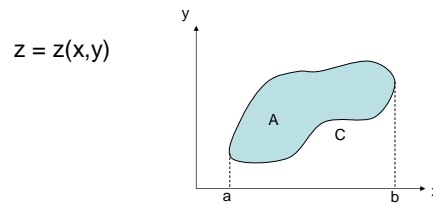


The fundamental theorem of calculus: $f = f(x)$

$$\int_{x_1}^{x_2} \left(\frac{df}{dx} \right) dx = \int_{x_1}^{x_2} df = f(x_2) - f(x_1)$$

In 1-D only one way to integrate this.

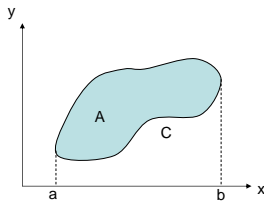
In 2-D infinite paths possible



For an exact differential

$$\oint_C dz = \int_a^b dz + \int_b^a dz = \int_a^b dz - \int_a^b dz = 0$$

Green's theorem



$$\oint_C Mdx + Ndy = \iint_A \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx dy$$

In general, the differential $dz = Mdx + Ndy$ is 'exact' if:

$$1. \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$2. \oint_C dz = 0$$

$$3. \int_a^b dz \quad \text{Is path independent}$$

e.g. state variables, $P = P(V,T)$

Inexact differentials do not satisfy these conditions

$$\text{e.g. } \vec{dz} = ydx - xdy$$

Sum of two inexact differentials can be exact.

Heat: Spontaneous flow of energy caused by the temperature difference between two objects

Work: Any other kind of energy transfer

Internal Energy: Total energy in a system

First law of thermodynamics (in words)

Increase in internal energy = Heat added + work done on the system

$$\Delta U = \Delta Q + \Delta W$$

(ΔW is the work done ON the system)