Derivation of G for a van der Waals gas using reduced variables: $\tilde{V}, \tilde{T}, \tilde{p}$

$$p = \frac{RT}{V - b} - \frac{a}{V^2} = -\left(\frac{\partial F}{\partial V}\right)_T$$

$$\Rightarrow \quad F = -RT \ln(V-b) - \frac{a}{V} + f(T)$$

$$G = F + pV = -RT\ln(V-b) - \frac{a}{V} + pV + f(T)$$

$$p_c = \frac{a}{27b^2}, \quad V_c = 3b, \quad T_c = \frac{8a}{27Rb}$$

$$G = -R\tilde{T}T_c \ln(\tilde{V}V_c - b) - \frac{a}{\tilde{V}V_c} + \tilde{p}\,p_c\tilde{V}V_c + f(\tilde{T})$$

$$= -\cancel{R}\tilde{T}\frac{8a}{27\cancel{R}b}\ln(\tilde{V}.3b - b) - \frac{a}{3b\tilde{V}} + \tilde{p}.\frac{a}{\cancel{27}b^{\cancel{2}}/9}.\tilde{V}.\cancel{3b}$$
$$+ f(\tilde{T})$$

$$= +\frac{a}{3b}\left[-\tilde{T}\cdot\frac{8}{\cancel{27}9}\ln(3\tilde{V}-1)b - \frac{1}{\cancel{3}\tilde{V}} + \frac{\tilde{p}\tilde{V}}{\cancel{9}3}\right] + f(\tilde{T})$$

$$= \frac{a}{3b}\left[-\frac{8}{9}\tilde{T}\ln(3\tilde{V}-1) - \frac{1}{\tilde{V}} + \frac{\tilde{p}\tilde{V}}{3}\right] + g(\tilde{T}).$$

Using reduced VdW equation

$$\boxed{\tilde{p} = \frac{8\tilde{T}}{3\tilde{V}-1} - \frac{3}{\tilde{V}^2} \Rightarrow \frac{\tilde{p}\tilde{V}}{3} = \frac{8}{3}\frac{\tilde{T}\tilde{V}}{(3\tilde{V}-1)} - \frac{1}{\tilde{V}}}$$

$$\Rightarrow G = \frac{a}{3b}\left[-\frac{8}{9}\tilde{T}\ln(3\tilde{V}-1) - \frac{2}{\tilde{V}} + \frac{8}{3}\frac{\tilde{T}\tilde{V}}{(3\tilde{V}-1)}\right] + g(\tilde{T})$$