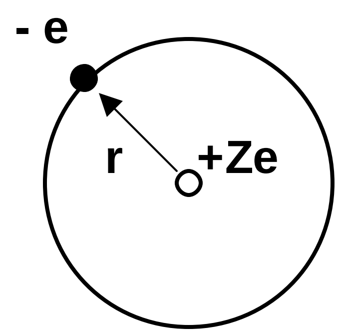
Bohr Model of the Atom

Disclaimer: These lecture notes are not meant to replace the course textbook. The content may be incomplete. Some topics may be unclear. These notes are only meant to be a study aid and a supplement to your own notes. Please report any inaccuracies to the professor.

Classical Model of the Hydrogen Atom



Consider a single electron in a circular orbit around the nucleus. Such a model is often referred to as the **planetary model**. The nucleus has charge $+Ze$, so we are attempting to describe **H, He^{+}, Li^{++}, …** The centripetal force must be balanced by the Coulomb force:

$$F_e = \frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{m_e v^2}{r}$$

This implies that the classical velocity is:

$$\boxed{v = \sqrt{\frac{Ze^2}{4\pi\epsilon_0 m_e r}}}$$

If we assume that the radius of the atom is $r = 0.5\times10^{-10}\ \text{m}$, then the velocity is $v = 2\times10^{6}\ \text{m/s}\ \approx 0.01c$. This is still non-relativistic. The centripetal acceleration is:

$$a_r = \frac{v^2}{r} = 10^{23}\ \text{m/s}^2$$

which is huge! Now a charged object which accelerates radiates electromagnetic radiation according to classical electromagnetism. The formula (from an advanced EM course) is:

$$P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{1}{m_e{}^2 c^3}\left(\frac{dp}{dt}\right)^2$$

$$F = \frac{dp}{dt} = ma \quad \Rightarrow \quad P = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0}\frac{a^2}{c^3}$$

So the power radiated at the first instant is about 6×10^{-8} watts. However, the initial kinetic energy of the electron is

$$E_K = \frac{1}{2} m_e v^2 = 2 \times 10^{-15} \text{ J}$$

So, the electron radiates all of its kinetic energy in much less than a second (about a nanosecond when solved precisely. This is about one million orbits.

Classical physics predicts that the atom should collapse in a fraction of a second!

Failures of Classical Physics

We have learned of several notable failures of classical physics in describing some phenomena:

1. **Ultraviolet catastrophe of the black-body radiation spectrum**
2. **Lack of dependence with light intensity of the photoelectric effect**
3. **Lack of quantization of atomic emission and absorption spectra**
4. **Collapse of atoms**

Thus, new laws of physics must be derived for atomic physics. The **Bohr Model** of the atom is the first step towards the new Quantum Mechanics

Bohr's Postulates

In 1913, Bohr proposed several postulates in an attempt to describe the atom.

1. Atoms have **stationary states** of definite energy. These states do not radiate any energy, as is necessary to explain the stability of atoms.

2. The emission and absorption of electromagnetic energy can only occur for transitions between stationary states. This radiation is emitted as a photon with an energy equal to the energy difference between the two stationary states:

   $$\Delta E = E_2 - E_1 = hf \qquad h = \text{Planck's constant}$$

3. The classical laws of physics apply *except* for electromagnetic radiation

4. The angular momentum of the system is **quantized**:

   $$L = n\frac{h}{2p} \equiv n\hbar \qquad n = 1,2,3,\ldots$$

   n is the **principal quantum number** and $\hbar$ ("h-bar") is Planck's constant divided by $2p$

The Bohr Model of the Atom

Let’s now explore the consequence of the quantization of angular momentum. The angular momentum is a vector quantity:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

The magnitude of ***L*** for circular motion is:

$$L = |\mathbf{L}| = |\mathbf{r} \times \mathbf{p}| = mvr$$

Bohr’s 4th postulate tells us:

$$L = m_e v r = n\hbar$$
$$\Rightarrow v = \frac{n\hbar}{m_e r}$$

We can equate this velocity to the classical derivation for an electron in a circular orbit:

$$v = \sqrt{\frac{Ze^2}{4\pi\epsilon_0 m_e r}} = \frac{n\hbar}{m_e r}$$
$$\Rightarrow \frac{Ze^2}{4\pi\epsilon_0 m_e r} = \frac{n^2\hbar^2}{m_e^2 r^2}$$

Thus,

$$r_n = n^2 \frac{4\pi\epsilon_0 \hbar^2}{Ze^2 m_e} \equiv n^2 \frac{a_0}{Z}$$
$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e} = 0.53 \times 10^{-10}\text{ m} \quad \text{The Bohr radius}$$

In other words, quantization of angular momentum implies that only particular circular orbits are allowed in the atom. The **Ground State** is the state when $n = 1$, the lowest level and smallest radius. This derived size of the atom is about what was expected at the time from indirect evidence.

The energy for an electron in circular orbit about a nucleus can be calculated classically:

$$V = \frac{-Ze^2}{4pe_0 r} \quad \text{is the potential energy}$$

$$E_K = \frac{1}{2}mv^2 = \frac{r}{2}\frac{mv^2}{r} = \frac{Ze^2}{8pe_0 r} \quad \text{is the kinetic energy}$$

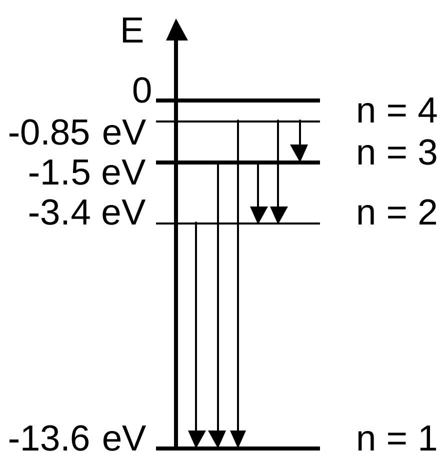
$$\Rightarrow E = -\frac{Ze^2}{8pe_0 r} \quad \text{is the total energy}$$

It is negative to denote that the system is bound (zero energy corresponds to when the electron and nucleus are infinitely separated). Plugging in for radius:

$$E_n = -\frac{Z^2 E_0}{n^2}$$

$$E_0 = \frac{e^2}{8pe_0 a_0} = \frac{m_e e^4}{2\hbar^2 (4pe_0)^2} = 13.6 \text{ eV}$$

Thus, the energy of the atom is quantized. The atom can exist only in certain states of definite energy. This quantization is what is necessary to explain the discrete nature of atomic spectral lines. We can draw an energy level diagram:



Consider a transition between 2 energy levels (labeled *u* and *l*). Then by Bohr's 2nd postulate, the radiated energy is the energy difference between the two levels:

$$\Delta E = E_\ell - E_u = hf$$
$$= -Z^2 E_0 \left(\frac{1}{n_\ell^{\,2}} - \frac{1}{n_u^{\,2}} \right)$$

$$\boxed{\frac{1}{\lambda} = \frac{f}{c} = \frac{Z^2 E_0}{hc} \left(\frac{1}{n_u^{\,2}} - \frac{1}{n_\ell^{\,2}} \right)}$$

Derived the Rydberg-Ritz Equation !

The predicted Rydberg constant is

$$\frac{E_0}{hc} \equiv R_\infty = \frac{m_e e^4}{4\pi c \hbar^3 (4\pi\varepsilon_0)^2} = 1.09737 \times 10^7 \text{ m}^{-1}$$

This is slightly different than the measured value of $1.09678 \times 10^7 \text{ m}^{-1}$, but that is because we assumed that the nucleus is infinitely massive. We need to replace the electron mass in the above equation with the **reduced mass:**

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_N}$$

In other words, the electron and nucleus orbit each other around a common center-of-mass. Skipping the details, the net effect is to use the effective mass μ above. The effect is minimal for heavy nuclei, but measurable for light nuclei.

We now can understand the line spectra from hydrogen (and hydrogen-like ions) in terms of transitions among quantized orbits

- Lyman Series $\Rightarrow$ transitions to $n = 1$, the **ground state**
- Balmer Series $\Rightarrow$ transitions to $n = 2$, the first excited state
- Paschen Series $\Rightarrow$ transitions to $n = 3$, the second excited state