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Bonus Question 4

Due Friday, Oct. 4

The Schrodinger equation describing the hydrogen atom involves the radially symmetric potential $U(r) = -\alpha\hbar c / r$. To solve it we write the equation in spherical coordinates and use separation of variables to create separate equations for *r*, θ and ϕ, yielding the time independent wavefunction $\psi(r,\theta,\phi) = R_{nl}(r) Y_l^m(\theta,\phi)$, where $R_{nl}(r)$ depends on the integers *n* and *l*, with $n > 0$ and $0 \le l < n$ is the eigenvalue describing the total angular momentum. $Y_l^m(\theta,\phi) \propto P_l^m(\cos\theta) e^{im\phi}$ is a spherical harmonic, where $P_l^m(\cos\theta)$ is an associated Legendre polynomial, *l* was described previously and *m* is the *z* component of angular momentum ($-l \le m \le l$). Only the radial wavefunction $R_{nl}(r)$ depends on the potential. The functions are normalized as follows (note the r^2 term)

$$\int_0^\infty R_{nl}^2(r) r^2 \, dr = 1$$

$$\int \left| Y_l^m \right|^2 d\Omega = 2\pi \int_{-1}^{1} P_l^m(\cos\theta)^2 \, d\cos\theta = 1$$

1. (3 pts) Write down the radial Schrodinger equation for this potential. You can google the answer, best described on Wikipedia.
2. (3 pts) Write down the radial solutions R_{10}, R_{20}, R_{21} and R_{30}.
3. (4 pts) Plot the probability distributions for the solutions in (2), e.g. $r^2 \left| R_{nl}(r) \right|^2$.