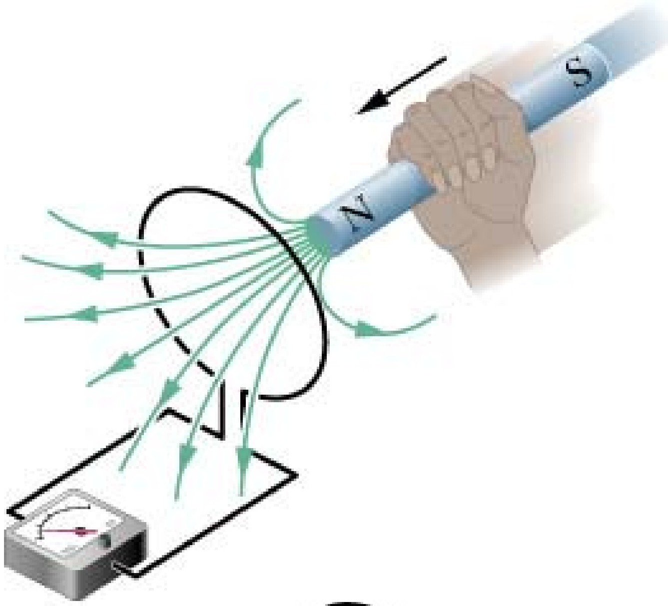
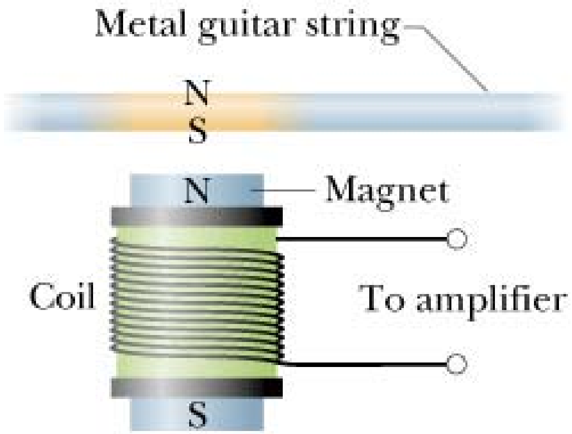
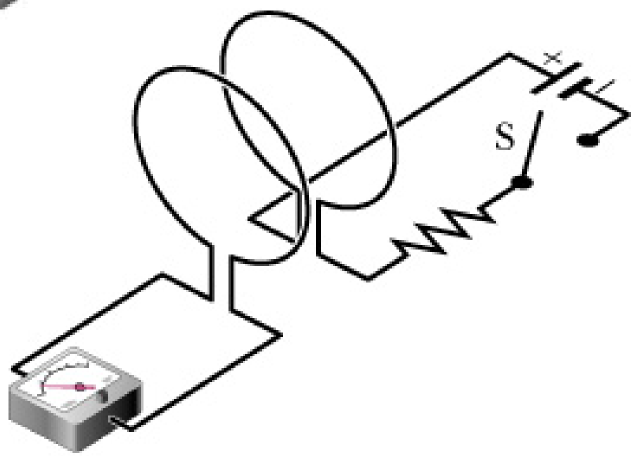
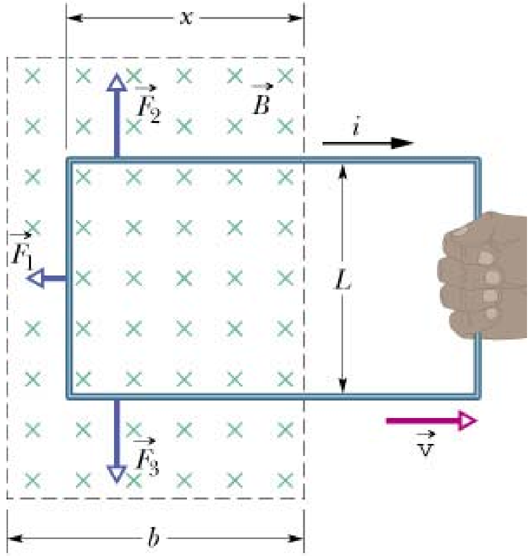
Chapters 34,36: Electromagnetic Induction





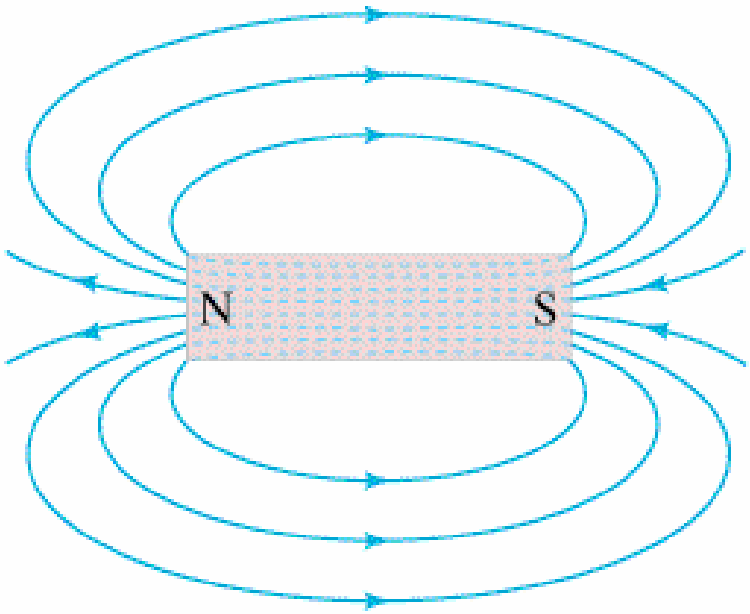




Topics

➔Electromagnetic Induction

- Magnetic flux
- Induced emf
  - Faraday's Law
  - Lenz's Law
  - Motional emf
- Magnetic energy
- Inductance
- RL circuits
- Generators and transformers



Reading Quiz 1

➔ Magnetic flux through a wire loop depends on:

- 1) thickness of the wire
- 2) resistivity of the wire
- 3) geometrical layout of the wire
- 4) material that the wire is made of
- 5) none of the above

$$\Phi_B = \int_A \mathbf{B} \cdot d\mathbf{A}$$

Flux depends only on geometrical properties

Reading Quiz 2

➔ An induced emf produced in a motionless circuit is due to

- 1) a static (steady) magnetic field
- 2) a changing magnetic field
- 3) a strong magnetic field
- 4) the Earth's magnetic field
- 5) a zero magnetic field

Faraday's law

Reading Quiz 3

➔ Motional emf relates to an induced emf in a conductor which is:

- ◆ 1) long
- ◆ 2) sad
- ◆ 3) stationary
- ◆ 4) insulated
- ◆ 5) moving

Potential difference proportional to velocity

Reading Quiz 4

- ➔ Faraday's law says that
  - a) an emf is induced in a loop when it moves through an electric field
  - b) the induced emf produces a current whose magnetic field opposes the original change
  - c) the induced emf is proportional to the rate of change of magnetic flux

Faraday's law

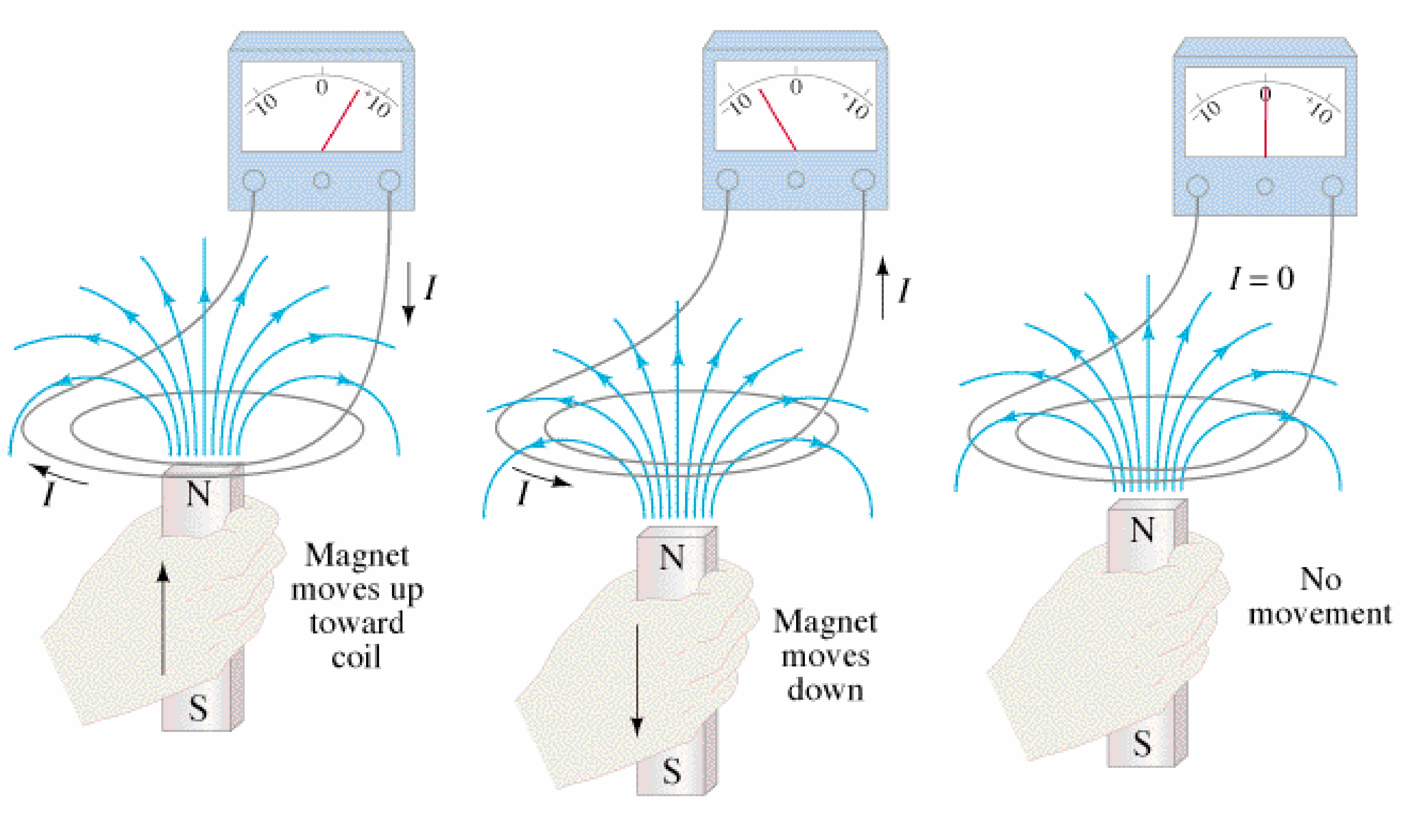
Reading Quiz 5

- A generator is a device that:
  - a) transforms mechanical into electrical energy
  - b) transforms electrical into mechanical energy
  - c) transforms low voltage to high voltage

Electromagnetic Induction

- ➔ Faraday discovered that a changing magnetic flux leads to a voltage in a wire loop
  - ◆ Induced voltage (emf) causes a current to flow !!
- ➔ Symmetry: electricity ⟷ magnetism
  - ◆ electric current → magnetic field
  - ◆ magnetic field → electric current
- ➔ We can express this symmetry directly in terms of fields
  - ◆ Changing E field → B field (“displacement current”)
  - ◆ Changing B field → E field (Faraday’s law)
- ➔ These & other relations expressed in Maxwell’s 4 equations
  - ◆ (Other 2 are Gauss’ law for E fields and B fields)
  - ◆ Summarizes all of electromagnetism
  - ◆ See Chapter 32

Experimental Observation of Induction



This effect can be quantified by Faraday's Law

Magnetic Flux

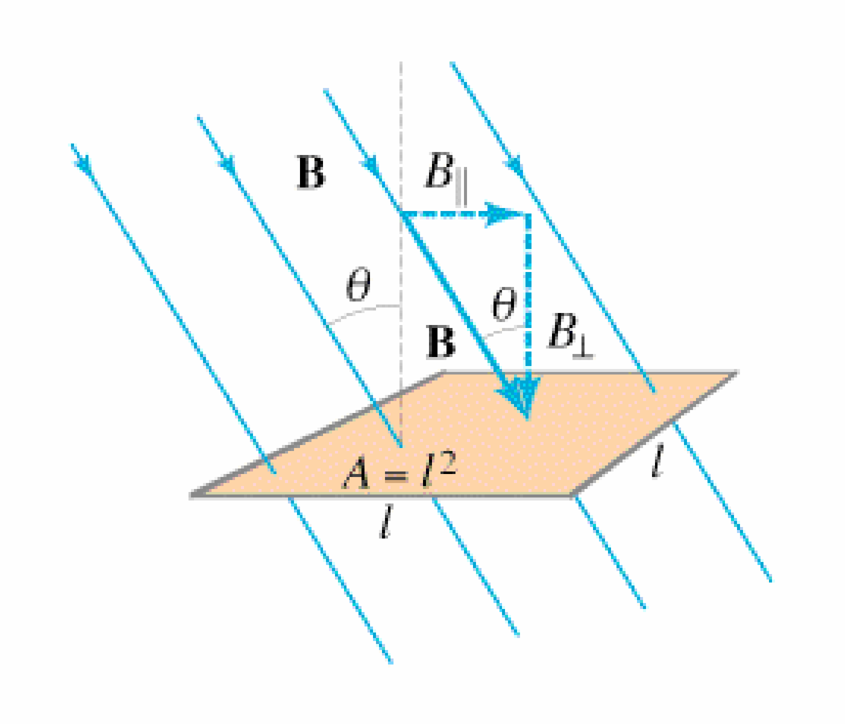
- ➔ Define magnetic flux Φ_B

$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$

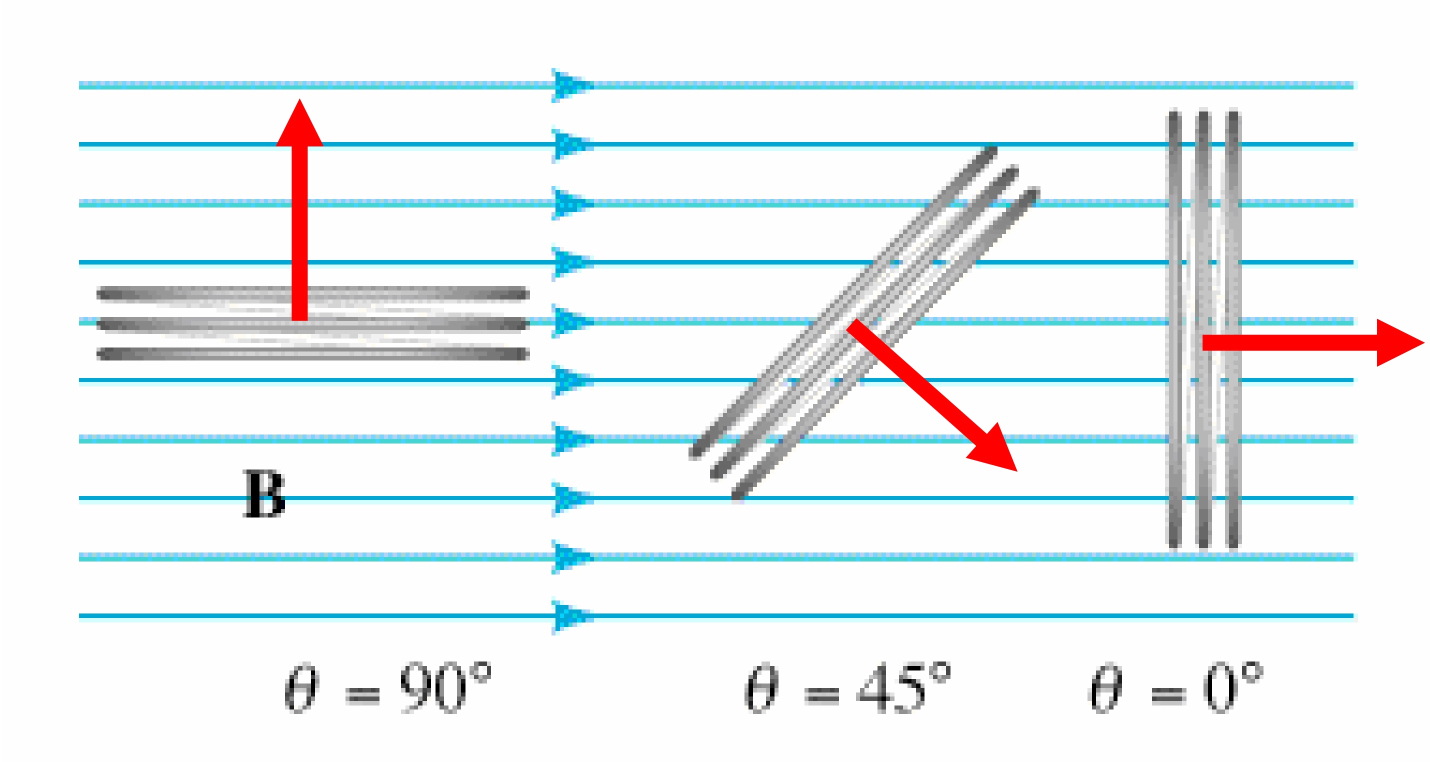
  - ◆ θ is angle between B and the normal to the plane
  - ◆ Flux units are T-m^2 = "webers"

- ➔ When B field is not constant or area is not flat
  - ◆ Integrate over area

$$\Phi_B = \int_A \mathbf{B} \cdot d\mathbf{A}$$



$$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$$



$$\Phi_B = 0 \qquad \Phi_B = \frac{1}{\sqrt{2}} BA \qquad \Phi_B = BA$$

Faraday's Law of Induction

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

- $\mathcal{E}$: induced emf
- N: number of loops
- $\frac{d\Phi_B}{dt}$: rate of change of flux with time

- The faster the change, the larger the induced emf
- Flux change caused by changing B, area, or orientation
- The induced emf is a *voltage*

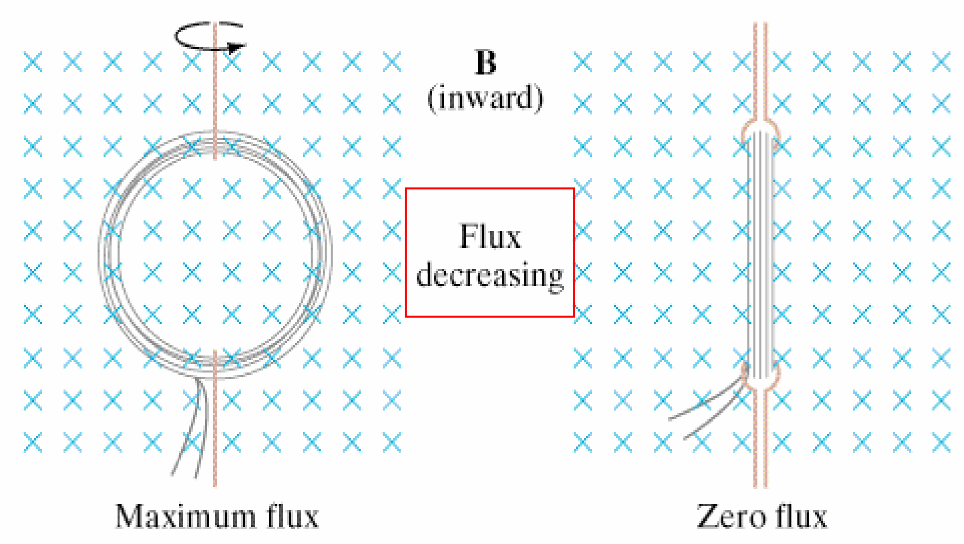
Faraday's Law & Flux Change

- Rotating coil

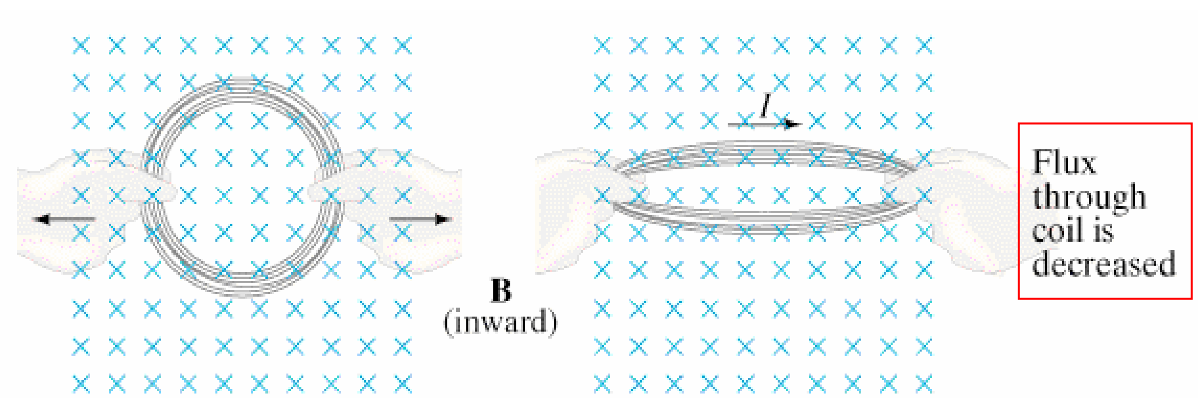
$$\Phi_B = BA\cos\omega t$$

$$E = -N\frac{d\Phi_B}{dt} = N\omega BA\sin\omega t$$

  - ϕ_B is maximum when coil faces up
  - E is maximum when coil faces sideways



- Stretched coil
  - B constant, θ constant
  - Area shrinks
  - $\Rightarrow$ Flux decreases



Faraday's Law of Induction

$$E = -N\frac{d\Phi_B}{dt}$$

induced emf

number of loops

rate of change of flux with time

- Minus sign from Lenz's Law:
- Induced current produces a magnetic field which *opposes* the original change in flux

Comment on Lenz's Law

- Why does the induced current oppose the change in flux?
- Consider the alternative
  - If the induced current reinforced the change, then the change would get bigger, which would then induce a larger current, and then the change would get even bigger, and so on . . .
  - This leads to a clear violation of conservation of energy!!

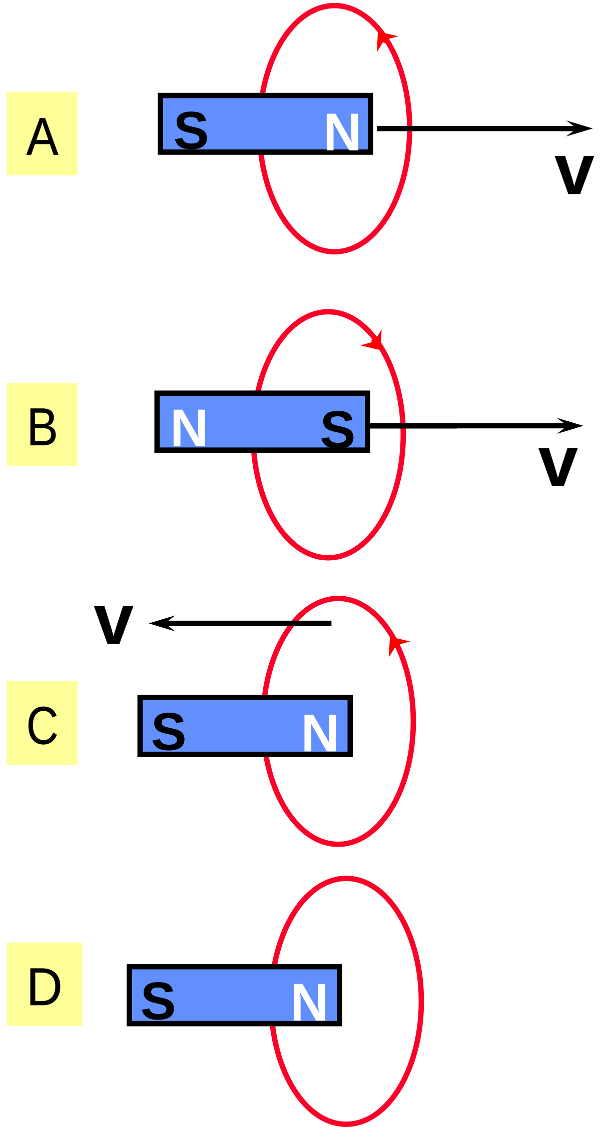
Direction of Induced Current

Bar magnet moves through coil
- Current induced in coil

Reverse pole
- Induced current changes sign

Coil moves past fixed bar magnet
- Current induced in coil as in (A)

Bar magnet stationary inside coil
- No current induced in coil

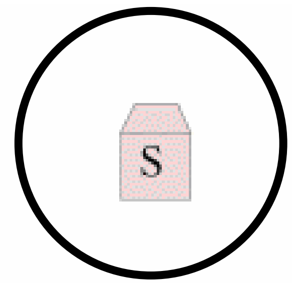


ConcepTest: Lenz's Law

➔ If a North pole moves towards the loop from above the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Must counter flux change in downward direction with upward B field

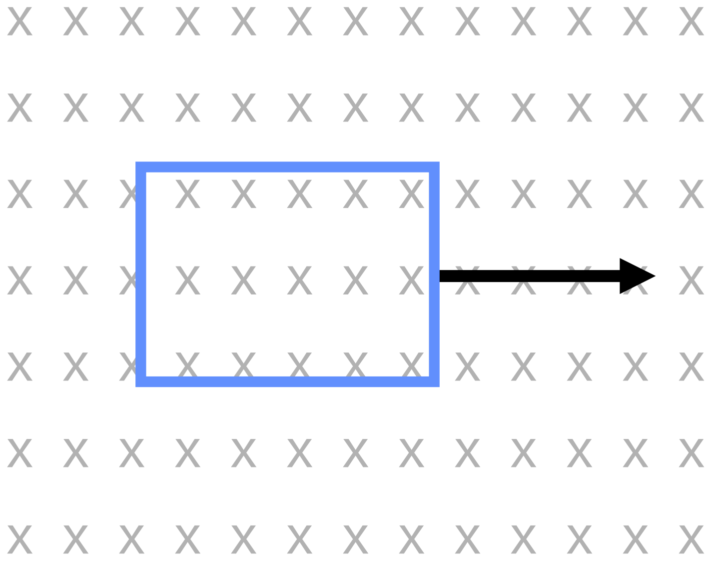


ConcepTest: Induced Currents

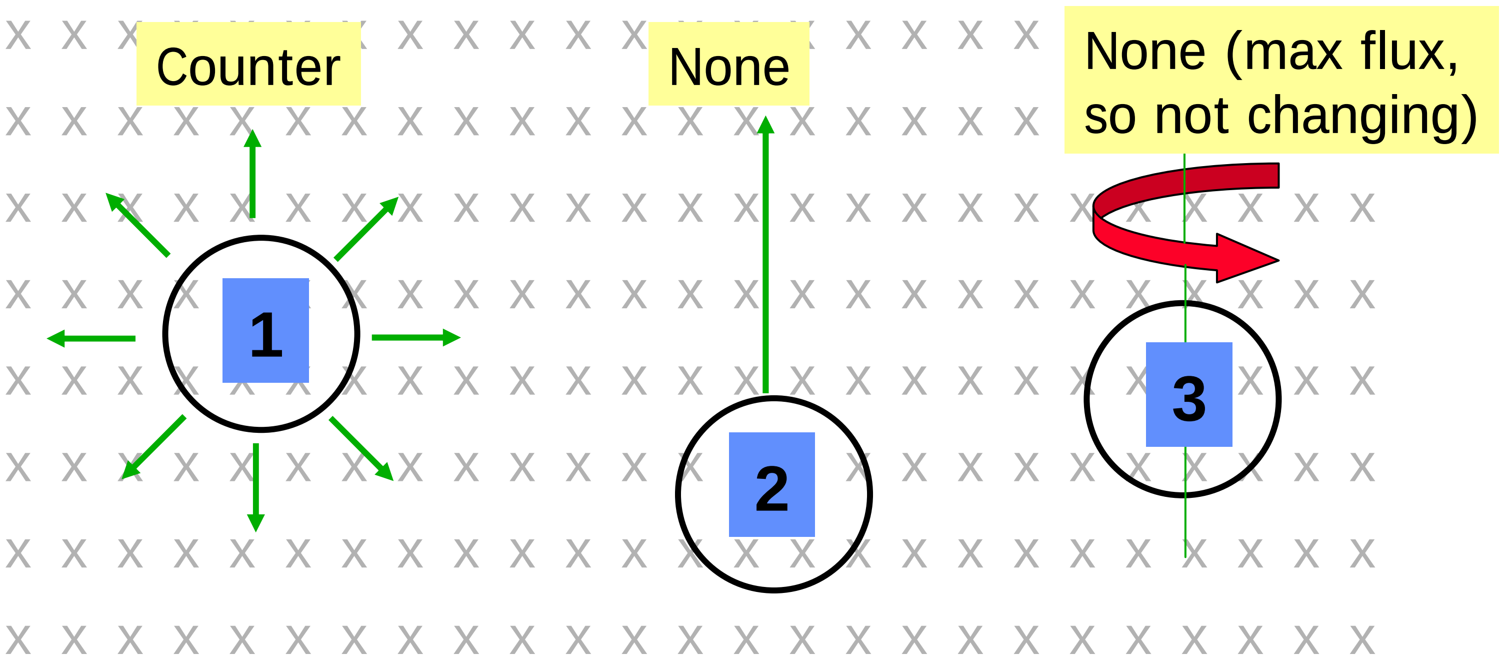
➔ A wire loop is being pulled through a uniform magnetic field. What is the direction of the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

No change in flux, no induced current



ConcepTest: Induced Currents



In each of the 3 cases above, what is the direction of the induced current?

(Magnetic field is into the page and has no boundaries)

(a) clockwise

(b) counter-clockwise

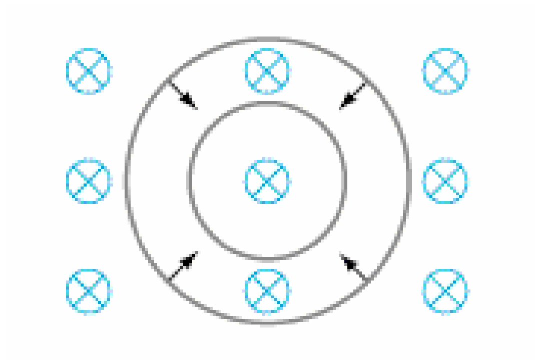
(c) no induced current

ConcepTest: Lenz's Law

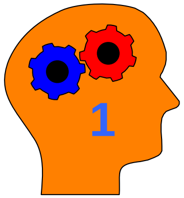
➔ If a coil is shrinking in a B field pointing into the page, in what direction is the induced current?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Downward flux is decreasing, so need to create downward B field



Induced currents



➔A circular loop in the plane of the paper lies in a 3.0 T magnetic field pointing into the paper. The loop's diameter changes from 100 cm to 60 cm in 0.5 s

- What is the magnitude of the average induced emf?
- What is the direction of the induced current?
- If the coil resistance is 0.05Ω , what is the average induced current?

$$|V| = \frac{d\Phi_B}{dt} = 3.0 \times \left| \frac{\pi\left(0.3^2 - 0.5^2\right)}{0.5} \right| = 3.016 \text{ Volts}$$

- Direction = clockwise (Lenz's law)

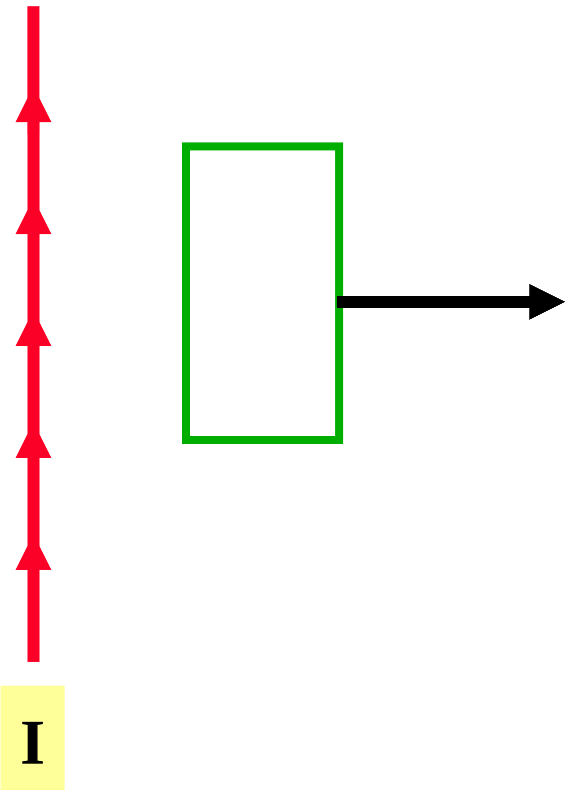
- Current = 3.016 / 0.05 = 60.3 A

ConcepTest: Induced Currents

➔ A wire loop is pulled away from a current-carrying wire. What is the direction of the induced current in the loop?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Downward flux through loop decreases, so need to create downward field

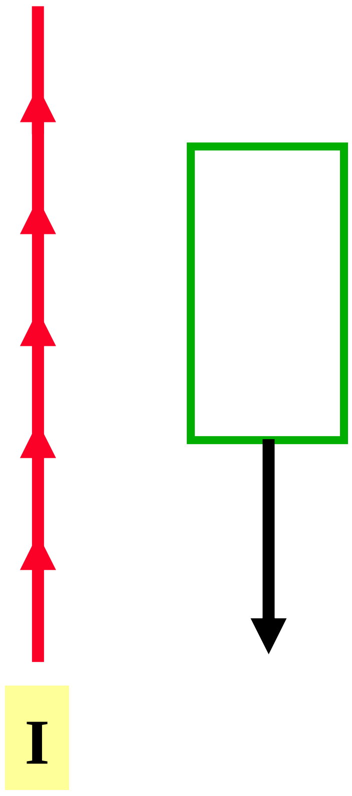


ConcepTest: Induced Currents

➔ A wire loop is moved in the direction of the current. What is the direction of the induced current in the loop?

- (a) clockwise
- (b) counter-clockwise
- (c) no induced current

Flux does not change when moved along wire

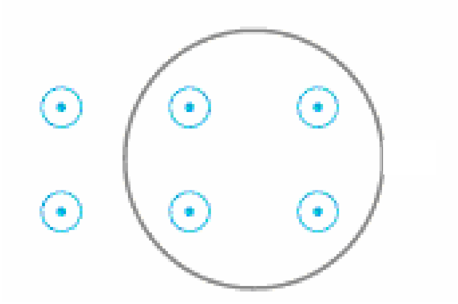


ConcepTest: Lenz's Law

➔ If the B field pointing out of the page suddenly drops to zero, in what direction is the induced current?

- ◆ (a) clockwise
- ◆ (b) counter-clockwise
- ◆ (c) no induced current

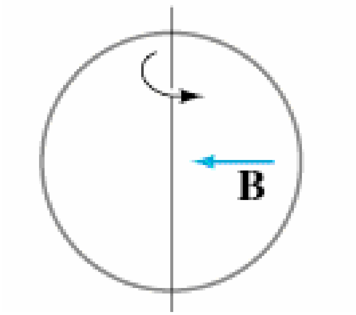
Upward flux through loop decreases, so need to create upward field



➔ If a coil is rotated as shown, in a B field pointing to the left, in what direction is the induced current?

- ◆ (a) clockwise
- ◆ (b) counter-clockwise
- ◆ (c) no induced current

Flux into loop is increasing, so need to create field out of loop



ConcepTest: Induced Currents

➔ Wire #1 (length L) forms a one-turn loop, and a bar magnet is dropped through. Wire #2 (length 2L) forms a two-turn loop, and the same magnet is dropped through. Compare the magnitude of the induced currents in these two cases.

- (a) $I_1 = 2\, I_2$
- (b) $I_2 = 2\, I_1$
- (c) $I_1 = I_2 \neq 0$
- (d) $I_1 = I_2 = 0$
- (e) Depends on the strength of the magnetic field

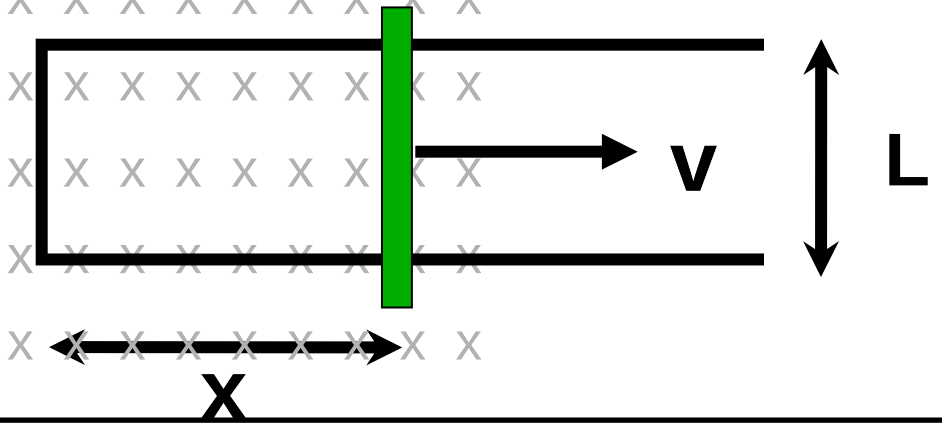
Voltage doubles, but R also doubles, leaving current the same

Motional EMF

➔Consider a conducting rod moving on metal rails in a uniform magnetic field:

$$\left|\mathcal{E}\right| = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = \frac{d(BLx)}{dt} = BL\frac{dx}{dt}$$

Current will flow counter-clockwise in this "circuit". Why?



$$\left|\mathcal{E}\right| = BLv$$

Force and Motional EMF

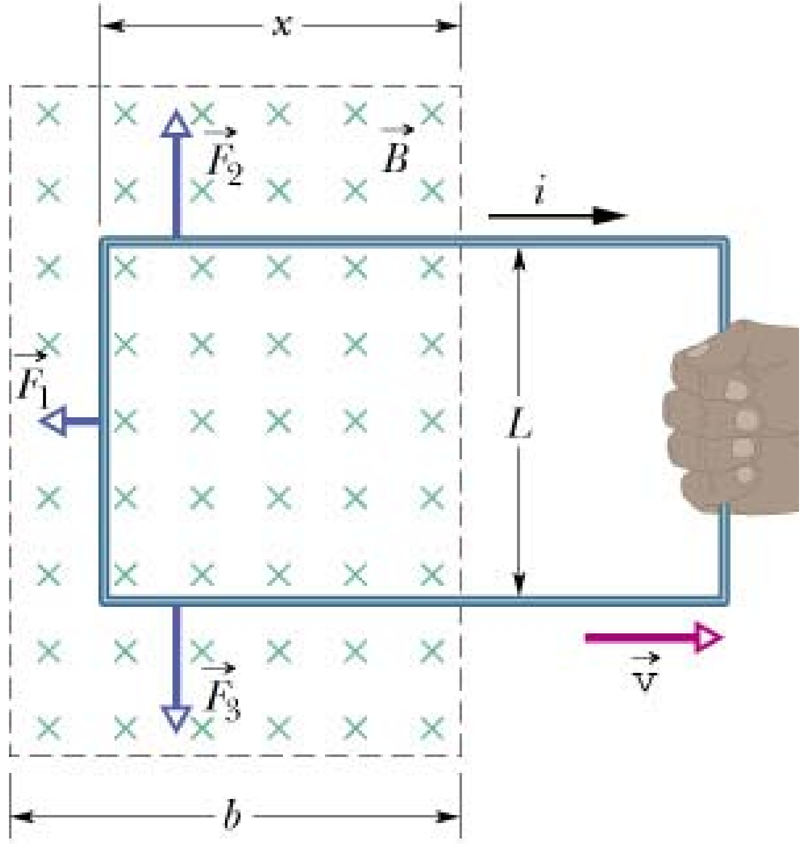
- Pull conducting rod out of B field
- Current is clockwise. Why?

$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

- Current within B field causes force

$$F = iLB = \frac{B^2 L^2 v}{R}$$

  - Force opposes pull (RHR)
  - Also follows from Lenz's law
- We must pull with this force to maintain constant velocity



Power and Motional EMF

➔ Force required to pull loop: $F = iLB = \frac{B^2L^2v}{R}$

➔ Power required to pull loop: $P = Fv = \frac{B^2L^2v^2}{R}$

➔ Energy dissipation through resistance

$$P = i^2R = \left(\frac{BLv}{R}\right)^2 R = \frac{B^2L^2v^2}{R}$$

➔ Same as pulling power! So power is dissipated as heat

- Kinetic energy is constant, so energy has to go somewhere
- Rod heats up as you pull it

Example

- Pull a 30cm x 30cm conducting loop of aluminum through a 2T B field at 30cm/sec. Assume it is 1cm thick.
  - Circumference = 120cm = 1.2m, cross sectional area = 10^{-4} m^2
  - R = ρL/A = 2.75 x 10^{-8} * 1.2 / 10^{-4} = 3.3 x 10^{-4}Ω
- EMF

$$\mathcal{E} = BLv = 2 \times 0.3 \times 0.3 = 0.18\,\text{V}$$

- Current

$$i = \mathcal{E} / R = 0.18 / 3.3 \times 10^{-4} = 545\,\text{A}$$

- Force

$$F = iLB = 545 \times 0.3 \times 2 = 327\,\text{N}$$

74 lbs!

- Power

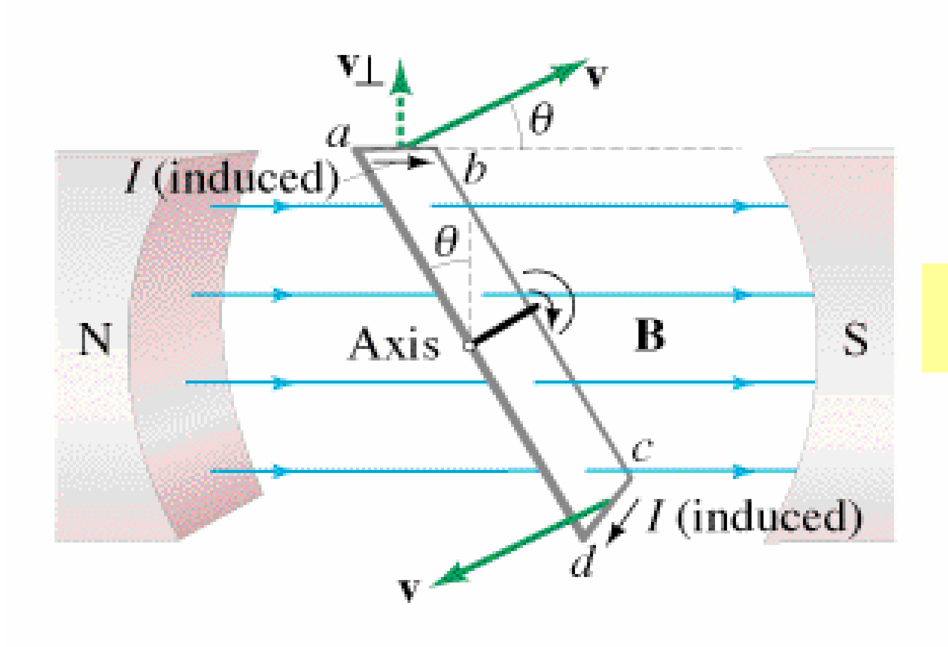
$$P = i^2 R = 98\,\text{W}$$

→ About 0.33^0 C per sec (from specific heat, density)

Electric Generators

➔Rotate a loop of wire in a uniform magnetic field:

- changing θ $\Rightarrow$ changing flux $\Rightarrow$ induced emf
- $\Phi_B = B A \cos\theta = B A \cos(\omega t)$



Rotation: $\theta = \omega t$

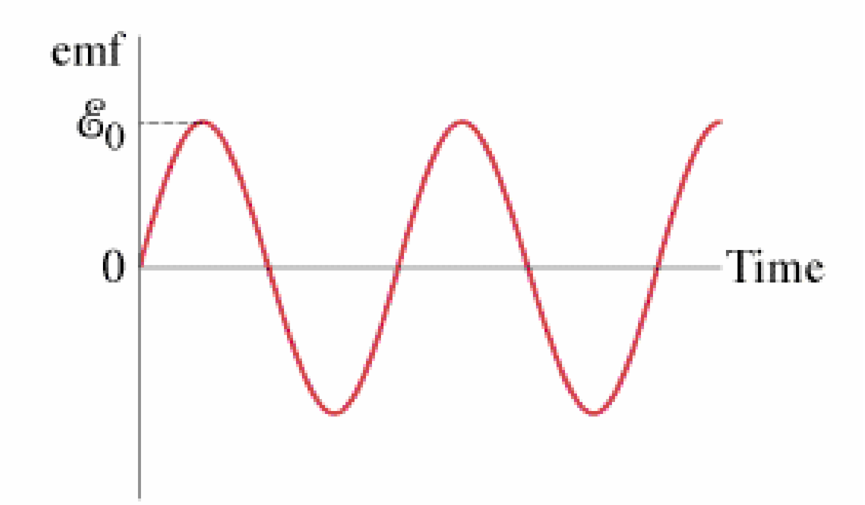
Electric Generators

➔ Flux is changing in a sinusoidal manner

- Leads to an alternating emf (AC generator)

$$\left|\mathcal{E}\right| = N\frac{d\Phi_B}{dt} = NBA\frac{d\cos(\omega t)}{dt} = NBA\omega\sin(\omega t)$$

- This is how electricity is generated
- Water or steam (mechanical power) turns the blades of a turbine which rotates a loop
- Mechanical power converted to electrical power

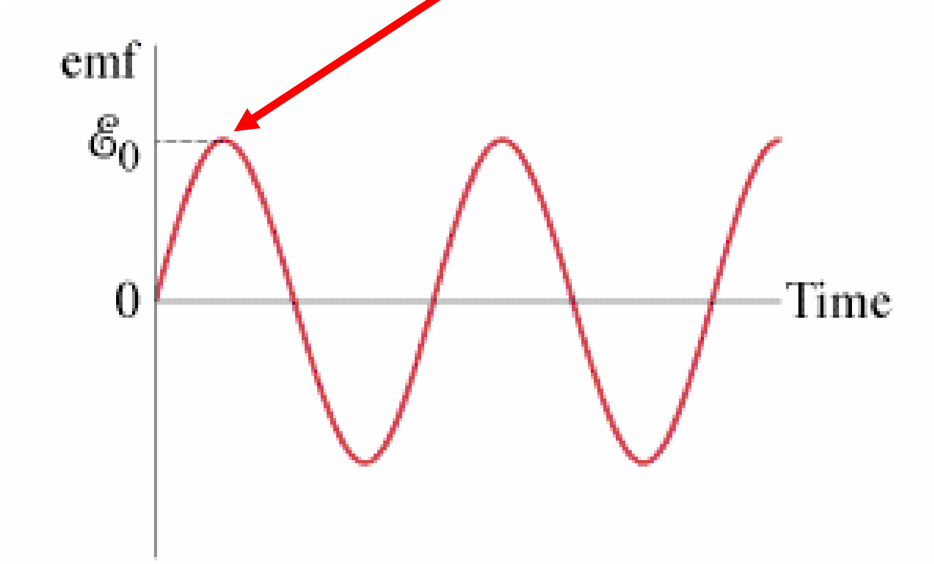


ConcepTest: Generators

➔A generator has a coil of wire rotating in a magnetic field. If the B field stays constant and the area of the coil remains constant, but the rotation rate increases, how is the maximum output voltage of the generator affected?

- (a) Increases
- (b) Decreases
- (c) Stays the same
- (d) Varies sinusoidally

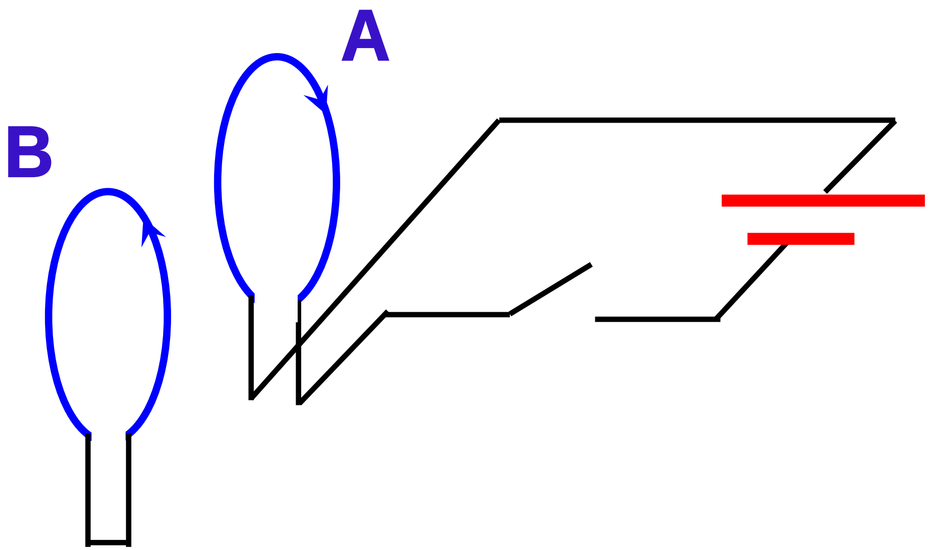
$$\left|\mathcal{E}\right| = NBA\omega \sin(\omega t)$$



Induction in Stationary Circuit

- Switch closed (or opened)
  - Current induced in coil B (directions as shown)

- Steady state current in coil A
  - No current induced in coil B



Inductance

- Inductance in a coil of wire defined by $L = \frac{N\Phi_B}{i}$
- Can also be written $Li = N\Phi_B$
- From Faraday's law $\mathcal{E} = -N\frac{d\Phi_B}{dt} = -L\frac{di}{dt}$
  - This is a more useful way to understand inductance
- Inductors play an important role in circuits when current is changing!

Self - Inductance

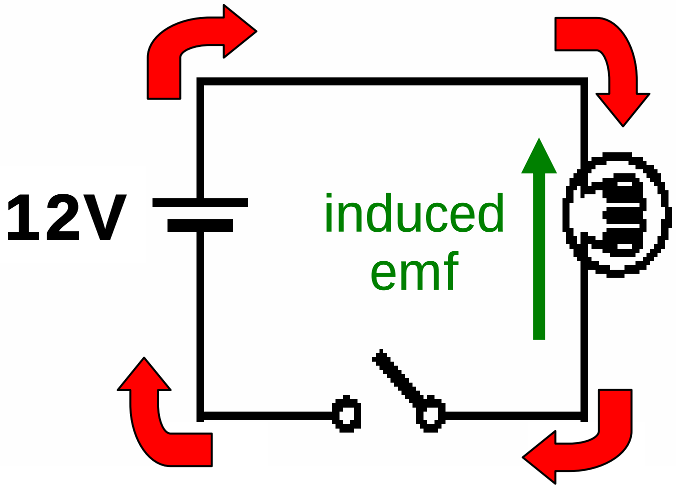
➔Consider a single isolated coil:

- Current (red) starts to flow clockwise due to the battery
- But the buildup of current leads to changing flux in loop
- Induced emf (green) opposes the change

This is a self-induced emf (also called "back" emf)

$$\mathcal{E} = -N\frac{d\Phi}{dt} = -L\frac{di}{dt}$$

L is the self-inductance
units = "Henry (H)"



Inductance of Solenoid

➔Total flux (length *l*)

$$B = \mu_0 i n$$

$$N\Phi_B = (nl)(BA) = \mu_0 n^2 A l i$$

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -\mu_0 n^2 A l \frac{di}{dt} = -L\frac{di}{dt}$$

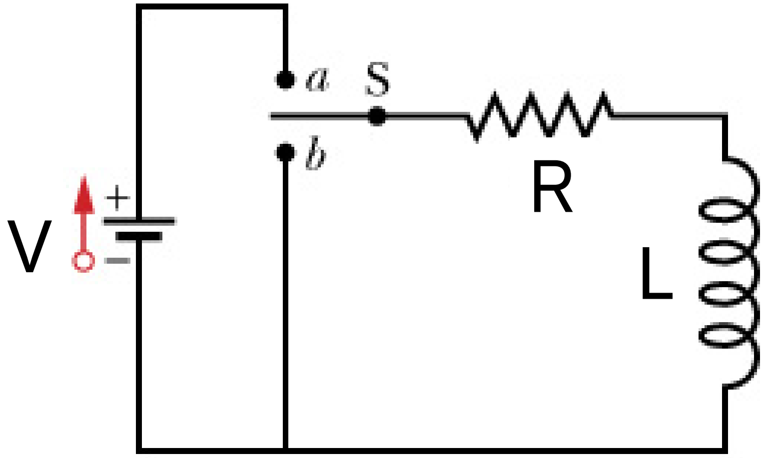
$$L = \mu_0 n^2 A l$$

To make large inductance:

- Lots of windings
- Big area
- Long

LR Circuits

- ➔ Inductance and resistor in series with battery of EMF *V*
- ➔ Start with no initial current in circuit
  - ◆ Close switch at t = 0
  - ◆ Current is initially 0 (initial increase causes voltage drop across inductor)
- ➔ Find $i(t)$
  - ◆ Resistor: $\Delta V = Ri$
  - ◆ Inductor: $\Delta V = L\,di/dt$



$$V - Ri - L\,di/dt = 0$$

Analysis of LR Circuit

➔ Differential equation is $di/dt + i\left(R/L\right) = V/R$

➔ General solution: $i = V/R + Ke^{-tR/L}$

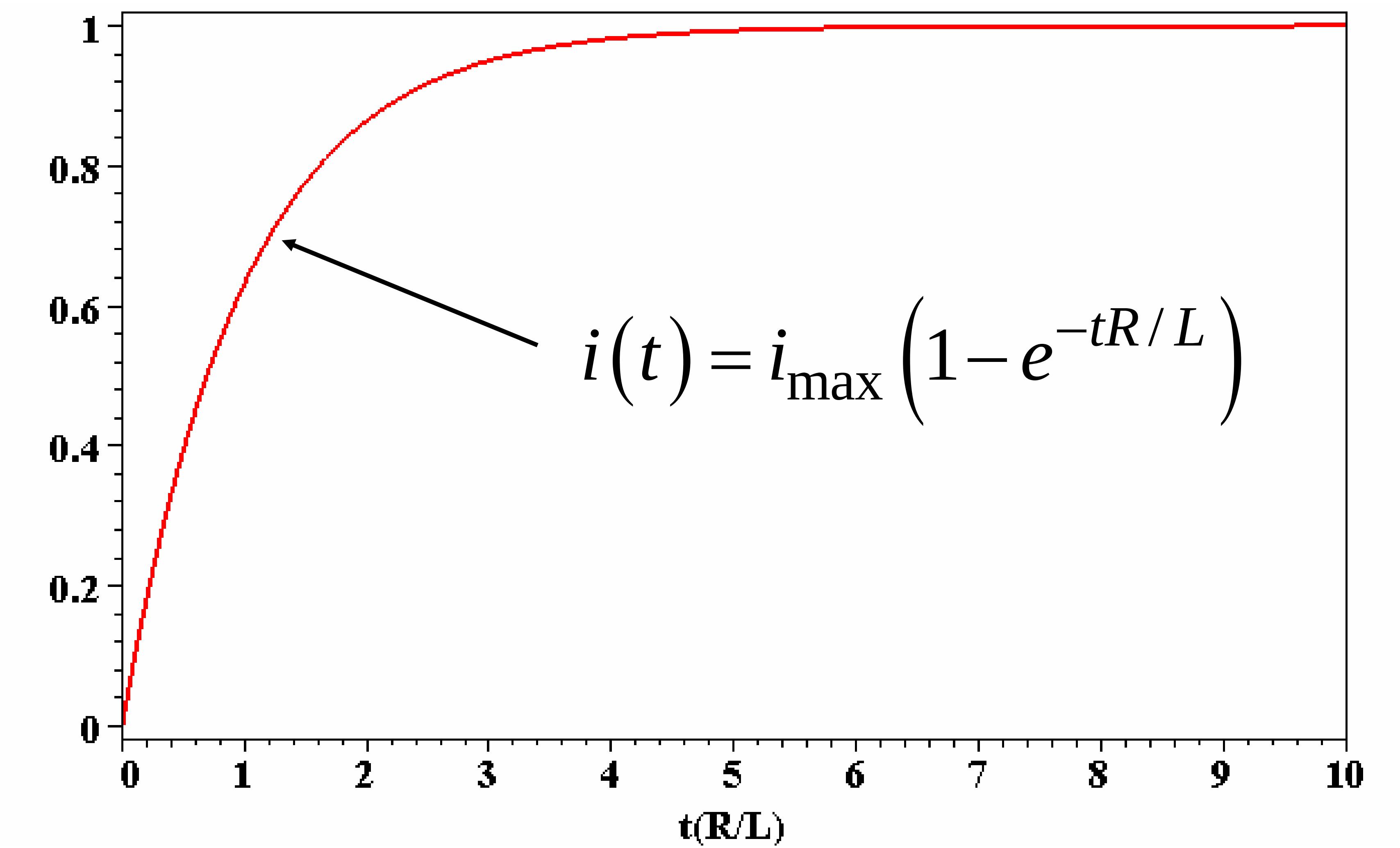
- (Check and see!)
- K = –V/R (necessary to make i = 0 at t = 0)

$$i = \frac{V}{R}\left(1 - e^{-tR/L}\right)$$

← Rise from 0 with time constant τ = L / R

↑ Final current (maximum)

Current vs Time in RL Circuit (Initially Zero Current in Inductor)



L-R Circuits (2)

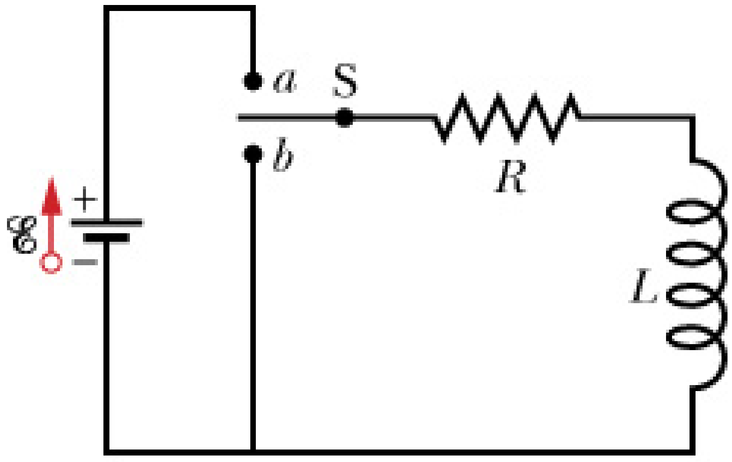
➔Switch off battery: Find $i(t)$ if current starts at i_0

$$0 = L\,di/dt + Ri$$

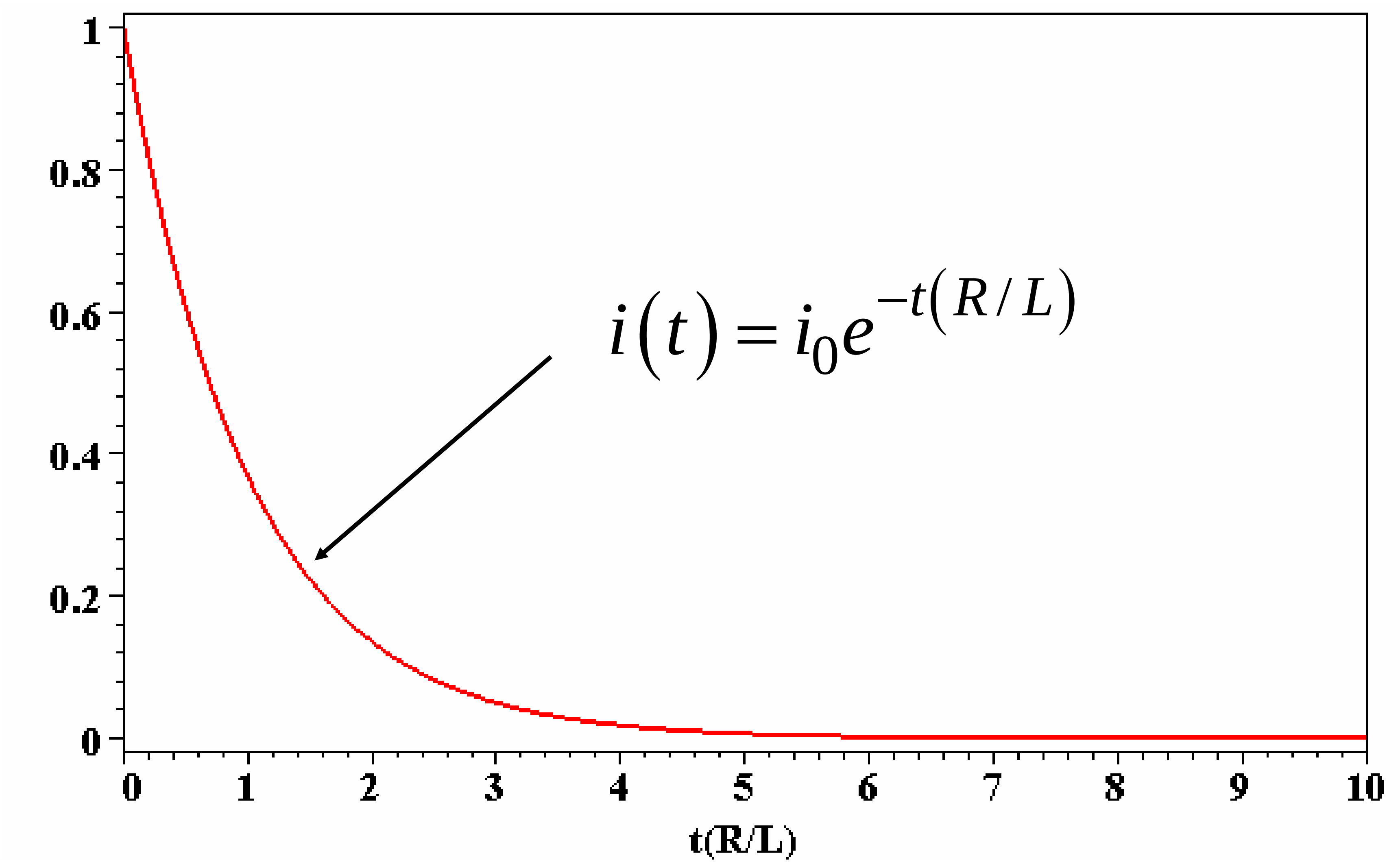
$$i = i_0 e^{-tR/L}$$

Exponential fall to 0 with time constant $\tau = L / R$

Initial current (maximum)



Current vs Time in RL Circuit (For Initial Current i_{max} in Inductor)



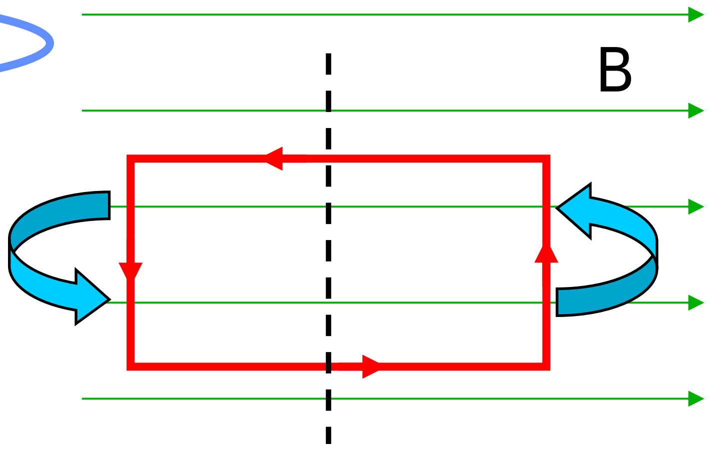
Exponential Behavior

- ➔ τ = L/R is the "characteristic time" of any RL circuit
  - Only t / τ is meaningful
- ➔ $t = \tau$
  - Current falls to 1/e = 37% of maximum value
  - Current rises to 63% of maximum value
- ➔ $t = 2\tau$
  - Current falls to $1/e^2$ = 13.5% of maximum value
  - Current rises to 86.5% of maximum value
- ➔ $t = 3\tau$
  - Current falls to $1/e^3$ = 5% of maximum value
  - Current rises to 95% of maximum value
- ➔ $t = 5\tau$
  - Current falls to $1/e^5$ = 0.7% of maximum value
  - Current rises to 99.3% of maximum value

ConcepTest: Generators and Motors

➔A current begins to flow in a wire loop placed in a magnetic field as shown. What does the loop do?

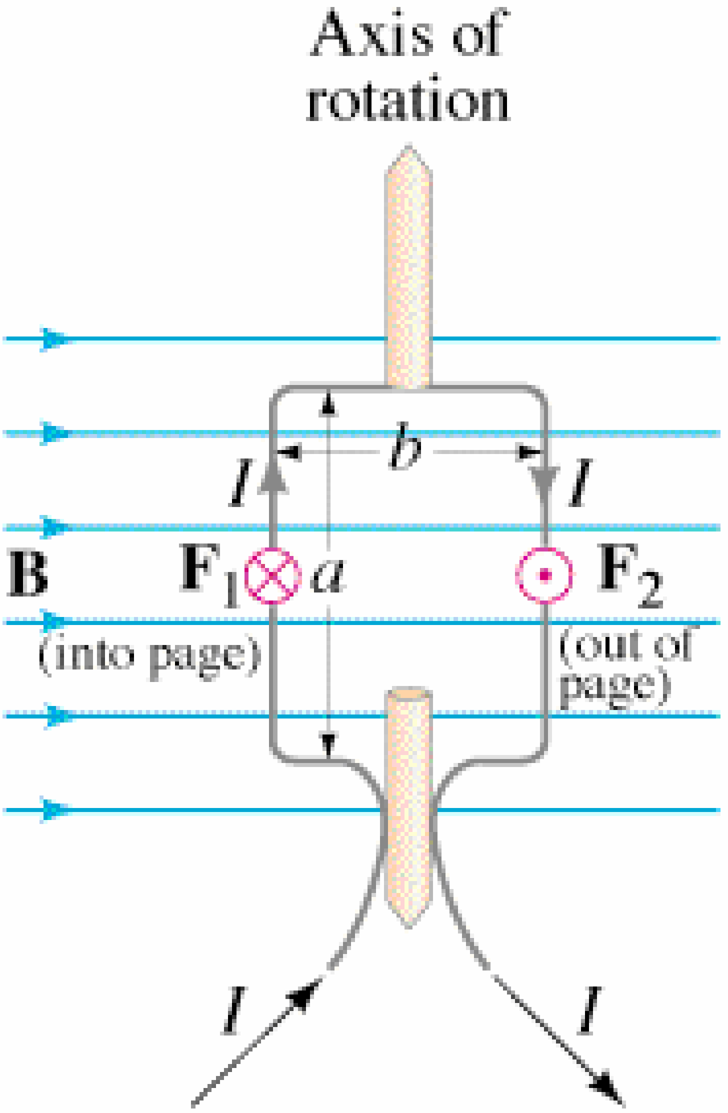
- (a) moves to the right
- (b) moves up
- (c) rotates around horizontal axis
- (d) rotates around vertical axis
- (e) moves out of the page



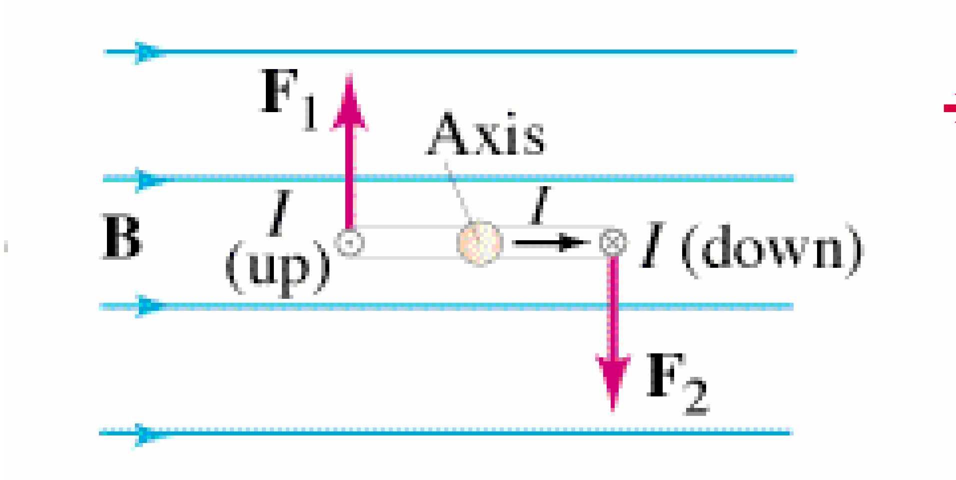
This is how a motor works !!

Electric Motors

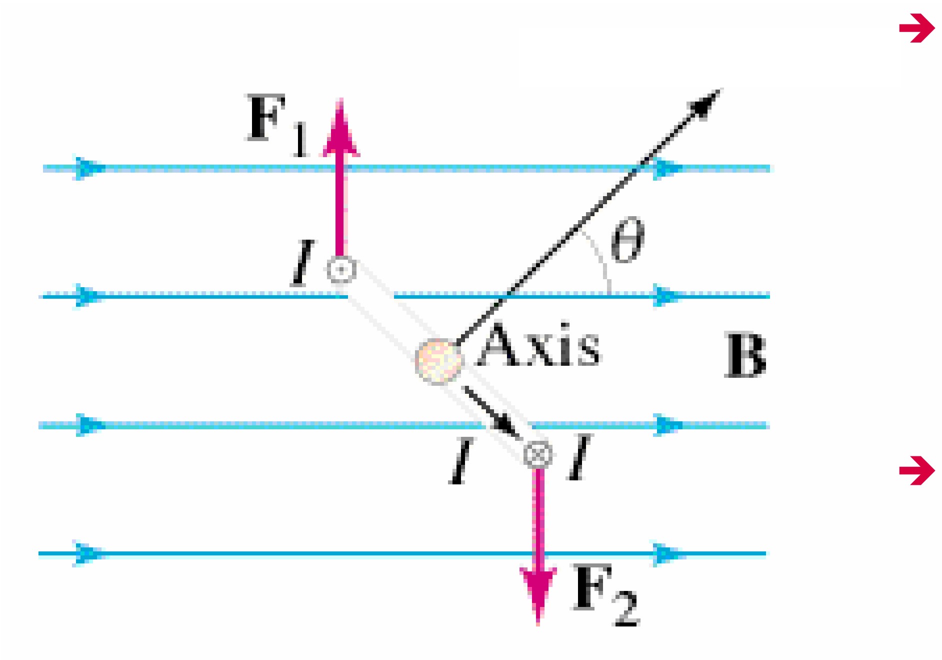
- Current is supplied from an external source of emf (battery or power supply)
- Forces act to rotate the wire loop
- A motor is essentially a generator operated in reverse!



Motor



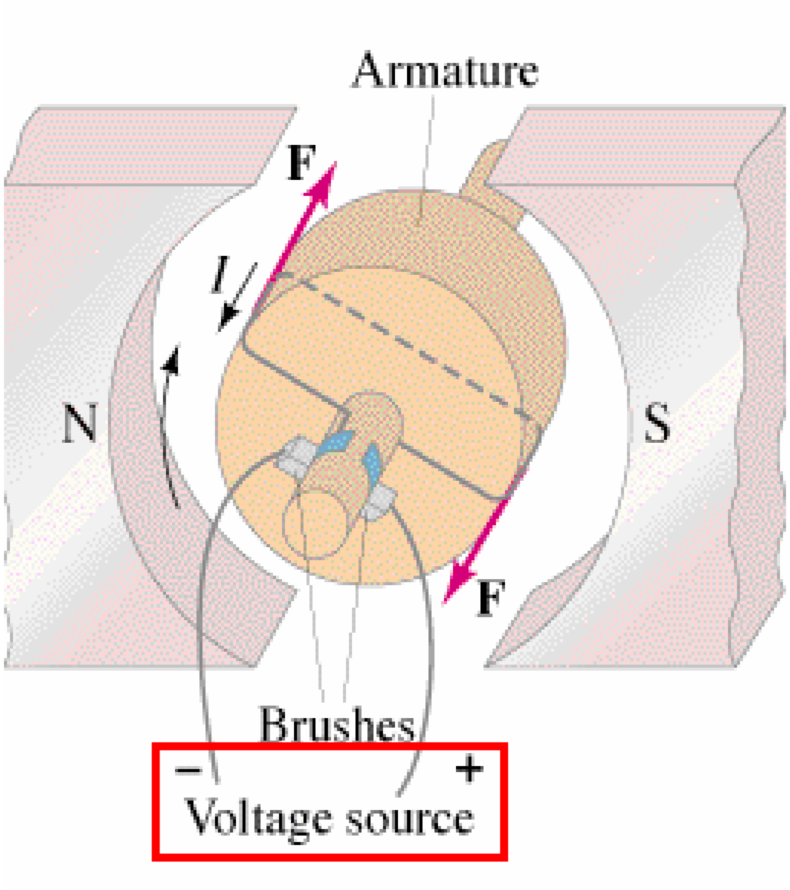
- Forces act to rotate the loop towards the vertical.



- When loop is vertical, current switches sign and the forces reverse, in order to keep the loop in rotation.

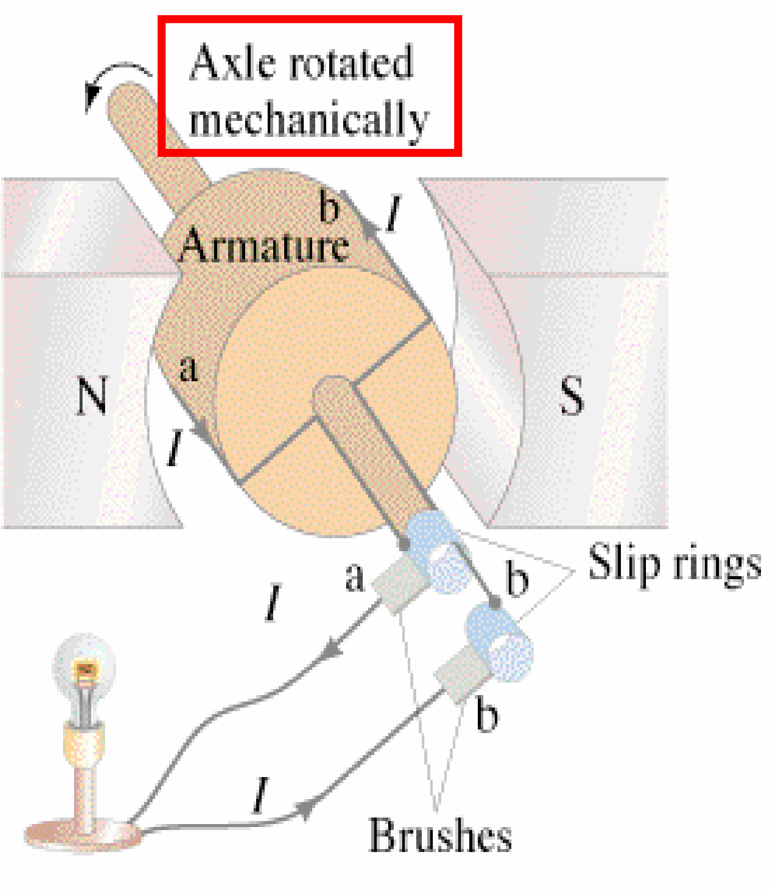
- This is why alternating current is necessary for a motor to operate.

Motors



Electrical ⇒ mechanical energy

Generators



Mechanical ⇒ electrical energy

Energy Stored in Magnetic Field

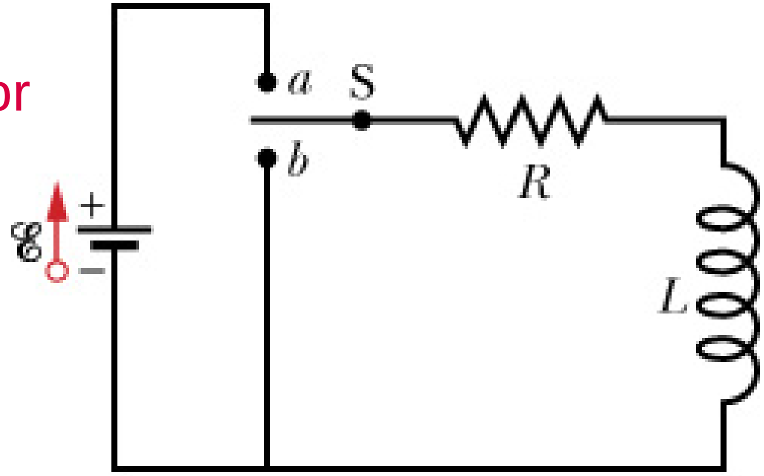
➔Just like electric fields, magnetic fields store energy

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$ ← Electric field

$$u_B = \frac{B^2}{2\mu_0}$$ ← Magnetic field

➔Let's see how this works

Energy of an Inductor



- ➔ How much energy is stored in an inductor when a current is flowing through it?
- ➔ Start with loop rule

$$\mathcal{E} = iR + L\frac{di}{dt}$$

- ➔ Multiply by I to get power equation

$$\mathcal{E}i = i^2 R + Li\frac{di}{dt} \qquad P_L = Li\frac{di}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}Li^2\right)$$

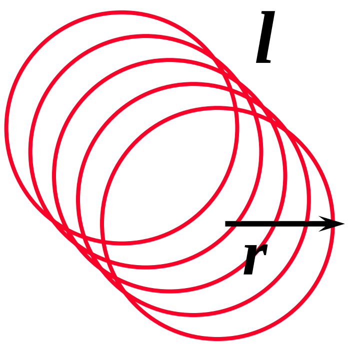
$$P_{\text{in}} = P_R\left(\text{heat}\right) + P_L\left(\text{store}\right)$$

- ➔ P_L = rate at which energy is being stored in inductor
  - ◆ Energy stored in inductor $U_L = \tfrac{1}{2}Li^2$
  - ◆ Similar to capacitor: $U_C = \dfrac{q^2}{2C}$

Energy in Magnetic Field (2)

➔Apply to solenoid (constant B field)

$$U_L = \tfrac{1}{2} L i^2 = \tfrac{1}{2}\left(\mu_0 n^2 l A\right) i^2$$



N turns

➔Use formula for B field: $B = \mu_0 n i$

$$U_L = \frac{B^2}{2\mu_0} l A$$

➔Calculate energy density: $u_B = \dfrac{U_L}{V} \quad V = Al$

$$u_B = \frac{B^2}{2\mu_0}$$ B field

$$u_E = \tfrac{1}{2}\varepsilon_0 E^2$$ E field

➔This is generally true even if B is not constant

Energy Calculation Examples

➔Calculate u_B for earth field, $B = 5 \times 10^{-5}$ T

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(5\times10^{-5}\right)^2}{2\times4\pi\times10^{-7}} \simeq 10^{-3}\ \text{J/m}^3$$

➔Calculate u_B for neutron star, $B = 10^8$ T

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(10^{8}\right)^2}{2\times4\pi\times10^{-7}} \simeq 4\times10^{21}\ \text{J/m}^3$$

➔Calculate u_B for magnetar, $B = 10^{11}$ T

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(10^{11}\right)^2}{2\times4\pi\times10^{-7}} \simeq 4\times10^{27}\ \text{J/m}^3$$

$$\rho = \frac{u_B}{c^2} \simeq 4\times10^{10}\ \text{kg/m}^3$$

← Use $E = mc^2$

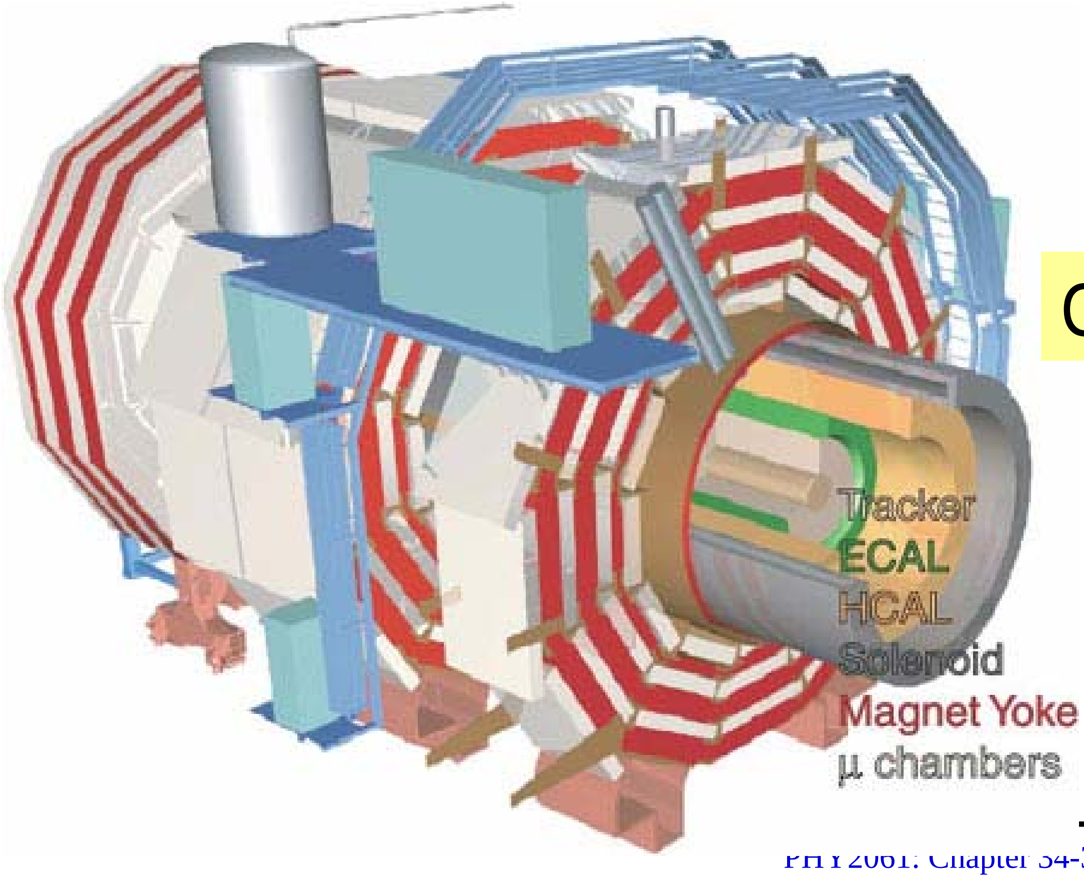
Web Sites

- ➔ Original magnetar discovery
  - http://science.nasa.gov/newhome/headlines/ast20may98_1.htm
  - http://www.firstscience.com/site/articles/solarflares.asp
- ➔ More recent magnetar discovery (Feb. 2005)
  - http://www.physorg.com/news3112.html
- ➔ Online articles on magnetars
  - http://solomon.as.utexas.edu/~duncan/magnetar.html
  - http://www.space.com/scienceastronomy/magnetar_formation_050201.html
  - http://apod.gsfc.nasa.gov/apod/ap041126.html
- ➔ Articles on neutron stars (second one has videos_
  - http://www.astro.umd.edu/~miller/nstar.html
  - http://antwrp.gsfc.nasa.gov/htmltest/rjn_bht.html

Gigajoule Magnet at CERN

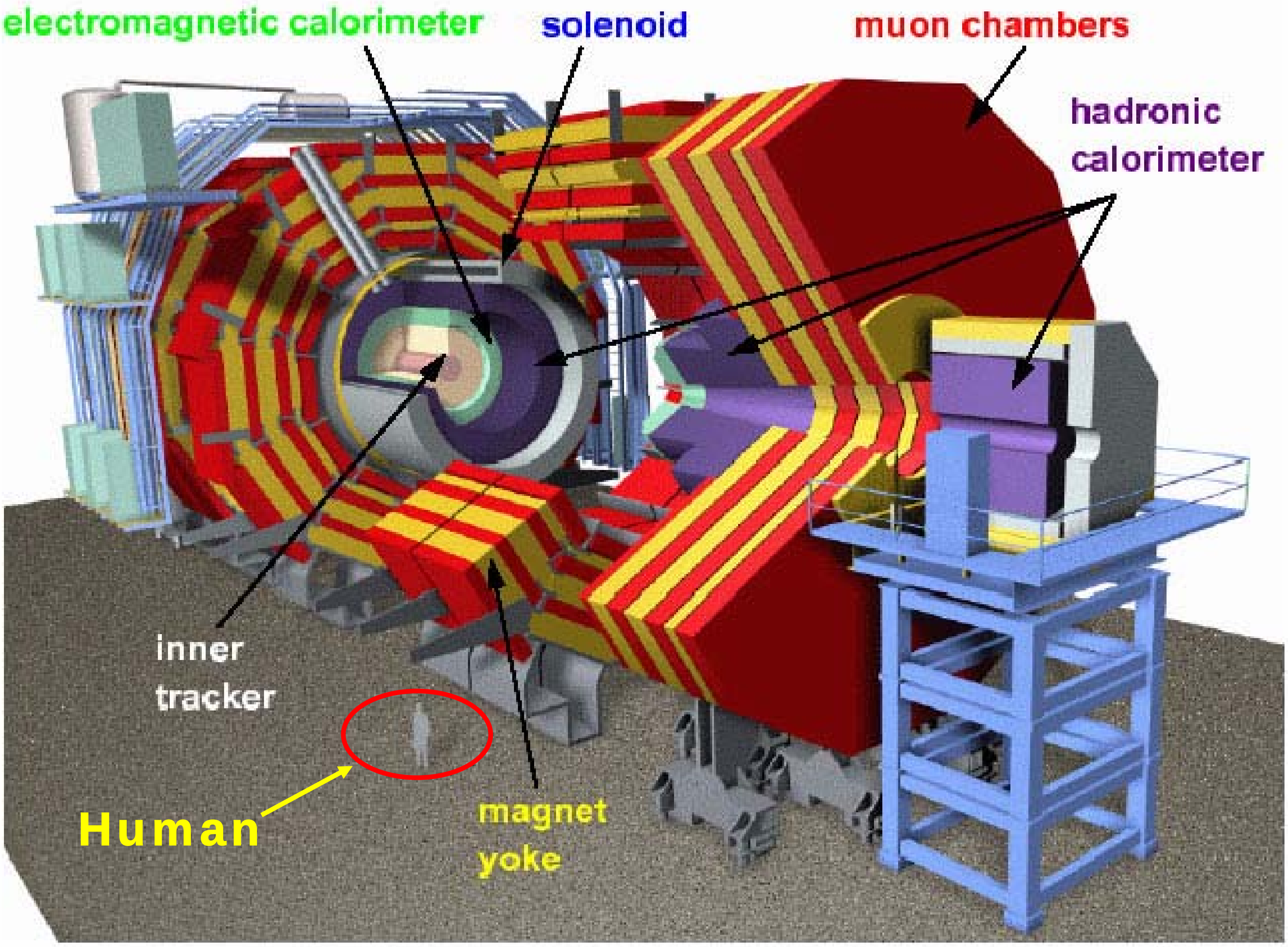
➔ CMS experiment at CERN

- p-p collisions at world's highest energy in 2007
- Hope to discover new particles, find the origin of mass and new fundamental forces



Compact Muon Solenoid

Compact Muon Solenoid



CMS Experiment Magnet

- Large central solenoid magnet to study particle production
  - B = 4T, R = 3.15 m, L = 12.5 m
  - $U_B = 2.6 \times 10^9$ J = 2.6 gigajoules!!

$$U_B = \frac{B^2}{2\mu_0} lA = \frac{4^2}{2 \times 4\pi \times 10^{-7}} \left(\pi \times 3.15^2\right)(12.5)$$

http://www.spacedaily.com/news/energy-tech-04b.html

CMS Articles and Pictures

- Home page
  - http://cmsinfo.cern.ch/Welcome.html/
- Pictures of detector
  - http://cmsinfo.cern.ch/Welcome.html/CMSdetectorInfo/CMSdetectorInfo.html
  - http://cmsinfo.cern.ch/Welcome.html/CMSmedia/CMSmedia.html
- Interesting article on solenoid, with pictures
  - http://cmsinfo.cern.ch/Welcome.html/CMSdocuments/MagnetBrochure/MagnetBrochure.pdf
- Other documents & pictures about CMS
  - http://cmsinfo.cern.ch/Welcome.html/CMSdocuments/CMSdocuments.html