Abstract. The electron energy balance equation is modified to take into account the effect of transfer of hot electron power from TES bolometer absorber combined with the sensor to the biasing circuit with the electron current for the case of non-Andreev contacts. Estimation calculations have shown that the power flow connected with said transfer is negligibly small in comparison with the hot electron power transfer to the thin metal film structure and substrate through electron-phonon interactions for studied earlier molybdenum-copper bi-layer thin film structures in 0.08 – 0.4 K temperature range [1]. The obtained equation was used to estimate a ratio of current decrement to incident radiation power (current responsivity) of the TES bolometers as well. There were no significant changes in the TES bolometers current responsivity found at fixed bias voltage across the absorber for the studied structures except the case at absorber lengths 1 μ and less when the current responsivity gain of order of several tens have been calculated.
The super low temperature hot-electron bolometer with normal metal absorber coupled into planar antenna for high sensitive submillimeter and far infrared waveband radiation detection was proposed in [2]. High efficiency of the bolometer supposed to be provided by very week thermal coupling of hot electrons with phonons owing to super low operation temperature \( T_{\text{oper}} \leq 0.3 \pm 0.1K \) and, additionally, by application of superconductor with critical temperature \( T_c \) higher than operating temperature for antenna. Then the Andreev reflection [3] of electrons at the interface between absorber and antenna traps the hot electrons and hence absorbed radiation energy in the absorber. First experimental study of such bolometer was fulfilled in [4, 5], where, in particular, a correlation between Joule power \( P_J = U \cdot I \) dissipated in absorber when the bias current \( I \) heats electrons and fifth powers of electron and phonon temperatures difference \( (T_e^5 - T_{ph}^5) \) were measured (see, for instance, Fig. 1 [4]). It was found that this correlation is well described by relation

\[
P_J = U \cdot I = \Sigma \nu (T_e^5 - T_{ph}^5)
\]

for \( T_e \leq 0.45 \) K where \( \Sigma = 3.7 \) nW K\(^{-5}\) \( \mu \text{m}^{-3} \) [4] and 3.0 nW K\(^{-5}\) \( \mu \text{m}^{-3} \) [5] - material parameter obtained from graph and describing thermal conductivity from electrons to phonons, \( \nu \) – absorber volume. Good agreement of experimental points and approximating straight line, at least for \( T_e \leq 0.45 \) K was accepted as the evidence of
the absence of any other electron power dissipation except connected with electron-phonon interaction one and hence as the evidence of Andreev electron reflection contribution at the interface between absorber and antenna.

Fig. 1. Correlation between $U \cdot I$ and $(T_e^5 - T_{ph}^5)$, Cu absorber $6 \times 0.3 \times 0.075 \mu m^3$ [4].

In a similar way to the super low temperature normal metal absorber bolometer it was supposed that Andreev electron reflection will give contribution to the operation of the transition edge sensor bolometer when absorber of radiation is fabricated of material with superconducting transition and combined with the sensor
and antenna is fabricated of superconductor with critical temperature $T_c$ higher than operating temperature of device [6, 7].

In this paper we compare a current to radiation power responsivity of TES bolometer with Andreev and non-Andreev contacts. To take into account hot electron power flow-out from the absorber-sensor to the bias circuit one may take as a basis the expression for thermal capacity of electrons in metals [8]

$$C_v = \frac{\pi^2}{2} \cdot \frac{Nk^2T_e}{E_F(0)} \text{ J/K}, \quad (2)$$

where $N$ is amount of electrons for which the thermal capacity is to be determined, $k \approx 1,38 \cdot 10^{-23} \text{ J/K}$ is Boltzman constant, $T_e \text{ K}$ is electron temperature, $E_F(0)$ is Fermi energy equal for metals at low temperatures $E_F(0) = 1…10 \text{ eV}$ [8]. We consider $N$ as an amount of electrons entering absorber-sensor and leaving it simultaneously per second as a current. If so, we may express $N$ through the current $I_A$:

$$N = I / e \text{ s}^{-1}, \quad (3)$$

where $I \text{ C/s}$ is electrical current, $e \approx 1.6 \cdot 10^{-19} \text{ C}$ is electron charge. Taking into account that the temperature of entering electrons is $T_{ph} \text{ K}$ i.e. the temperature of the thin metal film and substrate (phonons) and the temperature of leaving (hot) electrons is $T_e$ we may obtain from (2) and (3) expressions for power leaving the absorber-sensor together with hot electrons and power entering it together with bias current:
\[ P_e = \frac{\pi^2}{2} \cdot \frac{I k^2 T_e^2}{eE_F(0)} = \beta I T_e^2, \quad P_{ph} = \frac{\pi^2}{2} \cdot \frac{I k^2 T_{ph}^2}{eE_F(0)} = \beta I T_{ph}^2 \quad \text{W}, \quad (4) \]

where
\[ \beta = \frac{\pi^2}{2} \cdot \frac{k^2}{eE_F(0)}. \quad (5) \]

Adding the difference \(P_e - P_{ph}\) to the right side of (1) what is the electron energy balance equation for the bolometer with Andreev contacts case we obtain the electron energy balance equation for the case when the effect of hot electron power flow-out from the absorber-sensor to the bias circuit is present (non-Andreev contacts):
\[ P_J = U \cdot I = \Sigma \nu (T_e^5 - T_{ph}^5) + \beta I (T_e^2 - T_{ph}^2). \quad (6) \]

To estimate second member in right side of (6) in comparison with first one we have used results of measurements of \(R(T)\) dependences and calculated IV and PV curves of bi-layer Mo/Cu samples (Fig.2 and 3 and table I [1]) using (1) and measured \(R(T)\) dependences for TES bolometers with 8 x 0.8 \(\mu\)m\(^2\) which can be constructed on the bases of said samples. Calculations have shown that the value of second member in right side of equation (6) is not higher than 1% of the first member for all values of \(R(T_e), U, T_e, T_{ph}\) (two latter \(\sim\) \(T_c\)) in operating points given in Table I and for minimal of abovementioned value of Fermi energy \(E_F(0) = 1\text{ eV} = 1.6 \cdot 10^{-19}\text{ J.}\) This
correlation remains at the reducing of transverse dimensions of possible bolometers down to $0.8 \times 0.08 \mu m^2$. Described situation means that the hot electron power flow-out with the electrical current in our considered Mo/Cu structures case is more than two orders less intensive of the hot electron power flow-out owing to the electron-phonon interactions. It is easy to explain: second power flow-out takes place through relatively small contact areas when first one takes place through whole absorber-sensor volume. By this reason IV- and power-voltage curves calculated in [1] for bolometers with Andreev contacts will not differ noticeably from similar curves when contacts are not of Andreev type but ordinary.

Now with the purpose to estimate the current to radiation power responsivity and $NEP$ of considering bolometers for the case when two mechanisms of hot electron power flow-out are acting we add to (1) in a similar way with [1] a radiation power $P_{rad}$ to Joule power $U \cdot I$ and small additions $\Delta I$ and $\Delta T$ for current and temperature:

$$U(I + \Delta I) + P_{rad} = \Sigma v [(T_e + \Delta T)^5 - T_{ph}^5] + \beta (I + \Delta I) [(T_e + \Delta T)^2 - T_{ph}^2].$$

We assume that like in [1] the fixed bias voltage $U$ is applied to absorber-sensor what provides the negative electrothermal feedback action in electron system [9]. The equation for small values can be extracted from (7):

$$U \Delta I + P_{rad} \approx 5 \Sigma m T_e^4 \Delta T + \beta (T_e^2 - T_{ph}^2) \Delta I + 2 \beta I T_e \Delta T.$$  

(8)
Fig. 2. Results of measurements $R(T)$ dependences of Mo/Cu samples b, c and d which main parameters are given in the Table I [1].

Fig. 3. Calculated current-voltage (solid lines) and power-voltage (dashed lines) characteristics of constructed TES bolometers based on data of three measured Mo/Cu bi-layer structures at $T = 0.4$ K, $T = 0.27$ K and $T = 0.08$ K for bolometers based on samples b, c and d respectively [1].
### Table I

<table>
<thead>
<tr>
<th>Sample thicknesses, nm</th>
<th>Sample</th>
<th>$T_c$, K</th>
<th>$R_n$, Ohm</th>
<th>$\alpha = \frac{T}{R} \cdot \frac{dR}{dT}$</th>
<th>$U$, V</th>
<th>$S_I$, A/W</th>
<th>NEP, W/Hz$^{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mo</td>
<td>Cu</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 - 6) - for 15 x 1.5 mm$^2$ measured samples, (7 - 9) – for 8 x 0.8 µm$^2$ possible bolometers based on b - d samples. $S_I =</td>
<td>\Delta I</td>
<td>: P_{rad} = 1 : U$, $NEP = \sqrt{i_{\text{noise}}^2} : S_I$, $\sqrt{i_{\text{noise}}^2}$ is the rms noise current of SQUID readout-amplifier next to the bolometer. In our case $\sqrt{i_{\text{noise}}^2} \approx 4 \cdot 10^{-12}$ A/Hz$^{1/2}$.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a 12 0 0,93 67 1070 7</td>
<td>b 15 35 0,4 2,9 150 10$^{-7}$</td>
<td>c 12 35 0,27 2,6 320 10$^{-8}$</td>
<td>d 12 100 0,08 0,6 510 10$^{-9}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10$^{7}$</td>
<td>10$^{-8}$</td>
<td>10$^{-9}$</td>
<td>10$^{9}$</td>
<td>4$\cdot$10$^{-19}$</td>
<td>4$\cdot$10$^{-20}$</td>
<td>4$\cdot$10$^{-21}$</td>
</tr>
</tbody>
</table>

After simple transformation $\frac{U}{R + \Delta R} \approx \frac{U}{R} - \frac{U \Delta R}{R^2} = I - I \cdot \frac{\Delta R}{R} = I + \Delta I$ we have $\Delta I = -I \cdot \frac{\Delta R}{R}$ and $\frac{\Delta R}{R} = -\frac{\Delta I}{I}$. Then taking into account the relation $\alpha \approx (T_c / R) \cdot (\Delta R / \Delta T_c)$ describing the sharpness of superconducting transition of absorber material one obtains $\Delta T \approx \frac{1}{\alpha} \frac{T \Delta R}{R} \approx -\frac{1}{\alpha} T \frac{\Delta I}{I}$. Substituting obtained expression for $\Delta T$ to (8) and taking into
account values of $\alpha$ given in Table I one may see that members containing $\Delta T$ are negligibly small in comparison with other ones and we can write:

$$U \Delta I + P_{узл} \cong \beta (T_e^2 - T_{ph}^2) \Delta I.$$  \hspace{1cm} (9)

We consider at first the case when contacts to the absorber-sensor are made of a superconductor with high critical temperature providing the Andreev reflection of electrons in the absorber-sensor from these contacts. In this case the member in right side of (9) is absent and we have:

$$U \Delta I + P_{rad} \cong 0.$$  \hspace{1cm} (9')

One can obtain from (9') the expression for $S_I$ in case of Andreev contacts (see Table I).

In this given point we consider in more details the action of said above negative electrothermal feedback in electron system of absorber-sensor [9]. The fixed bias voltage and very sharp dependence of the absorber-sensor resistance on electron temperature (see [1]) leads to the arising of an electron thermostat. When a deviation of electron temperature takes place in this thermostat by any reason this deviation leads to the variation of the absorber-sensor resistance and consequently of the current through it. This current variation has such direction that the change of dissipated Joule power $U \Delta I$ compensates the variation of electron temperature. For instance when the reason of electron temperature variation is the incident radiation power $P_{rad}$ absorbed by the absorber the Joule power change $U \Delta I$ is equal to $P_{rad}$ with opposite sign. The described mechanism of negative electrothermal feedback
was discovered by Irvin [9]. Given here consideration will be useful in subsequent discussion.

We return to the equation (9). One factor is more now in the electron thermostat operation [see (6)]. This is the hot electron power flow-out with the electrical current, i.e. \( \beta I(T_e^2 - T_{ph}^2) \). The corresponding member \( \beta(T_e^2 - T_{ph}^2)\Delta I \) has appeared in (9). One may obtain the expression for bolometer current responsivity from (9) when the hot electron power flow-out with the electrical current takes place:

\[
S_I = \frac{-\Delta I}{U \Delta I - \beta(T_e^2 - T_{ph}^2)\Delta I} = \frac{1}{U(1-\eta)},
\]

where

\[
\eta = \frac{\beta(T_e^2 - T_{ph}^2)}{U}.
\]

One may see from (10) that in considered case the bolometer current responsivity is gained in comparison with the case when the hot electron power flow-out to the bias circuit is absent owing the blocking it by Andreev reflection. This gain is explained by the action of negative electrothermal feedback. The member \( \beta(T_e^2 - T_{ph}^2)\Delta I \) in (9) reduces the hot electron power flow-out with the electrical current owing to the reducing of this current for value \( \Delta I \). This means that hot electron temperature is increasing. As a result of this increasing the negative electrothermal feedback increases the value of current reducing more. Something similar to iterative process is arising and stops when the power equilibrium will be restored, i.e. the equation
(9) will be satisfied. To estimate $\eta$ determined by (11) and then current responsivity gain determined by (10) one has to know the Fermi energy $E_F (0)$ and to calculate $\beta$. We estimate $\eta$ for lower value $E_F (0)$ of given above ones, i.e. $E_F (0) = 1 \text{ eV} = 1,6 \cdot 10^{-19} \text{ J}$. Results of estimation using temperatures and bias voltages given in Table I are summarized in Table II. The bolometers with dimensions $l \times w \sim 0.1 \times 0.2$ and $0.8 \times 0.08 \ \mu\text{m}^2$ based on the structures c and d respectively have $\eta \approx 0.98$ and gain $\approx 50$ (Fig. 4). In case of Andreev contacts the second member in (6) is absent and, consequently, $\eta = 0$ and the regenerative phenomenon is absent. In other cases $\eta$ is small in comparison with unit and, consequently, current responsivities and NEP’s are practically the same as in the absence of the hot electron power flow-out to biasing circuit with electrical current owing to Andreev electron reflection (Table I).

<table>
<thead>
<tr>
<th>Samples</th>
<th>Transverse dimensions of absorber-sensor $l \times w, \mu\text{m}^2$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>b - d</td>
<td>$\sim 80 \times 8$</td>
<td>$\sim 0.01 ... 0.02$</td>
</tr>
<tr>
<td>b - d</td>
<td>$\sim 8 \times 0.8$</td>
<td>$\sim 0.1 ... 0.2$</td>
</tr>
<tr>
<td>c</td>
<td>$\sim 0.1 \times 0.2$</td>
<td>$\rightarrow 1$</td>
</tr>
<tr>
<td>d</td>
<td>$\sim 0.8 \times 0.08$</td>
<td>$\rightarrow 1$</td>
</tr>
</tbody>
</table>
Fig. 4. Regenerative gain as the function of absorber length and operating temperature for samples c (0.3 K) and d (0.1 K).
CONCLUSION

- Modified electron energy balance equation containing the member taking into account the transfer of hot electron power from TES bolometer absorber-sensor to the biasing circuit with the electron current is derived.

- Analysis made on the basis of this equation has shown that the hot electron power flow-out from the TES bolometer absorber-sensor to the bias circuit in case of bi-layer Mo/Cu thin film structures is negligibly small in comparison with the hot electron power flow-out from electron system to the metal film and substrate through electron-phonon interactions. This hot electron power flow-out to the bias circuit has not noticeable influence on IV- and power-voltage characteristics of TES bolometers.

- Situation with said two hot electron power flows-out takes place in case of bolometer with normal metal absorber because main parameters (materials, temperatures, dimensions) are similar to the
case considered in this paper. This means that good agreement of experimental points and approximating straight line in mentioned above experiments with the normal metal absorber bolometer is not sufficient to be accepted as the evidence of the absence of any other electron power dissipation except connected with electron-phonon interaction one and hence as the evidence of Andreev electron reflection contribution at the interface between absorber and antenna.

- The hot electron power flow-out from the TES bolometer absorber-sensor to the bias circuit does not deteriorate the bolometer current responsivity. On the contrary, at rather small transverse dimensions and low temperatures of bolometers it leads to the regenerative gain of the current responsivity. When Andreev contacts are used the regenerative gain is absent.

- To achieve a practical realization of the regenerative gain phenomenon the thorough investigation of material characteristics as well as fabrication technology and design development are needed.
REFERENCES