An Introduction to Particle Accelerators

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Introduction
Particle accelerators for HEP

• **LHC**: the world biggest accelerator, both in energy and size (as big as LEP)
  - Grand start-up and perfect functioning at injection energy in September 2008
  - First collisions expected in 2009
Particle accelerators for HEP

The next big thing. After LHC, a Linear Collider of over 30 km length, will probably be needed (why?)
Medical applications

• Therapy
  – The last decades: electron accelerators (converted to X-ray via a target) are used very successfully for cancer therapy)
  – Today's research: proton accelerators instead (hadron therapy): energy deposition can be controlled better, but huge technical challenges

• Imaging
  – Isotope production for PET scanners
Advantages of proton / ion-therapy

Protons and ions spare healthy tissues

200 MeV - 1 nA protons

4800 MeV - 0.1 nA carbon ions which can control radioresistant tumours

charged hadron beam that loses energy in matter

tumour target

(Dose Distribution Curve)

Absorbed Relative Dose (%) 100 80 60 40 20 0
linac proton light ion (carbon)

Body surface 5 10 Depth from body surface (cm)

http://global.mitsubishielectric.com/bui/particlebeam/index_b.html

X rays protons or carbon ions

( Slide borrowed from U. Amaldi )
Proton therapy accelerator centre

HIBAC in Chiba

( Slide borrowed from U. Amaldi )

What is all this? Follow the lectures... :)

Synchrotron Light Sources

- the last two decades, enormous increase in the use of synchrony radiation, emitted from particle accelerators
- Can produce very intense light (radiation), at a wide range of frequencies (visible or not)
- Useful in a wide range of scientific applications
Basic concepts
An accelerator

- Structures in which the particles will move
- Structures to accelerate the particles
- Structures to steer the particles
- Structures to measure the particles
Lorentz equation

• The two main tasks of an accelerator
  – Increase the particle energy
  – Change the particle direction (follow a given trajectory, focusing)

• Lorentz equation:
  \[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = q\vec{E} + q\vec{v} \times \vec{B} = \vec{F}_E + \vec{F}_B \]

• \( \vec{F}_B \) does no work on the particle
  – *Only \( \vec{F}_E \) can increase the particle energy*

• \( \vec{F}_E \) or \( \vec{F}_B \) for deflection? \( \vec{v} \) \( c \) \( \vec{B} \) Magnetic field of 1 T (feasible) same bending power as an electric field of \( 3 \times 10^8 \) V/m (NOT feasible)
  – *\( \vec{F}_B \) is by far the most effective in order to change the particle direction*
Acceleration techniques: DC field

- The simplest acceleration method: DC voltage
- Energy kick: $DE = qV$
- Can accelerate particles over many gaps: electrostatic accelerator
- Problem: breakdown voltage at $\sim 10$MV
- DC field still used at start of injector chain
Acceleration techniques: RF field

- Oscillating RF (radio-frequency) field

- "Widerøe accelerator", after the pioneering work of the Norwegian Rolf Widerøe (brother of the aviator Viggo Widerøe)

- Particle must sees the field only when the field is in the accelerating direction
  - Requires the synchronism condition to hold: $T_{\text{particle}} = \frac{1}{2} T_{\text{RF}}$
  
  \[ L = \frac{1}{2} v T \]

- Problem: high power loss due to radiation
Acceleration techniques: RF cavities

- Electromagnetic power is stored in a resonant volume instead of being radiated
- RF power feed into cavity, originating from RF power generators, like Klystrons
- RF power oscillating (from magnetic to electric energy), at the desired frequency
- RF cavities requires bunched beams (as opposed to coasting beams)
  - particles located in bunches separated in space
From pill-box to real cavities

- pill-box
- openings for beam passage
- nose cones for improving $R_s$
- spherical body for improving $Q$

LHC cavity module

ILC cavity

(from A. Chao)
Why circular accelerators?

- Technological limit on the electrical field in an RF cavity (breakdown)
- Gives a limited $E$ per distance
- Circular accelerators, in order to re-use the same RF cavity
- This requires a bending field $F_B$ in order to follow a circular trajectory (later slide)
The synchrotron

- Acceleration is performed by RF cavities
- (Piecewise) circular motion is ensured by a guide field $F_B$
- $F_B$: Bending magnets with a homogenous field
- In the arc section: $F_B = m \frac{v^2}{\rho} \Rightarrow \frac{1}{\rho} = \frac{qB}{p} \Leftrightarrow \frac{1}{\rho}[m^{-1}] \approx 0.3 \frac{B[T]}{p[GeV/c]$
- RF frequency must stay locked to the revolution frequency of a particle (later slide)
- Synchrotrons are used for most HEP experiments (LHC, Tevatron, HERA, LEP, SPS, PS) as well as, as the name tells, in Synchrotron Light Sources (e.g. ESRF)
Digression: other accelerator types

- **Cyclotron:**
  - constant B field
  - constant RF field in the gap increases energy
  - radius increases proportionally to energy
  - limit: relativistic energy, RF phase out of synch
  - In some respects simpler than the synchrotron, and often used as medical accelerators

- **Synchro-cyclotron**
  - Cyclotron with varying RF phase

- **Betatron**
  - Acceleration induced by time-varying magnetic field

- *The synchrotron will be the only circular accelerator discussed in this course*
Digression: other accelerator types

**Linear accelerators for linear colliders**

- will be covered in lecture about *linear colliders* at CERN
Particle motion

We separate the particle motion into:

- **longitudinal motion**: motion tangential to the reference trajectory along the accelerator structure, $u_s$

- **transverse motion**: degrees of freedom orthogonal to the reference trajectory, $u_x, u_y$

$u_s, u_x, u_y$ are unit vectors in a **moving coordinate system**, following the particle
Longitudinal dynamics
for a synchrotron

Longitudinal Dynamics: degrees of freedom tangential to the reference trajectory

\( u_s \): tangential to the reference trajectory
RF acceleration

• We assume a cavity with an oscillating RF-field: \( E_z = \hat{E}_z \sin(\omega_{RF}t) \)

• In this section we neglect the transit-transit factor
  – we assume a field constant in time while the particle passes the cavity

• Work done on a particle inside cavity:

\[
W = \int F\,dz = q \int E_z\,dz = q \int \hat{E}_z \sin(\omega_{RF}t)\,dz = q\hat{V} \sin(\omega_{RF}t)
\]
Synchrotron with one cavity

- The energy kick of a particle, $\Delta E$, depends on the RF phase seen, $f$

$$\Delta E = W = q \hat{V} \sin(\omega_{RF} t) = q \hat{V} \sin \phi$$

- We define a “synchronous particle”, $s$, which always sees the same phase $f_s$ passing the cavity

$$w_{RF} = h w_{rs} \quad (h: \text{“harmonic number”})$$

- E.g. at constant speed, a synchronous particle circulating in the synchrotron, assuming no losses in accelerator, will always see $f_s=0$
Non-synchronous particles

- A synchronous particle $P_1$ sees a phase $f_s$ and get a energy kick $D E_s$

- A particle $N_1$ arriving early with $f = f_s - d$ will get a lower energy kick

- A particle $M_1$ arriving late with $f = f_s + d$ will get a higher energy kick

- **Remember:** in a synchrotron we have bunches with a huge number of particles, which will always have a certain energy spread!
Frequency dependence on energy

• In order to see the effect of a too low/high DE, we need to study the relation between the change in energy and the change in the revolution frequency ($\eta$: "slip factor")

\[
\eta = \frac{df_r}{f_r} \frac{dp}{p}
\]

• Two effects:
  1. Higher energy \(\checkmark\) higher speed (except ultra-relativistic)

\[
f_r = \frac{\beta c}{2\pi R}
\]

  2. Higher energy \(\checkmark\) larger orbit “Momentum compaction”
Momentum compaction

- Increase in energy/mass will lead to a larger orbit

We define the “momentum compaction factor” as: 

\[ \alpha = \frac{dR}{R} = \frac{dp}{p} \]

- \( \alpha \) is a function of the transverse focusing in the accelerator, \( \alpha = \langle D_x \rangle / R \)
  - \( \alpha \) is a well defined quantity for a given accelerator
Phase stability

- $h>0$: velocity increase dominates, $f_r$ increases

- Synchronous particle stable for $0^\circ<f_s<90^\circ$
  - A particle $N_1$ arriving early with $f=f_s-d$ will get a lower energy kick, and arrive relatively later next pass
  - A particle $M_1$ arriving late with $f=f_s+d$ will get a higher energy kick, and arrive relatively earlier next pass

- $h<0$: stability for $90^\circ<f_s<180^\circ$

- $h=0$ at the **transition** energy. When the synchrotron reaches this energy, the RF phase needs to be switched rapidly from $f_s$ to $180-f_s$
Transverse dynamics: degrees of freedom orthogonal to the reference trajectory

\( u_x \): the horizontal plane
\( u_y \): the vertical plane
Bending field

- Circular accelerators: deflecting forces are needed
  \[ \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) = \vec{F}_E + \vec{F}_B \]

- Circular accelerators: piecewise circular orbits with a defined bending radius
  - Straight sections are needed for e.g. particle detectors
  - In circular arc sections the magnetic field must provide the desired bending radius:
    \[ \frac{1}{\rho} = \frac{eB}{p} \]

- For a constant particle energy we need a constant B field \( \text{dipole magnets with homogenous field} \)

- In a synchrotron, the bending radius, \( 1/eB/p \), is kept constant during acceleration (last section)
The reference trajectory

- An accelerator is designed around a reference trajectory (also called design orbit in circular accelerators)

- This is the trajectory an ideal particle will follow and consist of
  - a straight line where there is no bending field
  - arc of circle inside the bending field

- We will in the following talk about transverse deviations from this reference trajectory, and especially about how to keep these deviations small
Bending field: dipole magnets

- Dipole magnets provide uniform field in the desired region

- LHC Dipole magnets: design that allows opposite and uniform field in both vacuum chambers

- Bonus effect of dipole magnets: geometrical focusing in the horizontal plane

- \( \frac{1}{\rho} \): “normalized dipole strength”, strength of the magnet

\[
\frac{1}{\rho} = \frac{eB}{p} \Leftrightarrow \frac{1}{\rho} [m^{-1}] = 0.3 \frac{B[T]}{p[GeV/c]}
\]
Focusing field

- reference trajectory: typically centre of the dipole magnets

- Problem with geometrical focusing: still large oscillations and NO focusing in the vertical plane. The smallest disturbance (like gravity...) may lead to lost particle

- Desired: a restoring force of the type $F_{x,y} = -kx,y$ in order to keep the particles close to the ideal orbit

- A linear field in both planes can be derived from the scalar pot. $V(x,y) = gxy$
  - Equipotential lines at $xy = V_{\text{const}}$
  - $B$ Magnet iron surface
  - Magnet surfaces shaped as hyperbolas gives linear field
Focusing field: quadrupoles

- Quadrupole magnets give linear field in \( x \) and \( y \):
  \[
  B_x = -gy \\
  B_y = -gx
  \]

- However, forces are focusing in one plane and defocusing in the orthogonal plane:
  \[
  F_x = -qvgx \quad \text{(focusing)} \\
  F_y = qvgy \quad \text{(defocusing)}
  \]

- Opposite focusing/defocusing is achieved by rotating the quadrupole 90°.

- Analogy to dipole strength: normalized quadrupole strength:
  \[
  k = \frac{eg}{p} \iff k \left[ m^{-2} \right] \approx 0.3 \frac{g[T/m]}{p[GeV/c]}
  \]
Optics analogy

- Physical analogy: quadrupoles ≈ optics

- Focal length of a quadrupole: \( 1/f = kl \)
  - where \( l \) is the length of the quadrupole

- Alternating focusing and defocusing lenses will together give total focusing effect in both planes (shown later)
  - “Alternating Gradient” focusing
The Lattice

- An accelerator is composed of bending magnets, focusing magnets and non-linear magnets (later)

- The ensemble of magnets in the accelerator constitutes the “accelerator lattice”
Example: lattice components
Transverse beam size

RMS beam size:

$$\sigma(s) = \sqrt{\varepsilon_{rms} \beta(s)}$$

Beam quality
Lattice
Conclusion: transverse dynamics

• We have now studied the transverse optics of a circular accelerator and we have had a look at the optics elements,
  – the dipole for bending
  – the quadrupole for focusing
  – the sextupole for chromaticity correction

• All optic elements (+ more) are needed in a high performance accelerator, like the LHC
Synchrotron radiation
1) Synchrotron radiation

- Charged particles undergoing acceleration emit electromagnetic radiation

- Main limitation for circular electron machines
  - RF power consumption becomes too high

- The main limitation factor for LEP...
  - ...the main reason for building LHC!

- However, synchrotron radiations is also useful (see later slides)
Show RAD2D here

(anim)
Characteristic of SR: power

Lorentz invariant formula for power radiated by accelerated charged particles:

\[ P_S = \frac{e^2 c}{6\pi \varepsilon_0 (m_0 c^2)^2} \left( \frac{dp}{d\tau} \right)^2 \]

(This is Larmor’s non-relativistic formula with the substitutions \( dt \to d\tau \)
and \( p \to p' \)). Two cases:

1) Linear acceleration \( \frac{dv}{dt} \parallel v \):

\[ P_S = \frac{e^2 c}{6\pi \varepsilon_0 (m_0 c^2)^2} \left( \frac{dp}{dt} \right)^2 \]

\( \frac{dp}{dt} = \frac{dE}{dx} \) is in the order 10-100 MV/m in todays accelerators. \( P_S \) compared to power provided by the accelerator to increase the energy: \( \eta = \frac{P_S}{\frac{dE}{dt}} \sim 10^{-14} \)→ linear acceleration gives negligible radiation.

2) Circular acceleration \( \frac{dv}{dt} \perp v \):

\[ P_S = \frac{e^2 c}{6\pi \varepsilon_0 (m_0 c^2)^2} \left( \frac{dp}{dt} \right)^2 = \frac{e^2 c}{6\pi \varepsilon_0 (m_0 c^2)^4} \frac{E^4}{R^2} \]

Radiated power increase with \( E^4 \) (!).
Characteristics of SR: distribution

- Electron rest-frame: radiation distributed as a "Hertz-dipole"

\[
\frac{dP_s}{d\Omega} \propto \sin^2 \psi
\]

- Relativist electron: Hertz-dipole distribution in the electron rest-frame, but transformed into the laboratory frame the radiation form a very sharply peaked light-cone

We assume a photon emitted in the rest frame y-direction, while the particle is moving in the z-direction (acceleration in the x-direction), \( p' = [E/c, 0, p_y, 0] \)

\[
p' \gamma = \begin{bmatrix} \gamma & 0 & 0 & \beta \gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \beta \gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} E/c \\ 0 \\ p_y \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma E/c \\ 0 \\ p_y \\ \gamma \beta E/c \end{bmatrix}
\]

\[\Rightarrow \tan \theta = \frac{p_y}{p_z} = \frac{p_y}{\gamma \beta E/c} \approx \frac{E/c}{\gamma E/c} = \frac{1}{\gamma} \Rightarrow \text{The light-cone has an extremely small angle!}\]
Characteristics of SR: spectrum

- Broad spectra (due to short pulses as seen by an observer)
- But, 50% of power contained within a well defined "critical frequency"
  \[ \omega_c = \frac{3c\gamma^3}{2R} \]

Summary: advantages of Synchrotron Radiation
1. Very high intensity
2. Spectrum that cannot be covered easy with other sources
3. Critical frequency easily controlled
Example: European Synchrotron Radiation Facility (ESRF), Grenoble, France

Typical SR centre

Accelerator + Users

Some applications of Synchrotron Radiation:
- material/molecule analysis (UV, X-ray)
- crystallography
- archaeology...
Case: LHC
LHC injector system

- LHC is responsible for accelerating protons from 450 GeV up to 7000 GeV
- 450 GeV protons injected into LHC from the SPS
- PS injects into the SPS
- LINACS injects into the PS
- The protons are generated by a Duoplasmatron Proton Source
LHC layout

- circumference = 26658.9 m
- 8 interaction points, 4 of which contain detectors where the beams intersect
- 8 straight sections, containing the IPs, around 530 m long
- 8 arcs with a regular lattice structure, containing 23 arc cells
- Each arc cell has a FODO structure, 106.9 m long
LHC beam transverse size

\[ \beta_{typ} \approx 180 m, \beta^* = 0.55 m, \varepsilon \approx 0.5 nm \times rad \]

\[ \sigma_{arc} = \sqrt{\varepsilon \beta_{typ}} \approx 0.3 mm \]

\[ \sigma_{IP} = \sqrt{\varepsilon \beta^*} \approx 17 \mu m \]

beta in drift space:

\[ b(s) = b^* + \frac{(s-s*)^2}{b^*} \]
LHC cavities

- Superconducting RF cavities (standing wave, 400 MHz)
- Each beam: one cryostats with 4+4 cavities each
- Located at LHC point 4
## LHC main parameters

at collision energy

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle type</td>
<td>$p$, $Pb$</td>
</tr>
<tr>
<td>Proton energy $E_p$ at collision</td>
<td>$7000$ GeV</td>
</tr>
<tr>
<td>Peak luminosity (ATLAS, CMS)</td>
<td>$10 \times 10^{34}$ cm$^{-2}$s$^{-1}$</td>
</tr>
<tr>
<td>Circumference $C$</td>
<td>$26,658.9$ m</td>
</tr>
<tr>
<td>Bending radius $r$</td>
<td>$2804.0$ m</td>
</tr>
<tr>
<td>RF frequency $f_{RF}$</td>
<td>$400.8$ MHz</td>
</tr>
<tr>
<td># particles per bunch $n_p$</td>
<td>$1.15 \times 10^{11}$</td>
</tr>
<tr>
<td># bunches $n_b$</td>
<td>$2808$</td>
</tr>
</tbody>
</table>
References

• Bibliography:
  – K. Wille, The Physics of Particle Accelerators, 2000
  – ...and the classic: E. D. Courant and H. S. Snyder, "Theory of the Alternating-Gradient Synchrotron", 1957
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• Other references
  – USPAS resource site, A. Chao, USPAS January 2007
  – O. Brüning: CERN student summer lectures
  – N. Pichoff: Transverse Beam Dynamics in Accelerators, JUAS January 2004
  – U. Amaldi, presentation on Hadron therapy at CERN 2006
  – Several figures in this presentation have been borrowed from the above references, thanks to all!