

Study of the Optimum Momentum Resolution in the CMS Muon System

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July 24, 2000

Abstract

This project deals with the development of equations from simulation data to assign momentum to muons moving through the endcap region of the Compact Muon Solenoid (CMS) experiment at the Large Hadron Collider at The European Center for Nuclear Research. The layout of the CMS experiment, the endcap detector operation, and the details of developing momentum-assignment equations are described. A three-station momentum reconstruction function is presented that meets the CMS requirements for transverse momentum resolution.

1 Introduction

The Large Hadron Collider (LHC) currently being built near Geneva, Switzerland will study particle collisions of energies up to 14 trillion electron volts (TeV). The purpose of the LHC is to search for new particles at high energies and possibly confirm predictions of the Standard Model of particle physics [1]. The Compact Muon Solenoid (CMS) is one of the four detectors currently being developed for use in LHC experiments. The CMS detector is designed to search for the Higgs boson and will also operate at other energies to study collisions of ions [2]. The proton bunches of the CMS experiment will provide so many collisions in such a short time, about 40,000,000 per

second, that a trigger system must be employed to narrow collision data to those which are interesting. This trigger must determine properties of the muons escaping the detector at a high rate so that computers are able to collect data from important collisions for study off-line. The Level-1 Trigger will reduce the number of muon events to 100,000 per second, and the higher-level triggers will reduce the rate to 100 per second. The triggers are designed to quickly identify the muon track's transverse momentum so that low-momentum tracks can be rejected. Therefore, simulation studies must be done to provide the trigger with algorithms that calculate properties of the muons quickly and accurately.

One of the properties that the trigger must be able to calculate is the momentum of the muons moving through the detector. Since the CMS solenoid provides a strong magnetic field, the paths of the escaping muons are curved as they move through the detector regions. The trigger will reconstruct the transverse momentum (P_t) of a particular muon track based upon the amount of bending measured by the detector stations. The momentum resolution depends on multiple scattering in the iron as well as the position resolution measured by the detector. The reconstructed P_t resolution must be high in order for the trigger to reject most low- P_t muons and allow for an acceptable rate so that interesting muons can be studied. The reconstructed P_t should have as small an uncertainty as possible to efficiently reject low momentum muons.

2 Layout of the CMS

The CMS detector consists of a large superconducting solenoid coil surrounded by multiple layers of iron and muon detectors (See Fig. 1). The solenoid coil will produce a uniform magnetic field of 4 T in the central region [2]. This strong magnetic field bends the paths of escaping particles to a degree that allows for study at high

momentum and a “compact” design of the detector. Calorimeters surround the collision site inside the superconducting coil. Beyond the coil, alternating layers of iron and muon detectors line the barrel and the endcap regions of the CMS. The iron serves to prevent the magnetic field from extending beyond the detector. Cathode strip chambers and resistive plate chambers comprise the endcap detectors, and drift tubes and resistive plate chambers are used in the barrel region.

3 Operation of the Endcap Detectors

The endcap of the CMS system uses trapezoidal cathode strip chambers (CSC) with a 10° or 20° angular extent. Six CSC chambers together comprise one of the four stations (ME1-ME4). When placed together, the CSC chambers form a disk. In each CSC are cathode strips at ground voltage and anode wires at high potential. Inside the chamber is a gas that ionizes when a charged particle passes through it. After the gas is ionized, electrons drift to the positively charged anode wire which will in turn induce a positive charge on the nearby cathode strips. The altered charge on the cathode strips allows the azimuthal angle (ϕ) of the muon hit to be measured. The charge on the anode wires gives a measure of the radial position of the muon (θ). Since the chamber is fixed, the z coordinate will be known. The six layers of strips allow for a three-dimensional measurement of the muon path [3].

4 Momentum Reconstruction

The paths of charged muons moving through the four stations are bent in the direction of the ϕ angle due to the high magnetic field in the detector (See Fig. 2). The non-uniform magnetic field in the endcap makes finding the transverse momentum complicated. In general, the degree of bending is inversely proportional to the

momentum of the muons. The amount of bending is calculated by using the difference between the azimuthal angles ($\Delta\phi$) measured in the different stations of the detector. Since the magnetic field varies, the rapidity ($\eta = -\ln \tan \theta/2$) must be considered when assigning Pt (see Fig. 3). It should be possible to determine the Pt of a muon in a certain area of the detector based on the difference between the azimuthal angle values in any pair of the four stations.

4.1 Resolution Requirements and Simulation Data

The Level-1 Trigger requires a Pt uncertainty of 30% or lower to meet the rate requirements (See Fig. 4). However, to allow for a larger safety factor, a 20% uncertainty is desirable [4]. To achieve lower uncertainty, the momentum reconstruction equation can utilize both the rapidity (η) and change in azimuthal angle ($\Delta\phi$). Also, $\Delta\phi$ values from multiple detector stations can be used.

The CMS simulation package CMSIM 118 provides detailed simulations of the reconstruction of muon tracks. CMSIM uses GEANT to describe the detector geometry and the transportation of particles in the detectors [5]. Using the data from these simulations, we can develop equations that reconstruct momentum based on two, three, or four station $\Delta\phi$ measurements and the η region of the detector. This calculation should consider correlations between $\Delta\phi$ measurements from different pairs of stations. These equations can be tested using the CMSIM data to determine their momentum resolutions at different areas of the detector as well as at different momenta.

4.2 Two-station Pt Assignment

The amount of transverse bending a muon experiences as it moves through the detector region is generally inversely proportional to its transverse momentum (Pt). This

means that Pt can be reconstructed using as few as two detector stations by finding the change in azimuthal angle (ϕ) between the two stations. The reconstructed Pt is also a function of the η region of the muon track. Using CMSIM data, the change in azimuthal angle can be related to the momentum using the equation $\Delta f = a(\mathbf{h}) / Pt$, where a varies with η . This fits the simulation to a reasonable degree of accuracy. Since the reconstructed Pt does not fit quite as well at high and low Pt values another reconstruction equation $\Delta f = a / Pt + b / Pt^2$ is used.

4.3 Three-station Pt Assignment

To further improve the measurement of Pt, data from three detector stations can be used in the fit. This method uses the difference in the angle ϕ from station 1 to station 2 and the difference in ϕ from station 2 to station 3. There should not be as much bending in the third detector station as it is further away from the strong magnetic field of the solenoid. Even so, the change in ϕ from station 1 to station 2 is related to the corresponding muon track's angle change as it passes through station 2 and 3. Thus, the momentum assignment equation must take into account the relationship.

We estimate the momentum from the measurements using the method of maximum likelihood or maximum probability. A bivariate normal distribution, a curve that combines two normal variables that are not necessarily independent, is used for the likelihood function [6]. The bivariate normal takes the form

$$f(x_1, x_2) = \frac{1}{2\pi s_1 s_2 \sqrt{1 - r^2}} \exp\left(\frac{-1}{2(1 - r^2)} \left[\frac{(x_1 - \mathbf{m}_1)^2}{s_1^2} - \frac{2r(x_1 - \mathbf{m}_1)(x_2 - \mathbf{m}_2)}{s_1 s_2} + \frac{(x_2 - \mathbf{m}_2)^2}{s_2^2} \right]\right)$$

In this equation σ_i refers to the spread on each $\Delta\phi$, μ_i refers to the mean $\Delta\phi$ value, and ρ is the correlation coefficient relating the two $\Delta\phi$ values. The derivative of this function

can be set equal to zero to maximize the likelihood. Solving this relationship for $1/Pt$ provides an equation that gives the inverse of the momentum based on the change in ϕ from 1 to 2 and the change from 2 to 3 (see Appendix A.1). Then, CMSIM data can be used to determine the constants of the equation at various η regions of the detector. Using the simulation constants with the bivariate normal distribution provides an equation for the inverse of momentum dependent on the two $\Delta\phi$ values and the physical η value for the muon track.

4.3.1 Three-station Correlation Coefficient

Plots of $\Delta\phi$ from station 1 to station 2 ($\Delta\phi_{12}$) vs. $\Delta\phi$ from station 2 to station 3 ($\Delta\phi_{23}$) suggest a positive correlation between the two values (See Fig. 5). The correlation coefficient comparing $\Delta\phi_{12}$ to $\Delta\phi_{23}$ is in the form of

$$r = \frac{\langle (x - \mathbf{m}_x)(y - \mathbf{m}_y) \rangle}{\mathbf{s}_x \mathbf{s}_y},$$

where σ_x is the r.m.s. spread and μ_x is the mean of the $\Delta\phi_{12}$ normal curve in the simulation data and σ_y is the r.m.s. spread and μ_y is the mean of $\Delta\phi_{23}$. The Gaussian variable data can be taken from the CMSIM data to determine the correlation between the two $\Delta\phi$ values (see Fig. 6). They are about 60% correlated.

4.3.2 Three-station Parameterizations

The bivariate normal distribution involves mean and spread values for each detector pair. In order to obtain the spread and mean variables necessary to maximize the bivariate normal, simulation data must again be used. The two-station Pt reconstruction can be used to obtain the relationship between $\Delta\phi$ and Pt for each station pair [$\Delta\mathbf{f} = a(\mathbf{h})/Pt$]. The parameter a varies depending on the η region and is the same as in

the two-station fit. To obtain the spread variable, one can use the relative error of the simulation normal curves. The relative error of the simulation normal curves is the spread divided by the mean and can be calculated as a function of η . The two-station parameterized mean, a/P_t , can be multiplied by the relative error to get a parameterized spread value that varies with changing η . The three-station momentum assignment equation was able to produce $1/P_t$ resolution close to 20% at low P_t and in the η regions 1.2 to 2.0 (see Fig. 7 and Fig. 8). A plot of the first change in azimuthal angle versus the second change in azimuthal angle for the three station reconstruction function is shown in Figure 9.

4.4 Four-station P_t Assignment

In order to further improve the P_t measurement, an equation that depends on four detector stations to reconstruct momentum can be developed. This equation uses the mean momentum value from the change in ϕ angle from station 1 to 2, from station 2 to 3, and from station 3 to 4. As with the three-station assignment, one must consider correlations between the three mean $\Delta\phi$ values. The multivariate normal distribution is a generalization of the bivariate normal and can be applied to a three-mean set. The multivariate normal function is maximized with respect to P_t in the same manner as the three-station fit is treated. The three mean relationships are fitted into a likelihood function and maximized with respect to P_t . Then it is solved for $1/P_t$ to obtain an equation that relates the three $\Delta\phi$ values to the inverse of the muon track momentum (see Appendix A.2).

4.4.1 Four-station Correlation Coefficient

Finding the constants of the four-station Pt assignment follows a generalized form of the three-station assignment procedure. Since there are three $\Delta\phi$ values involved in the four-station equation, it requires three correlation coefficients, which can be determined from the CMSIM data. These values take the same form as the three-station coefficients:

$$\mathbf{r}_{xy} = \frac{\langle xy \rangle - \mathbf{m}_x \mathbf{m}_y}{\mathbf{s}_x \mathbf{s}_y}, \quad \mathbf{r}_{xz} = \frac{\langle xz \rangle - \mathbf{m}_x \mathbf{m}_z}{\mathbf{s}_x \mathbf{s}_z}, \quad \text{and} \quad \mathbf{r}_{yz} = \frac{\langle yz \rangle - \mathbf{m}_y \mathbf{m}_z}{\mathbf{s}_y \mathbf{s}_z}.$$

In these relationships x refers to $\Delta\phi$ 12, y to $\Delta\phi$ 23, and z to $\Delta\phi$ 34.

4.4.2 Four-station Parameterizations

The $\Delta\phi$ mean parameterizations remain the same as the two-station fit ($\Delta\mathbf{f} = a / Pt$) but must be extended to include all three $\Delta\phi$ values. The a value still varies based on η . The relative error is again used to compute the spread by multiplying the relative error, which varies with η , times the a/Pt mean for each of the three $\Delta\phi$ regions.

The four-station parameterization was unsuccessful in improving the momentum resolution beyond that of the three-station method. The lack of improvement in resolution with the addition of the third change in azimuthal angle could be due to the fact that there is not much bending between the third and fourth detector station because the magnetic field is lower in this region.

5 Conclusion

New parameterizations for Pt assignment have been made for two-, three-, or four-station measurements. These fits are based on the CMSIM 118 simulation data and can be used to guide momentum assignment at various parts of the CMS trigger. The a parameter that relates Pt to the change in $\Delta\phi$ has been determined as it varies with η . It is

also known how the relative error changes with η . The new three-station Pt assignment equation is successful in improving the Pt resolution at low momentum close to the goal of 20%. The next method to improve the Pt resolution would be to assign a direction vector at each station along with the position information, utilizing more information about the muon's path.

Acknowledgments

T.M. would like to thank the NSF for funding the Research Experience for Undergraduates summer program, Kevin Ingersent and Alan Dorsey for allowing him to participate, and would also like to thank Micah Stoutimore for his help on this project.

A.1 Transverse Momentum Assignment Equation for Three-Station Method

$$\frac{1}{pt} = \frac{-\frac{x}{c_1^2 a} - \frac{y}{b c_2^2} + \frac{px}{a c_1 c_2} + \frac{py}{b c_1 c_2} + \sqrt{\left(\frac{x}{c_1^2 a} + \frac{y}{b c_2^2} - \frac{px}{a c_1 c_2} - \frac{py}{b c_1 c_2}\right)^2 + 8(1-p^2)\left(\frac{x^2}{a^2 c_1^2} + \frac{y^2}{b^2 c_2^2} - 2\frac{pxy}{a c_1 b c_2}\right)}}{4-4p^2}$$

A.2 Transverse Momentum Assignment Equation for Four-Station Method

$$\begin{aligned} \frac{1}{pt} = & -\frac{1}{6}(a x c_2^2 b^2 k^2 d^2 (1-pyz^2) + (bx+ay) c_1 a c_2 b k^2 d^2 (pxz pyz - pxy) \\ & + (dx+az) c_1 a c_2^2 b k d (pxy pyz - pxz) + by a^2 c_1^2 d^2 k^2 (1-pxz^2) \\ & + (yd+zb) a^2 c_1^2 b c_2 d k (pxy pxz - pyz) + z d c_1^2 a^2 b^2 c_2^2 (1-pxy^2)) / (\\ & c_1^2 a^2 c_2^2 b^2 k^2 d^2 \%1) + \frac{1}{6} \text{sqrt}\left((a x c_2^2 b^2 k^2 d^2 (1-pyz^2) \right. \\ & + (bx+ay) c_1 a c_2 b k^2 d^2 (pxz pyz - pxy) + (dx+az) c_1 a c_2^2 b k d (pxy pyz - pxz) \\ & + by a^2 c_1^2 d^2 k^2 (1-pxz^2) + (yd+zb) a^2 c_1^2 b c_2 d k (pxy pxz - pyz) \\ & \left. + z d c_1^2 a^2 b^2 c_2^2 (1-pxy^2))\right)^2 / (c_1^4 a^4 c_2^4 b^4 k^4 d^4 \%1^2) + (\\ & 12 x^2 c_2^2 b^2 k^2 d^2 (1-pyz^2) + 24 xy c_1 a c_2 b k^2 d^2 (pxz pyz - pxy) \\ & + 24 xz c_1 a c_2^2 b^2 k d (pxy pyz - pxz) + 12 y^2 a^2 c_1^2 d^2 k^2 (1-pxz^2) \\ & + 24 yz a^2 c_1^2 b c_2 d k (pxy pxz - pyz) + 12 z^2 c_1^2 a^2 b^2 c_2^2 (1-pxy^2)) / (\\ & c_1^2 a^2 c_2^2 b^2 k^2 d^2 \%1) \end{aligned}$$

$$\%1 = 1 - pxy^2 - pxz^2 - pyz^2 + 2 pxy pxz pyz$$

References

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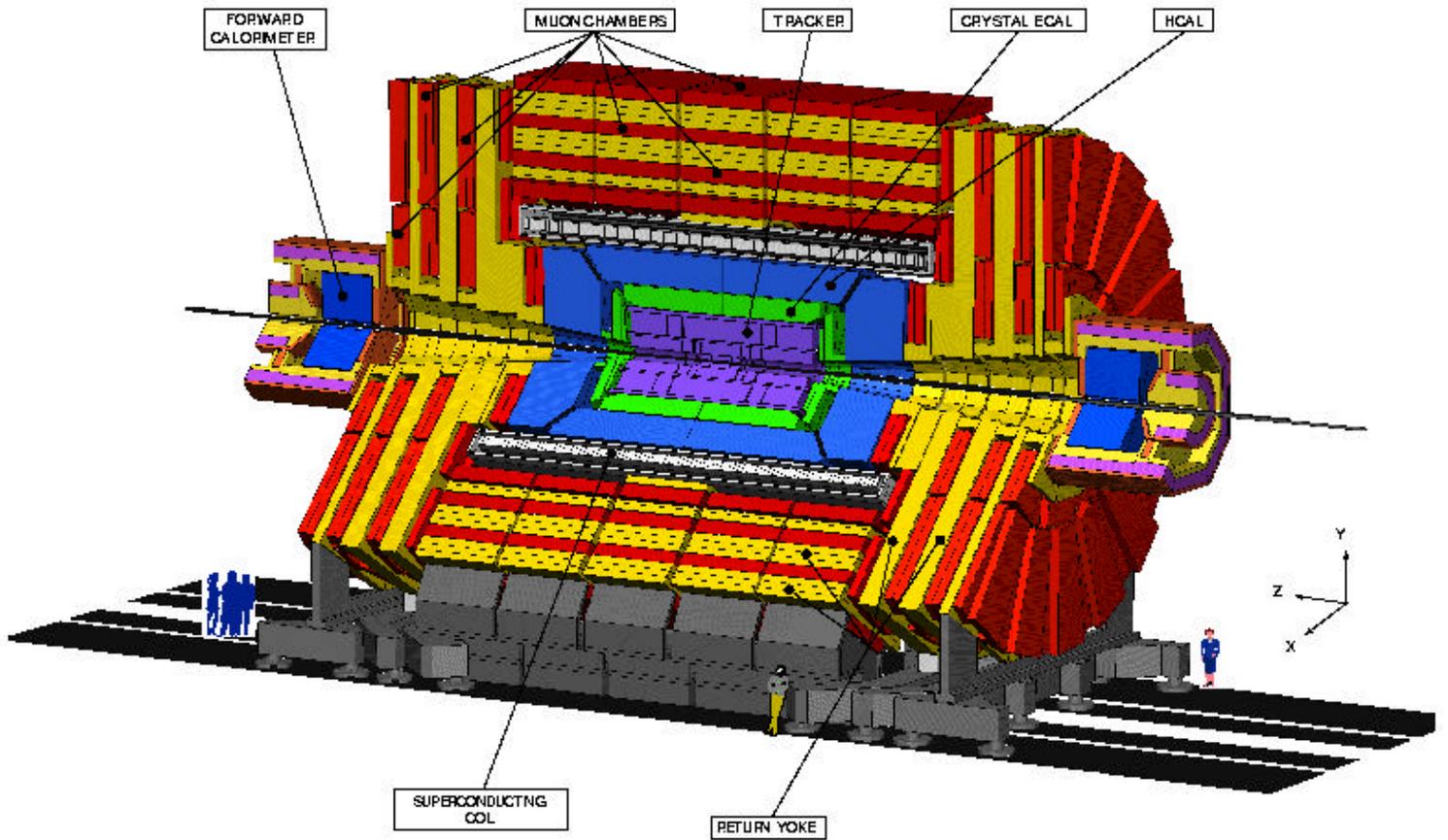


Figure 1: A cutout view of the CMS detector. The superconducting coil and the endcap muon detector stations are labeled. The drawing is from CERN CMS Information web site (<http://cmsinfo.cern.ch>).

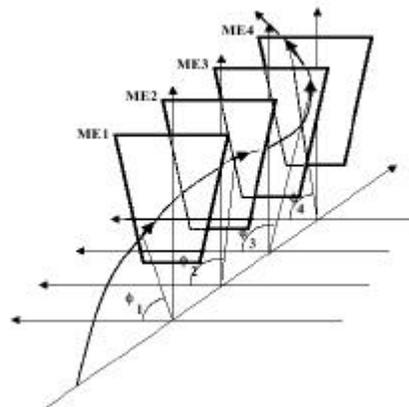


Figure 2: The muon path as it moves through the four CSC stations under the influence of the strong magnetic field. The ϕ angle of the bending can be seen. This drawing is from S. Wang, CMS Internal Note 2000/026.

CMS Longitudinal View

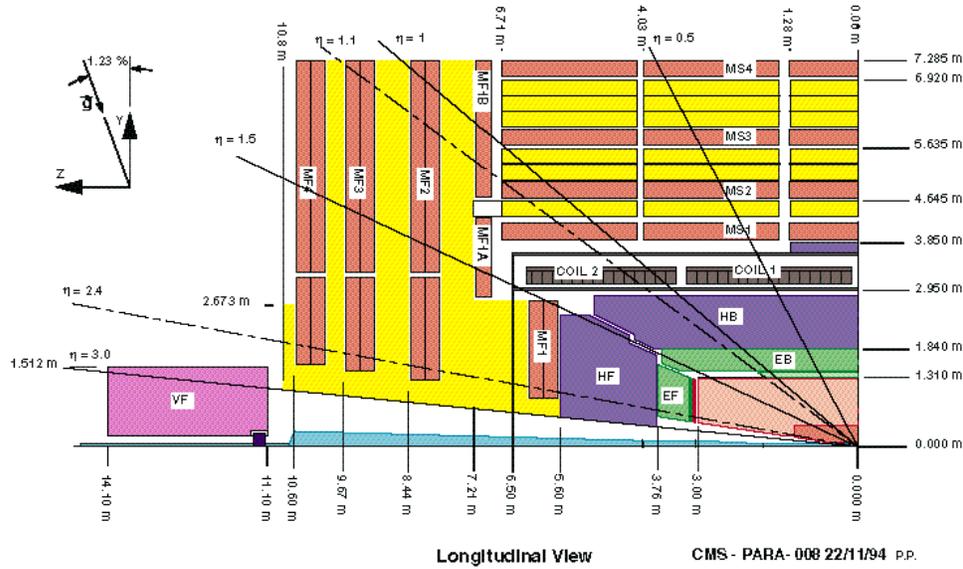


Figure 3: The cross-section view of the CMS detector shows the various η regions in the detector. The drawing is from CERN CMS Information web site (<http://cmsinfo.cern.ch>).

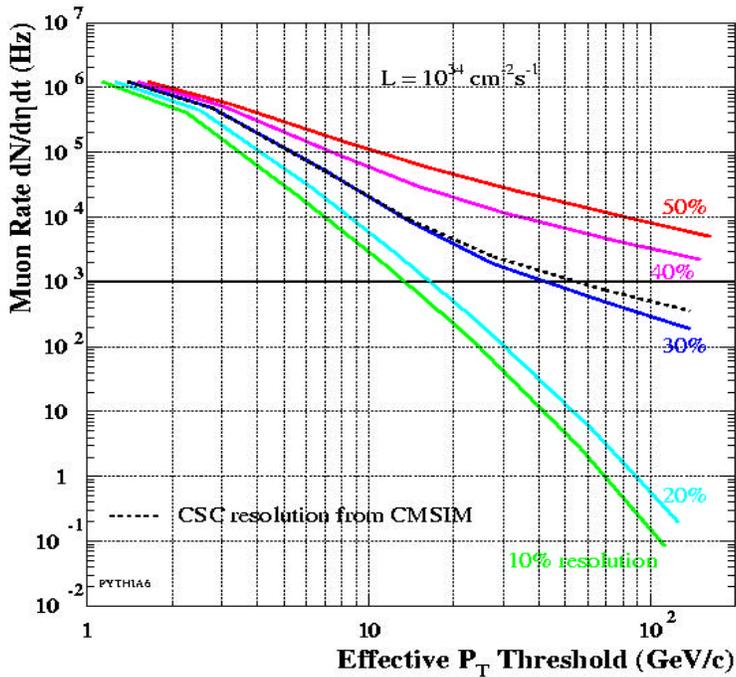


Figure 4: Rate requirements for the Level-1 Trigger. The transverse momentum resolution must be better than 30% to meet the desired rate requirements. This graph is from S. Wang, CMS Internal Note 2000/026.

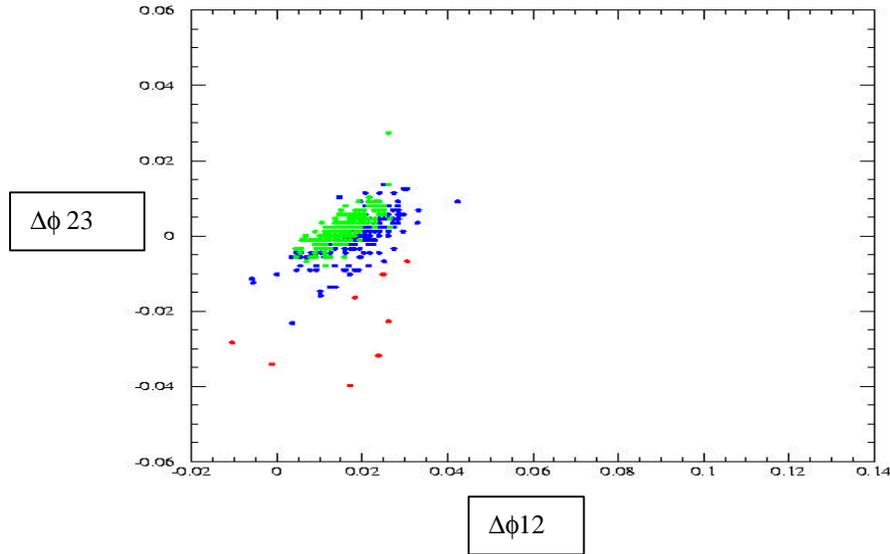


Figure 5: $\Delta\phi$ 23 vs. $\Delta\phi$ 12. Simulation data, comparing the two $\Delta\phi$ values over the h region 1.4 to 1.5.

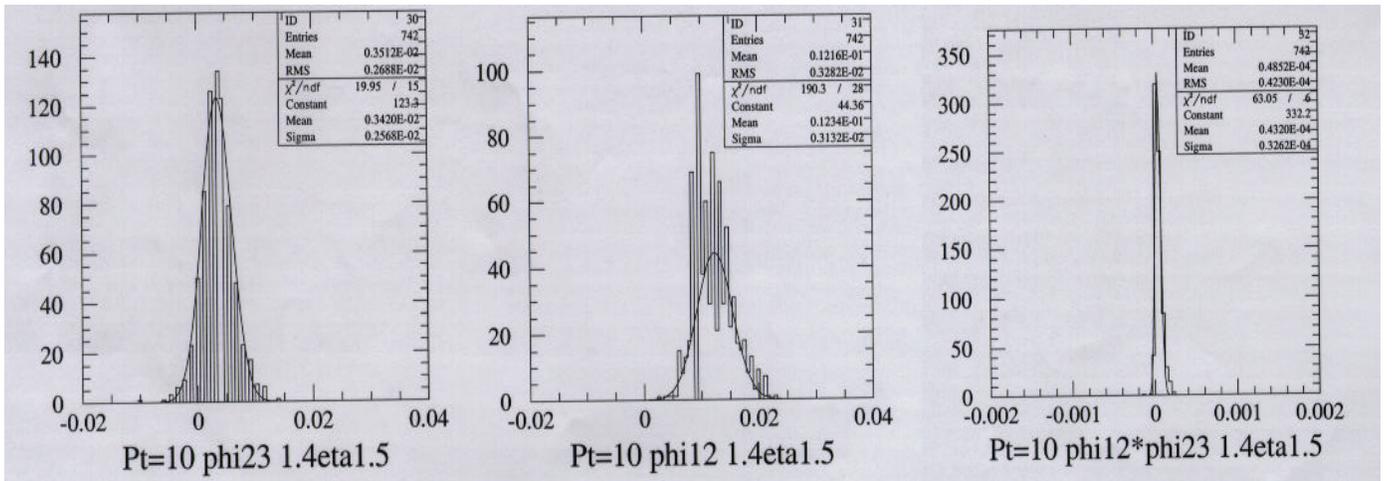


Figure 6: The normal curves to compute the correlation coefficient for $\Delta\phi$ 12 and $\Delta\phi$ 34 are shown. The normal variables from above are used in the equation

$$r_{yz} = \frac{\langle yz \rangle - m_y m_z}{s_y s_z}$$

to compute the correlation.

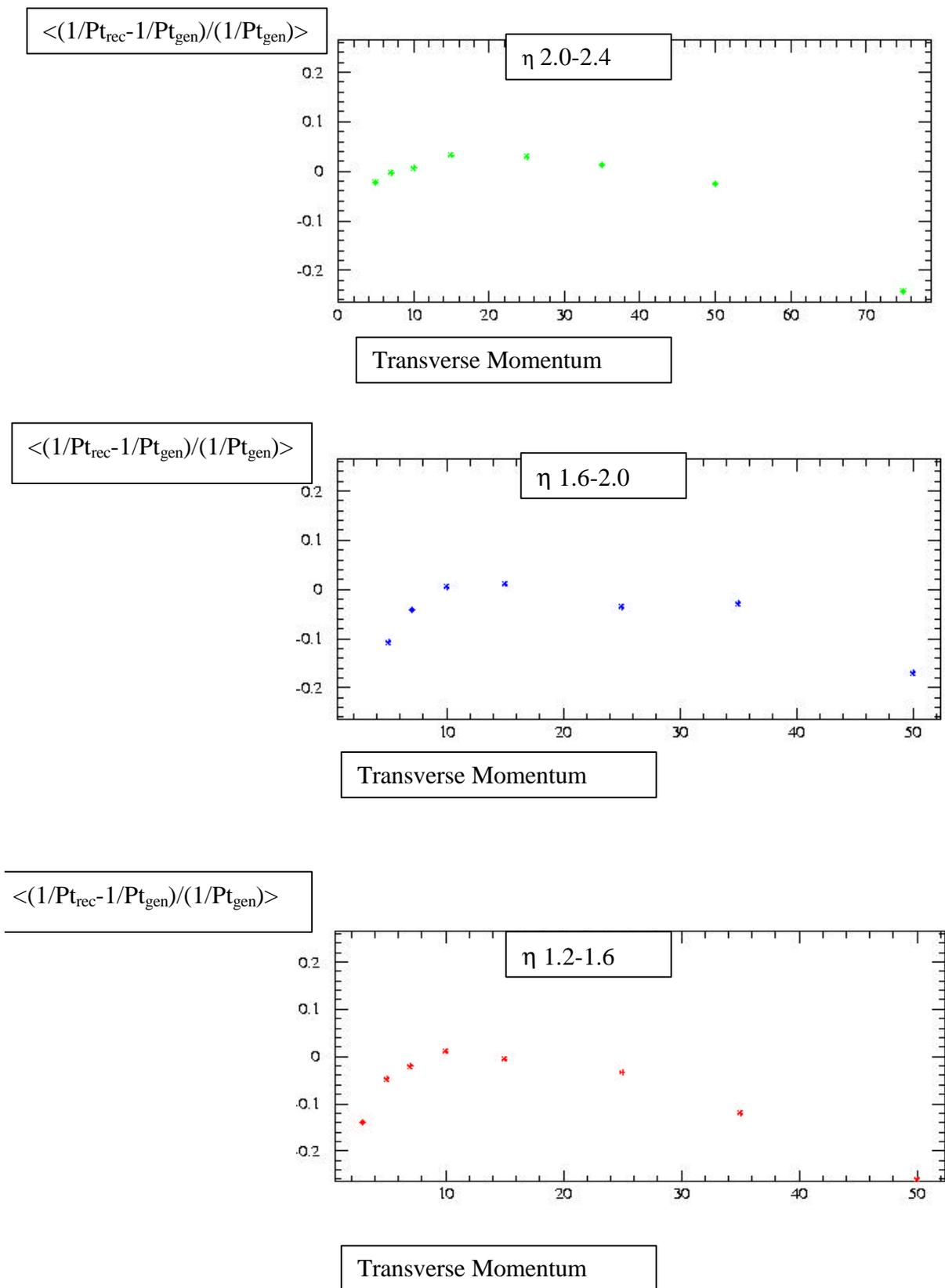


Figure 7: The difference between the reconstructed $1/P_t$ and the generated $1/P_t$ as a function of P_t for three different η regions. The difference is generally small for low P_t events.

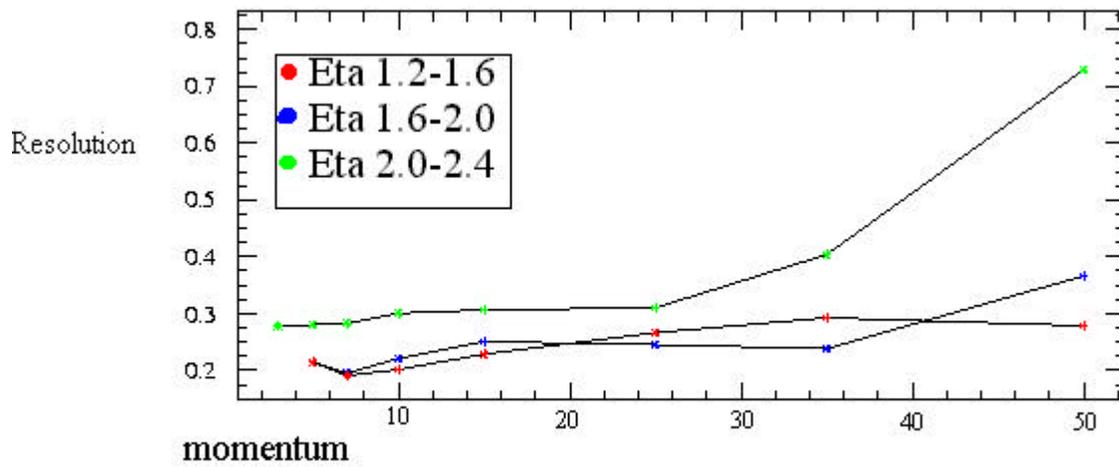


Figure 8: $1/P_t$ resolution as a function of P_t for different η regions.

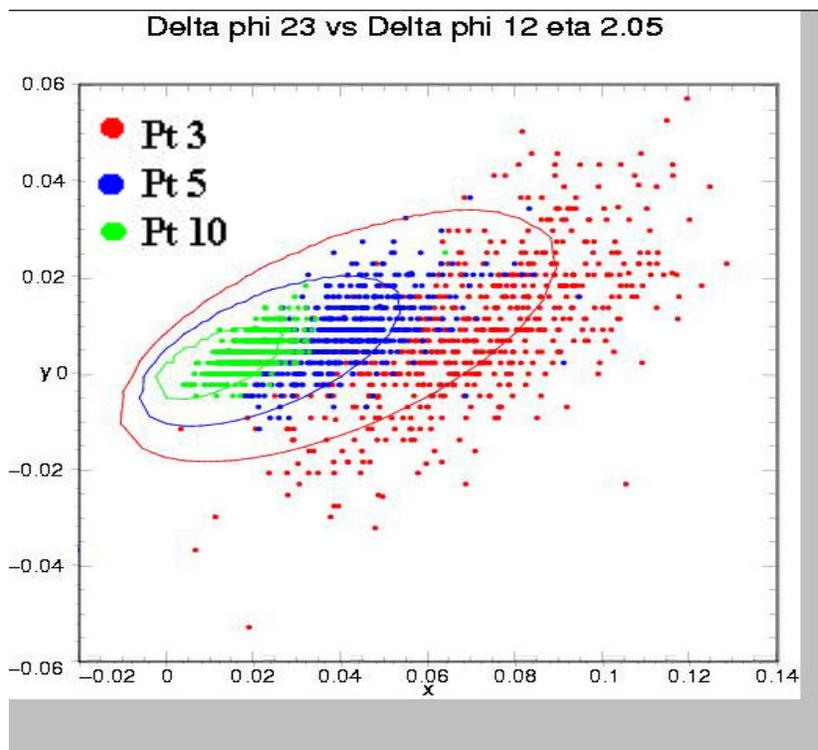


Figure 9: A plot of $\Delta\phi_{23}$ versus $\Delta\phi_{12}$ from simulation data. Overlaying the plot is a contour of equal P_t from the three-station parameterization (solid lines).