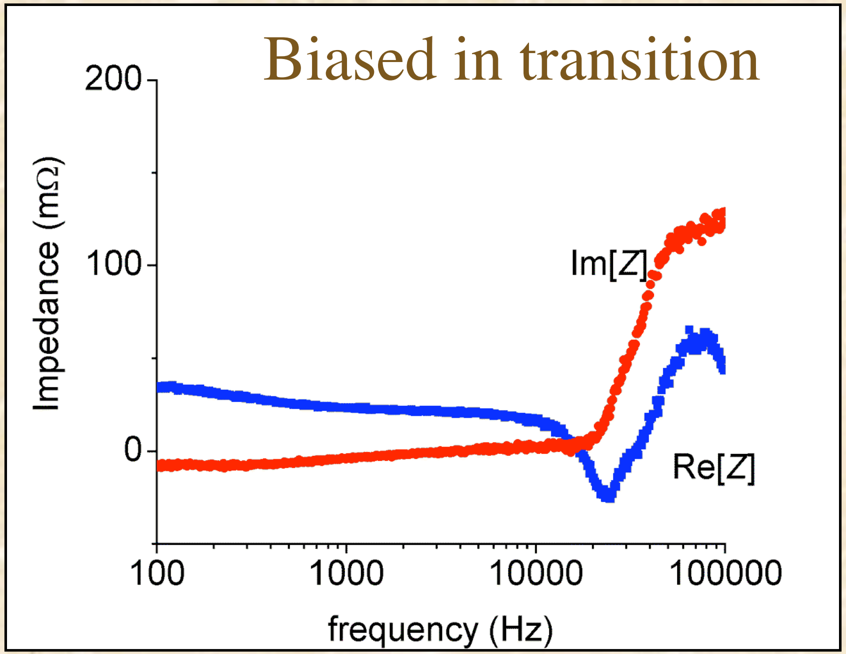
Impedance in the complex plane



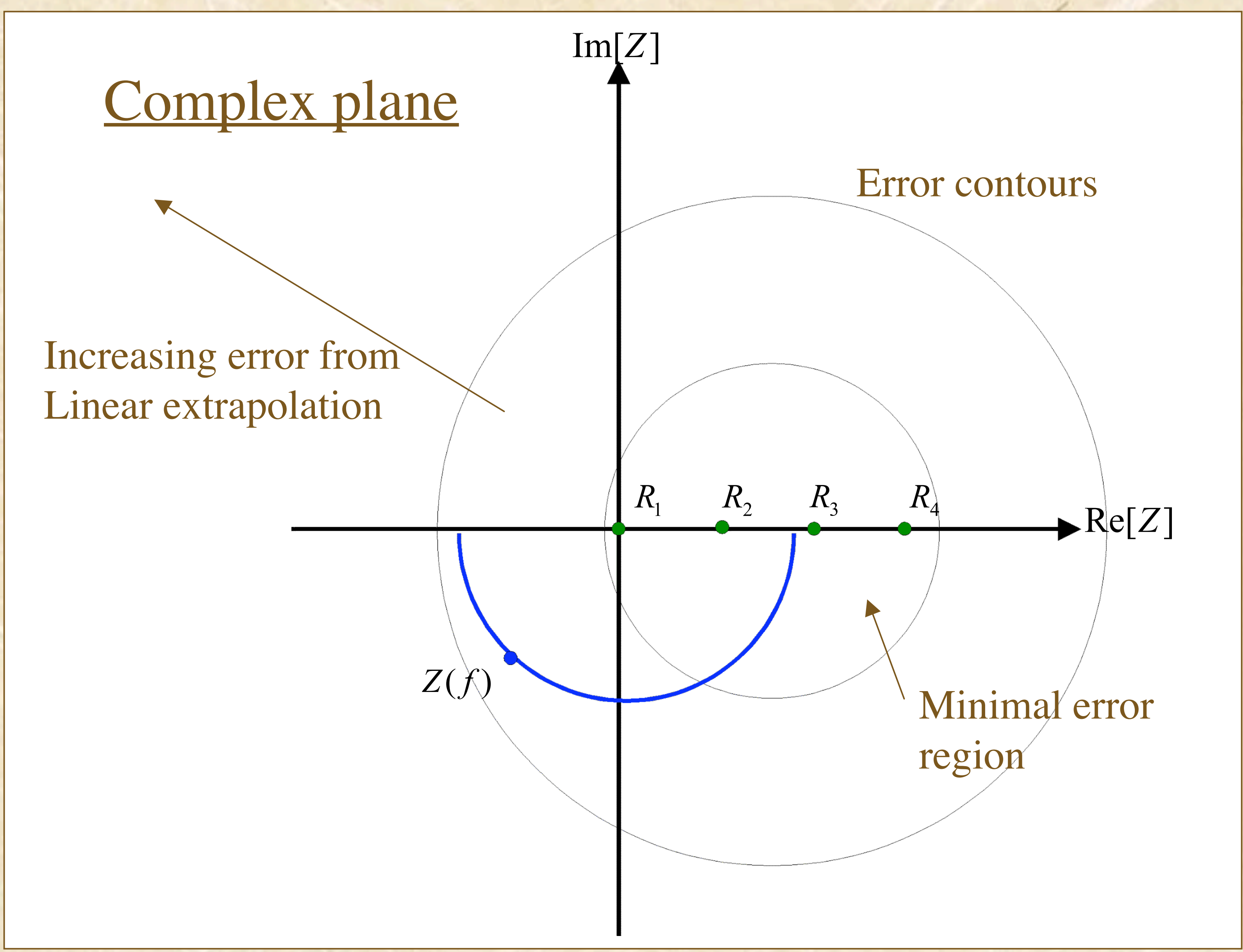
Thevenized TES data

- Tested technique on unsuspended TES--very fast time constant
- Real valued measurements worked out as expected
- Z(f) of biased TES in transition--but not as expected





Overview



Complex plane

Error contours

Increasing error from Linear extrapolation

Minimal error region

Requirements

- Measure either current or voltage through Z
- Vary the resistance of detector
- Stable gain and phase response from amplifiers
- Measure detector resistance (use low frequency data)

Summary

- Imperfect circuit models introduce **artifacts** in *Z(f)*
- Equivalent circuit method accounts for strays, distributed loads
- Includes gain and phase of amplifiers
- Allows accurate large bandwidth impedance measurements
- Useful for studying physics of calorimeters and bolometers