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PHZ4390
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# Homework 8

## Due Wednesday, Oct. 30

1. Calculate the mean and error of the following quantities (errors are all independent).
   a. (3 pts) $(4.5 \pm 0.2) + (2.5 \pm 0.1) - (3.2 \pm 0.4)$
   b. (3 pts) $(4.5 \pm 0.2)(2.5 \pm 0.1)(3.2 \pm 0.4)$
   c. (3 pts) $(4.5 \pm 0.2)(2.5 \pm 0.1) / (3.2 \pm 0.4)$
   d. (3 pts) $(4.5 \pm 0.2)^2 (2.5 \pm 0.1)^5 / (3.2 \pm 0.2)^4$
   e. (3 pts) $(3.9 \pm 0.5)^{1/4}$
2. (5 pts) Show explicitly that the mean of the Poisson distribution $f(n) = \mu^n e^{-\mu} / n!$ is $\mu$ and that the standard deviation is $\sigma = \sqrt{\mu}$.
3. (4 pts) Calculate the mean value and error of the ratio $a / a + b$, where $a = 9.2 \pm 0.8$ and $b = 4.6 \pm 0.8$. Hint: as in the note on statistics, expand the function using derivatives to first order in $\delta_a$ and $\delta_b$ and calculate the error on the ratio in terms of the individual (independent) errors.
4. The reaction $e^+e^- \to \mu^+\mu^-$ is measured in a $e^+e^-$ collider. The differential cross section (in natural units) is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(1 + \cos^2\theta\right)$$

   where $s$ is the center of mass energy squared and $\theta$ is the angle between the electron (moving along the +$z$ direction) and the outgoing $\mu^-$. The detector can only measure muon angles more than 30° from the beam (measured from either beam) because detector components near the beam interfere with the bending magnets and other accelerator elements. Assume that the center of mass energy is 10.55 GeV.
   a. (5 pts) What is the total cross section in nb (nanobarns)?
   b. (5 pts) What is the overall efficiency of detecting both muons? The efficiency is the fraction of events that can be measured. If muons could be measured at all angles the efficiency would be 1.0.
   c. (5 pts) How many years (1 year = $10^7$ sec to include down times) are required to record 400,000 events if the average luminosity is $\mathcal{L} = 1.5 \times 10^{31}$ cm$^{-2}$s$^{-1}$?

5. A CMS experimenter looks for the process $pp \to Z' + X, Z' \to \mu^+\mu^-$ at the LHC, where $Z'$ is a very heavy (800 – 2000 GeV) boson with properties similar to the *Z*. A theoretical model predicts that the cross section at a given mass (including the decay branching fraction) is 2.5 fb and the integrated luminosity is 12 $\text{fb}^{-1}$/year. The detection efficiency is determined from simulation to be 25%.

    a. (2 pts) If the $Z'$ exists and behaves as predicted by the model, what is the average number of events of this type that should be measured per year?

    b. (4 pts) Suppose the experiment is run and the background is negligible. If the theoretical model was correct, what is the probability of seeing 3 or fewer events after one year, according to the theoretical prediction?

    c. (4 pts) Refer to (b) again. What is the 90% upper limit of the cross section for this process if no events are measured? Is this consistent with the theoretical model? Why or why not?

6. An experiment has a tracker (which measures momentum) followed by an electromagnetic calorimeter (which measures energy). The tracker momentum resolution $\sigma_p$ is $\sigma_p / p = 0.0025 + 0.0015\,p$ for all tracks and the EM calorimeter energy resolution $\sigma_E$ is $\sigma_E / E = 0.0060 + 0.15 / \sqrt{E}$ for electrons and photons. Momentum and energy are measured in GeV.

    a. (2 pts) What is the tracker measurement error on *p* at 100 GeV?

    b. (5 pts) A track is identified as an electron when it is measured by both devices and satisfies $E/p = 1$ within experimental errors (the ratio is much less than 1 for muons or hadrons). What is the error of the $E/p$ measurement for 100 GeV electrons?

    c. (5 pts) If both devices are used, what is combined energy resolution of a 40 GeV $e^-$ emitted in the x-y plane? Recall from the note on statistics how to combine two measurements with errors.

    d. (5 pts bonus) If both devices are used, at what momentum (assuming in the x-y plane) is the combined fractional energy resolution of an electron the worst? What is the fractional error at that energy? This is a problem that is best done by using wxMaxima by either plotting the combined relative resolution or evaluating it for different energies to find where it is maximized.