Paul Avery
PHZ4390
Oct. 30, 2013

Homework 9

Due Wednesday, Nov. 6

1. Resonant production involves a process of the form $ab \to X \to 12$, where X is an intermediate particle with mass m_X and width Γ_X. The cross section involves two matrix elements and a propagator: $\mathfrak{M} \propto \mathfrak{M}(ab \to X) \times \text{propagator} \times \mathfrak{M}(X \to 12)$. Since matrix elements normally satisfy time invariance, we write this as $\mathfrak{M} \propto \mathfrak{M}(X \to ab) \times \text{propagator} \times \mathfrak{M}(X \to 12)$. We have to account for initial and final state spins and the spin of X. Using this and other QM rules, when we square the matrix element we get the following cross section vs E_{cm}:

$$\sigma(ab \to X \to 12) = \frac{2S_X + 1}{(2S_a + 1)(2S_b + 1)} \frac{16\pi}{s} \frac{\Gamma_X^2 / 4}{(E_{\text{cm}} - m_X)^2 + \Gamma_X^2 / 4} B_{ab} B_{12}$$

where $s = E_{\text{cm}}^2$, S_a, S_b and S_X are the spins of the incoming particles and X, respectively, and $B_{ab} = \Gamma_{ab} / \Gamma_X$ and $B_{12} = \Gamma_{12} / \Gamma_X$ are the branching fractions of X to $a + b$ and $1 + 2$, respectively. For e^+e^- annihilation, the intermediate state X always has spin 1, the same as the photon, leading to the simplified equation

$$\sigma(e^+e^- \to X \to 12) = \frac{12\pi}{s} \frac{\Gamma_X^2 / 4}{(E_{\text{cm}} - m_X)^2 + \Gamma_X^2 / 4} B_{ee} B_{12}$$

   a. (5 pts) Evaluate $\sigma(e^+e^- \to \mu^+\mu^-)$ in nb at the peak of the J/ψ and ψ(2S). Calculate the ratio of these cross sections to $\sigma(e^+e^- \to \mu^+\mu^-)$ calculated without the resonance.

   b. (5 pts) Evaluate $\sigma(e^+e^- \to \mu^+\mu^-)$ in nb at the peak of the ϒ(1S), ϒ(2S), and ϒ(3S). Calculate the ratio of these cross sections to $\sigma(e^+e^- \to \mu^+\mu^-)$ calculated without the resonance.

2. Refer to the previous problem. In real e^+e^- experiments, the beam energies are not exactly constant but fluctuate slightly, leading to an energy spread of the center of mass energy (a few MeV) that is much larger than the resonance widths of the lightest ψ and ϒ states (20 keV to 300 keV). Thus one cannot directly measure the true cross section on the resonance peak. However, the integral $\int \sigma \, dE_{\text{cm}}$ measured experimentally is the same as the true value.

    a. (5 pts) Show that measurements of $\int \sigma_{\text{tot}} \, dE_{\text{cm}}$ and B_{ee} (using $B_{12} = 1$ to get the total cross section σ_{tot}) gives a measurement of Γ_X. This method was used to actually determine the widths of the ψ and ϒ resonances. Note that, for narrow resonances, $s = E_{\text{cm}}^2$ is approximately constant and the integral is effectively from $-\infty$ to $+\infty$.

    b. (3 pts) Using PDG data, calculate $\int \sigma_{\text{tot}} \, dE_{\text{cm}}$ (in nb-GeV) for the J/ψ and ϒ(1S) resonances.

3. (5 pts) Why do experimental plots of the ψ and ϒ resonances show asymmetric peak shapes which are higher above the resonance than below the resonance? You can google the answer but be sure you understand it.

4. Look up the parameters of the ψ and ϒ resonances in the PDG.

    a. (3 pts) Why are the widths of the J/ψ, ψ(2S), ϒ(1S), ϒ(2S) and ϒ(3S) resonances so small, i.e., why are they so “narrow”?

    b. (2 pts) Why is the width of the ϒ(4S) resonance so much larger than the widths of the lighter three ϒ resonances?

    c. (3 pts) Why is the ϒ(4S) width still so narrow compared with hadrons like the ρ, K^* and Δ?

5. (5 pts) What is the value of R predicted to be for E_{cm} = 6 GeV, 15 GeV, 20 GeV and 200 GeV? Include the α_s correction from QCD that we discussed in class. A discussion and a plot showing the dependence of α_s vs energy can be found in M&S pp. 183-187.

6. (5 pts) From the information in the PDG (use the summary tables for the gauge bosons), calculate R at the peak of the Z^0.

7. (5 pts) What do the masses of the charmonium and bottomium systems help us measure? What additional information does the bottomonium system provide?