

Line Spectra and Rutherford Scattering

Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.

Line Spectra

The optical radiation from materials when burned or excited in electrical discharge is *not* continuous. Rather, a series of lines at particular wavelengths are emitted, which can be separated when the light is passed through a prism or a diffraction grating. The explanation of this emission from atoms was a mystery to scientists at the end of the 19th century, requiring a better understanding of atomic structure and new physical laws.

Nevertheless, empirical laws for the wavelengths emitted from **hydrogen** were derived by Balmer, Ritz, Rydberg, and others which can be summarized in the following equation:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n^2} - \frac{1}{m^2} \right) \quad n, m = 1, 2, 3, \dots \quad m > n$$

This equation is known as the Rydberg-Ritz equation, and the Rydberg constant for hydrogen is $R_H = 1.09678 \times 10^7 \text{ m}^{-1}$.

It is quite obvious that the optical spectrum of hydrogen is quantized with quantum numbers n and m in the above equation. When n is held fixed and m is varied, a series of lines are seen:

Lyman Series (1916)	$n = 1$	$m = 2, 3, 4, \dots$	$\lambda = 121.6, 102.6, \dots \text{ nm}$	ultraviolet
Balmer Series (1885)	$n = 2$	$m = 3, 4, 5, \dots$	$\lambda = 656.5, 486.3 \dots \text{ nm}$	visible
Paschen Series (1908)	$n = 3$	$m = 4, 5, 6, \dots$	$\lambda = 1876, 1282, \dots \text{ nm}$	infrared
Brackett Series (1922)	$n = 4$	$m = 5, 6, 7, \dots$		infrared
Pfund Series (1924)	$n = 5$	$m = 6, 7, 8, \dots$		infrared

For example, to calculate the first Balmer line of hydrogen (H_α):

$$\frac{1}{\lambda_{23}} = (1.09678 \times 10^7 \text{ m}^{-1}) \left(\frac{1}{2^2} - \frac{1}{3^2} \right) = 1.523 \times 10^6 \text{ m}^{-1}$$

$$\Rightarrow \lambda_{23} = 656.5 \text{ nm} \quad (\text{which appears red})$$

The main question, however, is why are the atomic spectra quantized?

Atomic Models

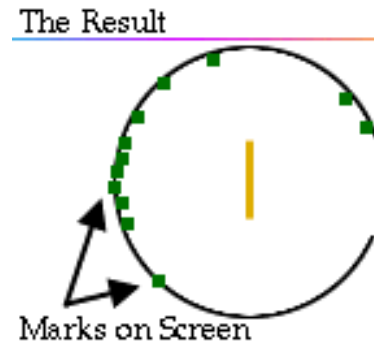
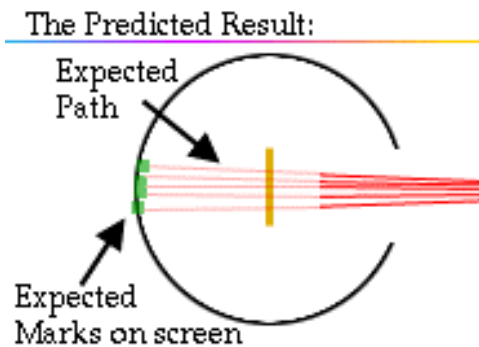
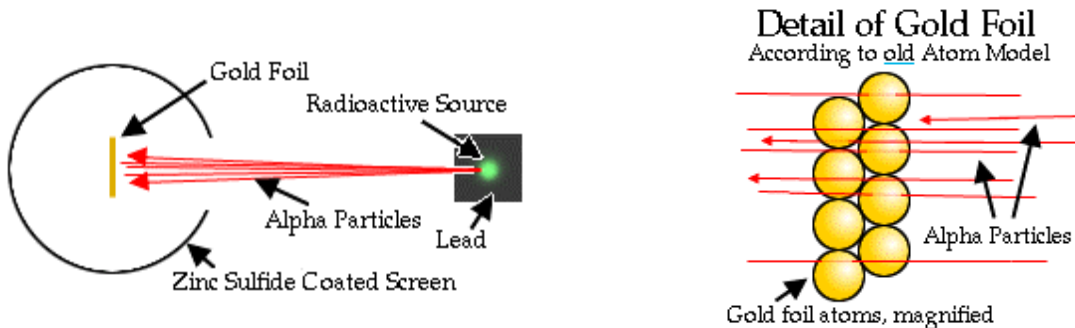
J.J. Thompson Model



Having demonstrated that the quantum of electricity, and a constituent of the atom, is the electron, Thompson proposed that electrons were distributed uniformly in a positive medium. This is sometimes referred to as the “plum-pudding” model. The trouble with this model, however, is that to explain atomic radiation you have to assume some sort of vibration of the electrons about their equilibrium positions, and there is no way to get the emission spectrum to be quantized.

Rutherford Scattering Experiments

The correct picture of atomic structure was deduced from experiments performed by Geiger and Marsden in 1909 in Rutherford’s lab in 1909. It was observed that α particles from radioactive sources sometimes back-scatter at more than 90° off thin gold foils, and this was contrary to what was expected:



The prediction was that the heavy α particles would smash into the light electrons embedded in Thompson's "plum-pudding" atom, and thus deflect only slightly. We can prove that they should not back-scatter off electrons using conservation of momentum and kinetic energy. We don't need relativity because the rest energy of an α particle is 3727 MeV, which its kinetic energy is typically less than 7 MeV.

Consider a head-on elastic collision with an electron at rest. Conservation of momentum implies:

$$M_a V_a = M_a V_a' + m_e v_e$$

Conservation of kinetic energy implies:

$$\frac{1}{2} M_a V_a^2 = \frac{1}{2} M_a V_a'^2 + \frac{1}{2} m_e v_e^2$$

Now we can square the first equation and divide by $2M_a$:

$$\frac{1}{2} M_a V_a^2 = \frac{1}{2} M_a V_a'^2 + \frac{1}{2} \frac{m_e^2}{M_a} v_e^2 + m_e V_a' v_e$$

comparing this to the second equation we see that:

$$\frac{1}{2} m v_e^2 = \frac{1}{2} \frac{m_e^2}{M_a} v_e^2 + m_e V_a' v_e$$

$$\Rightarrow V_a' = \frac{1}{2} \left(1 - \frac{m_e}{M_a} \right) v_e$$

Now we can plug this velocity into the very first equation to get:

$$v_e = \frac{2V_a}{1 + \frac{m_e}{M_a}}$$

and inserting this into the definition of V_a' yields:

$$V_a' = \frac{1 - \frac{m_e}{M_a}}{1 + \frac{m_e}{M_a}} V_a$$

For $m_e < M_a$, there is no way to get the velocity of the α particle to change sign and thus backscatter. Rather, we have:

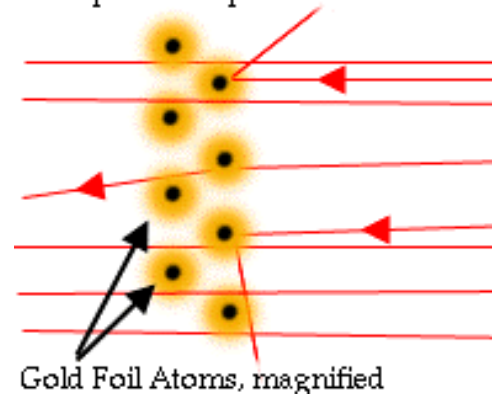
$$m_e \ll M_a$$

$$V_a' \approx V_a$$

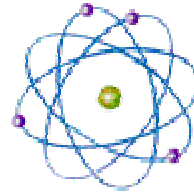
$$v_e \approx 2V_a$$

The conclusion of Rutherford is that the atom contains a small but massive positively charged **nucleus**. The α -particle backscatters because $M(\text{gold nucleus}) > M_a$.

The Positive Nucleus Theory Explains Alpha Deflection



Rutherford Model



Armed with the Rutherford model of the atom, one can calculate the scattering rate as a function of angle from the Coulomb scattering of the α -particle off a gold nucleus (i.e. the scattering of 2 charged particles using electromagnetism). The derivation of the **Rutherford Scattering** formulae will not be shown here, but suffice it to say that it was verified experimentally by Geiger and Marsden in 1913.

Thus, the Rutherford nucleus model is correct!

Since no deviation was observed from the predicted scattering rate, one can derive an upper limit on the size of the nucleus by calculating the distance of closest approach for a head-on collision. (If the α -particle had penetrated the nucleus, there would have been a deviation in the scattering rate). We set the initial kinetic energy equal to the Coulomb potential energy at closest approach (when the kinetic energy goes to zero):

$$E_k = \frac{Z_1 Z_2 e^2}{4\pi \epsilon_0 r_{\min}}$$

For α -particles from radioactive sources limited to a kinetic energy less than 10 MeV,
 $r_{\min} \approx 10^{-14}$ m.

This sets the size of the nucleus, and can be compared to the size of the atom itself, which we will show to be approximately 10^{-10} m. Thus, the size of the nucleus is 10,000 times smaller than the size of the atom, and thus the atom is mostly empty space!