Paul Avery
PHZ4390
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Homework 6

Due Friday Oct. 11

1. (5 pts) Round steel marbles packed as tightly as theoretically possible are poured into an empty form to create a wall 6 m thick. How many nuclear interaction lengths of material are in the wall thickness?
2. (5 pts) An experiment searching for proton decay through the GUT mediated mode $p \to \pi^0 + e^+$ is carried out using a large tank of water (400,000 tonnes) as the proton source. How many protons would decay per year if the proton lifetime is 10^{33} years and the predicted branching fraction to $\pi^0 + e^+$ is 30%? Assume that any proton in the water is eligible to decay. (Bound neutrons would have an almost identical decay process as protons, i.e., $n \to \pi^- + e^+$, but we are not considering those decays here.)
3. The flux of antineutrinos produced by the reactor in the Cowan-Reines experiment was approximately $10^{13}\ /\, \mathrm{cm}^2\ /\, \mathrm{s}$. They looked for the reaction $\bar{\nu}_e + p \to e^+ + n$ in a tank of water.
   a. (5 pt) What is the luminosity ($\mathrm{cm}^{-2}\ \mathrm{sec}^{-1}$) for the antineutrino flux striking a single target particle?
   b. (5 pts) If the cross section for the reaction is $1.0 \times 10^{-43}\ \mathrm{cm}^2$ and the flux all passes through the tank, what is the minimum mass of water (in kg) necessary to get an interaction rate of ~3 events / hour? Assume that any proton in the mixture (but no neutrons) can interact with an antineutrino.
4. The LHC has a total circumference of 26,659 m. It is planned to carry 2 counterrotating proton beams of 7 TeV apiece, with the current in each beam 0.54 A. Dipole magnets cover only a fraction ε_M of the full circumference of the LHC, with the rest filled by other equipment and straight sections, free from any bending. Thus the particles complete a turn in a true distance $2\pi r \varepsilon_M$ and the equivalent "bending radius" is $r\varepsilon_M$. The bending radius of the LHC dipole magnets is 2804 m.
   a. (2 pts) What fraction of the circumference is filled with dipole magnets
   b. (3 pts) What magnetic field is needed for each dipole if the proton energy in each beam is 7 TeV?
   c. (5 pts) How much energy (in MeV) is lost by each proton per turn? How much energy would an electron lose per turn at the same energy?
   d. (2 pts) How much RF power must be supplied to both proton beams to keep the energy in the ring constant?

5. A particle X decays into three different channels with the following branching fractions:

$$B\left(X \to K^+K^-\right) = 0.01$$

$$B\left(X \to K^+\pi^-\right) = 0.09$$

$$B\left(X \to \pi^+\pi^-\right) = 0.90$$

An experiment is set up to detect *X*-particle decays. One graduate student looks at a distribution of invariant masses of two kaons in the K^+K^- decay channel and finds that the distribution has a peak that she can fit with the following formula (mass m_{KK} is in GeV):

$$\frac{dN_{KK}}{dm_{KK}} = \frac{4500}{(m_{KK} - 6.5)^2 + 0.16}$$

a. (1 pt) Approximately how many total K^+K^- decays are measured?

b. (1 pt) What is the width (GeV) and lifetime of the *X* particle (sec)?

c. (1 pt) What are the partial widths Γ_{KK}, $\Gamma_{K\pi}$ and $\Gamma_{\pi\pi}$ in GeV?

d. (1 pt) Another student studies a distribution of $\pi^+\pi^-$ invariant masses. What is your prediction for the function that describes this measured distribution, assuming that the final state is measured with three times the efficiency as that of the K^+K^- state?

6. The CMS detector has a cylindrical tracking chamber of radius 1.1 m that sits inside a solenoidal magnetic field of 3.8 T. The interaction point is close to the geometric center of the chamber and the tracking chambers are arranged in cylinders ranging in radius from 6 cm to 950 cm. Charged particles orbit in helices, where the circular motion takes place in the *x–y* plane perpendicular to the magnetic field while the particle moves along *z* with constant momentum. A precise determination of a track's momentum and direction is made by measuring the position of the "hits" it leaves in the silicon layers it passes through and then fitting the hits to a helix. The momentum and its sign are measured from the radius of curvature.

a. (5 pts bonus) What is the radius of curvature (in m) of a 25 GeV π^- emitted 30° relative to the beam direction?

b. (5 pts bonus) What is the maximum momentum (in GeV) of a particle that is produced at the interaction point in the *x-y* plane and is fully contained within the measurement region of tracking chamber? Draw the figure to see how easy of a question this is.