A high energy, nearly monoenergetic photon beam can be created by “backscattering” a laser beam from a high energy electron beam. Basically, each laser photon has a collision with an oncoming electron in which a fraction of the electron’s energy is transferred to the photon.

1. (7 pts) Using 4-momentum conservation, show that the outgoing photon has an energy approximately given by

\[ E_\gamma = \frac{E_e}{1 + \frac{m_e^2}{4\omega E_e}} \]

where \( \omega \) is the laser photon energy (energy \( \sim \) eV), \( E_e \) is the initial electron energy (multi-GeV) and \( m_e \) is the electron mass. To do this problem, note that it is best to express the 4-momentum of the outgoing electron in terms of the other particles, then square it (getting rid of all its properties except for its mass). It’s also a good idea to write down the 4-vectors of the other three particles (all move along a single line but the directions are important). Note that you must be careful when evaluating \( E_e - p_e \) because the electron mass cannot be neglected there.

2. (3 pts) Plot the fractional energy \( E_\gamma / E_e \) as a function of \( \omega \), assuming the electron energy in the oncoming beam is 50 GeV. For what laser wavelength does the outgoing photon have 50% of the electron energy? Use wxMaxima or another tool if that’s more convenient. Which tool did you choose?