New Constraints on Neutrino Velocities

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Abstract

The OPERA collaboration has claimed that muon neutrinos with mean energy of 17.5 GeV travel 730 km from CERN to the Gran Sasso at a speed exceeding that of light by about 7.5 km/s or 25 ppm. However, we show that such superluminal neutrinos would lose energy rapidly via the bremsstrahlung of electron-positron pairs ($\nu \to \nu + e^- + e^+$). For the claimed superluminal neutrino velocity and at the stated mean neutrino energy, we find that most of the neutrinos would have suffered several pair emissions en route, causing the beam to be depleted of higher energy neutrinos. Thus we refute the superluminal interpretation of the OPERA result. Furthermore, we appeal to Super-Kamiokande and IceCube data to establish strong new limits on the superluminal propagation of high-energy neutrinos.

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I.   INTRODUCTION AND CONCLUSIONS

The OPERA collaboration has reported evidence of superluminal neutrino propagation[1]. The CNGS beam, consisting of pulses of muon neutrinos with mean energy of 17.5 GeV and an energy spread extending beyond 50 GeV, travels about 730 km from CERN to the OPERA detector in the Gran Sasso Laboratory. The group reports that the travel time of the ultrarelativistic neutrinos is about 60 ns less than expected. We phrase our discussion in terms of the parameter $\delta \equiv (v_\nu^2 - 1)$ wherein we take the speed of light in vacua to be unity. The OPERA claim is $\delta = 5 \times 10^{-5}$. Recognizing the potential impact of this result, the collaboration writes that it intends "to continue its studies to investigate possible. . . systematic effects that could explain the observed anomaly."

The OPERA claim (hereafter, the anomaly) is compatible with earlier studies of high-energy neutrinos such as MINOS[2], which yielded the result $\delta = 10.2 \pm 5.8 \times 10^{-5}$. However, observations of ~ 10 MeV neutrinos from supernova SN1987a provide the constraint[3–5] $\delta < 4 \times 10^{-9}$. Thus, the alleged anomaly must be energy dependent, decreasing rapidly from 10 GeV to 10 MeV. We note in passing that observations of neutrino oscillations allow one to deduce far more severe constraints on neutrino velocities at relevant energies[6, 7]. Lorentz-violating velocity differences as large as 10^{-20} between neutrinos of different species would have been readily detected and are excluded. Thus, the velocity anomaly, if correct, must pertain to the propagation of all three types of neutrino.

Let us assume that muon neutrinos with energies of order tens of GeV travel at superluminal velocity. As in all cases of superluminal propagation, certain otherwise forbidden processes are kinematically permitted, even in vacuum. In particular, we focus on the following analogs to Cherenkov radiation:

$$\nu_\mu \longrightarrow \begin{cases} \nu_\mu + \gamma & (a) \\ \nu_\mu + \nu_e + \overline{\nu}_e & (b) \\ \nu_\mu + e^+ + e^- & (c) \end{cases} \qquad (1)$$

These processes cause superluminal neutrinos to lose energy as they propagate and, as we shall see, process (c) places a severe constraint upon potentially superluminal neutrino velocities.

Process (b) is irrelevant because all three neutrino species are known to travel at virtually the same velocity. Process (a) is kinematically allowed for all neutrino energies provided

$v_\nu > 1$. However process (a) is induced by a W loop diagram and thus we find its effect on neutrino propagation to be smaller by a factor of α/π than that of process (c), whenever process (c) is kinematically allowed.

Process (c), pair bremsstrahllung, proceeds through the neutral current weak interaction. The threshold energy for this process is $E_0 = 2m_e/\sqrt{v_\nu^2 - v_e^2}$, where v_e is the maximal attainable velocity of an electron and m_e its mass. However, we know[6, 7] that $v_e = 1$ to a precision of at least 10^{-15}. Thus we may write $E_0 = 2m_e/\sqrt{\delta}$. Its value is about 140 MeV for the OPERA value of δ. It is process (c) that allows us to exclude the OPERA anomaly and place a strong constraint on neutrino superluminality.

We have computed both Γ, the rate of pair emission by an energetic superluminal neutrino, and dE/dx, the rate at which it loses energy in the high energy limit where the electron and neutrino masses may be neglected[1]:

$$\Gamma = k' \frac{G_F^2}{192\pi^3} E^5 \delta^3 \tag{2}$$

$$\frac{dE}{dx} = -k \frac{G_F^2}{192\pi^3} E^6 \delta^3 \tag{3}$$

where k and k' are numerical constants: $k = 25/448$, $k' = 1/14$. These expressions, aside from the numerical factors, follow from simple arguments. The factors of G_F arise from the low energy form of the weak interactions while those of energy then follow from dimensional analysis. The power of δ is related to the power of energy: the 4-momentum of the superluminal neutrino is timelike (relative to the speed of light) with a square of δE^2. We may therefore work in the neutrino "rest" frame with an effective "mass" of $\sqrt{\delta} E$. In this frame the powers of δ follow the powers of E^2. The relativistic dilation factor needed to boost back to the original frame is the ratio of the original energy divided by the effective "mass", $\gamma = 1/\sqrt{\delta}$. Applying the usual dilation factors to Γ and dE/dx gives our result.

Note that the mean fractional energy loss due to a single pair emission is $E^{-1}(dE/dx)/\Gamma = k/k' \approx 0.78$: about three-quarters of the neutrino energy is lost in each emission.

We integrate dE/dx assuming δ not to vary significantly in the relevant energy interval. We find that neutrinos with initial energy E_0, after traveling a distance L, will have energy

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[1] These expressions are leading order in δ. We have also neglected the vector-current coupling of the electron: $c_V = 0$ and $c_A = -1/2$.

E as given by:

$$E^{-5} - E_0^{-5} = 5k\delta^3 \frac{G_F^2}{192\pi^3} L \equiv E_T^{-5} \qquad (4)$$

The steeply falling (with energy) form of dE/dx means that neutrinos with initial energy greater than E_T rapidly approach a terminal energy, E_T, which is essentially independent of the initial neutrino energy. Adopting the OPERA result $\delta = 5 \times 10^{-5}$ and using the OPERA baseline of 730 km we find a terminal energy of about 12.5 GeV. Few, if any, neutrinos will reach the detector with energies in excess of 12.5 GeV. Thus the CNGS beam would be profoundly depleted and spectrally distorted upon its arrival at the Gran Sasso. Using the expression for Γ above we may also establish that *any* superluminal neutrino with the velocity claimed by OPERA of *any* specific initial energy much greater than 12.5 GeV has a negligible probability of arriving at the Gran Sasso without having lost most of its energy. The observation of neutrinos with energies in excess of 12.5 GeV cannot be reconciled with the claimed superluminal neutrino velocity measurement.

Our analysis yields strong new constraints on superluminal neutrino velocities. Super-Kamiokande has carefully studied atmospheric neutrinos that traverse the earth (upward-going in the detector) over an energy range extending from 1 GeV to 1 TeV[8–10]. These upward directed neutrinos, in traversing a distance of 10,000 km, would experience a depletion and spectral distortion as we have described above. The observation of such neutrinos with 1 TeV energy allows us to conservatively deduce that $\delta < 1.4 \times 10^{-8}$, similar to but slightly weaker than the lower energy neutrino velocity constraint deduced from SN1987a.

The IceCube collaboration has reported the observation of upward-going showers with reconstructed shower energies above 16 TeV[11]. Using a neutrino energy of 16 TeV and a minimum baseline of 500 km (which would be appropriate for a horizontal neutrino) we obtain a more stringent limit $\delta < 3.75 \times 10^{-10}$, superior to the SN1987a constraint by an order of magnitude. Finally IceCube has also reported events with energies in excess of 100 TeV. Observation of neutrinos with this energy and a baseline of at least 500 km implies a limit of $\delta < 1.7 \times 10^{-11}$.

While $\delta < 1.7 \times 10^{-11}$ is significantly better than previous bounds, a more careful analysis of the path-lengths and energies of the highest energy events from Super-Kamiokande, IceCube and other neutrino telescopes may enable an even stronger constraint.

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