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PHZ4390
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Homework 2

Due Monday, Sep. 9, 2013 (see me for help or hints on any problem)

1. A particle of mass m and total energy $E = 3m$ collides with a particle of mass $4m$ at rest. Let quantities with subscript "S" (E_S, p_S, v_S and γ_S) describe the motion of the entire two-particle system measured in the lab frame.
   a. (4 pts) What is $\sqrt{s}$ (invariant mass of system or total energy measured in CM)?
   b. (3 pts) What are E_S, p_S, γ_S and v_S?
   c. (3 pts) What should be the initial energy E to make the invariant mass equal to $11m$?

2. A π^0 (mass m) having total energy E in the lab frame decays into two photons. For the following problems, express the answers using E and m, except as noted.
   a. (2 pts) What is the momentum of each of the photons in the π^0 rest frame?
   b. (2 pts) What is the largest transverse momentum of the photon in the lab frame? Transverse momentum is the momentum perpendicular to the parent's direction of motion. (Hint: how does transverse momentum transform under an LT along the direction of the parent?)
   c. (3 pts) If one photon is emitted at $\theta^* = 90°$ in the π^0 center of mass relative to the direction of the π^0, what is the angle θ of the photon in the lab frame?
   d. (3 pts) Now assume that the moving π^0 emits two photons in the forward and backward direction. What are the energies and directions of the two photons as a function of E and m? You can simplify the answer by assuming $m \ll E$. Evaluate the photon energies in the lab frame for $m = m_{\pi^0} = 0.135$ GeV, $E = 5$ GeV and 50 GeV.

3. A nucleus of mass M at rest decays into a nucleus of mass $M - \Delta$ plus a phton.
   a. (5 pts) What is the energy of the photon in terms of M and Δ? Use 4-vectors to solve this exactly.
   b. (5 pts) What the energy of the photon using Newtonian physics with conservation of momentum and energy (kinetic energies add up to Δ in natural units)? Assume that $E = p$ for a photon (natural units). Expand as necessary to find the fractional difference from (a) when Δ is small. What is the fractional difference when $\Delta = 0.01M$?

4. (10 pts) Find the threshold energies in GeV for the reactions $p + p \to p + p + K^+ + K^-$ and $\pi^- + p \to p + W^-$, where the target proton is at rest. Use the PDG values for all masses, noting that pions and kaons are mesons and W bosons are "gauge particles".

5. For the reaction $A+B \to C+D$ we define the "Mandelstam invariants", s, t and u to be $s \equiv (p_A + p_B)^2 = E_{CM}^2$, $t \equiv (p_A - p_C)^2$ and $u \equiv (p_A - p_D)^2$, all of which are Lorentz invariant. As discussed in class, the invariant s is just the center of mass energy squared. The invariants t and u can be thought of as being the momentum transfer from A to C and from A to D, respectively.

   a. (5 pts) Prove that $s+t+u = m_A^2 + m_B^2 + m_C^2 + m_D^2$.

   b. (5 pts) When all masses are the same, show that the invariants, when measured in the CM frame, are $s = 4\left(\mathbf{p}^2 + m^2\right)$, $t = -2\mathbf{p}^2(1-\cos\theta)$ and $u = -2\mathbf{p}^2(1+\cos\theta)$ where $\mathbf{p}$ is the 3-momentum of A and θ is the angle of C relative to A.

6. Differential equations of the form $dy/dt = f(t,y)$ must be solved numerically when no analytical method exists. A multi-step "predictor-corrector" method is often used, in which a set evenly spaced points determined by another method are used by an extrapolation formula (predictor) to calculate the next point which is then corrected by another formula. For example, in the 4th order Adams-Bashforth method (accurate to h^4, where h is the time step), the predictor step has the form (with previously determined values y_0, y_1, y_2, y_3)

$$y_4 = y_3 + h\sum_{j=0}^{3} c_N^j f\left(t_j, y_i\right) = y_3 + h\left[-\frac{3}{8}f\left(t_0,y_0\right) + \frac{37}{24}f\left(t_1,y_1\right) - \frac{59}{24}f\left(t_2,y_2\right) + \frac{55}{24}f\left(t_3,y_3\right)\right]$$

where the time values are $t_i = t_0 + ih$. The 4th order coefficients c_N^j are given by ($N = 4$)

$$c_N^{N-j-1} = \frac{(-1)^j}{j!\left(N-j-1\right)!}\int_0^1 \prod_{\substack{n=0 \\ n\neq j}}^{N-1} \left(x+n\right)dx$$

You can verify that this works for $N = 4$. See the discussion and formulas in http://en.wikipedia.org/wiki/Linear_multistep_method.

(10 pts bonus) Find the *exact* coefficients for the $N = 6$ method, which is accurate to h^6. You can use wxMaxima, Sage, SymPy, Mathematica or Maple, but you must print out the program. See me for help.