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CBX 92–40
August 20, 1992

The Charged Particle Energy Correction in CLEO

1. Introduction

In May, 1991 I sent out a computer mailing detailing how I calculated the charged particle energy correction for CLEO 1.5 and CLEO 2. The correction consists of two parts: (1) a term that accounts for dE/dx losses in material preceding the main drift chamber and (2) a B field multiplication factor. The code implementing the correction is in the library `FTLIB` (this will be replaced soon by `KNLIB`) and is described in CBX 91–12. It requires the use of the KN track list if you are interested in 4–vectors.

I document here the method that I used to obtain the energy correction and to show how the K_s, Λ, D^0 and D^+ masses vary with momentum. This analysis uses a total of 1066 pb^{-1} of CLEO 2 data (4S2_5 and 4S3_1) and 320 pb^{-1} of CLEO 1.5 data. I was not able to present very detailed CLEO 2 results in my original note because I only had 120 pb^{-1} of 4S1 data available at the time.

2. Why the correction is needed

The track fitting program DUET only partially accounts for the effect of material on particle propogation. Multiple scattering is treated by allowing an $r-\phi$ kink at the inner boundary of the DR and then applying propogation of errors to the measurement covariance matrix to account for hit correlations (as described in CBX 91–74). No attempt is made at all to correct for the energy losses suffered by particles as they cross device boundaries and gas. CLEO relies instead on an "after the fact" correction, such as the DKSxxx routines described in CBX 91–12, to the particle energy/momentum, even though the method is not strictly valid and does not do a good job for low momentum tracks. This note describes how that correction is performed.

Before doing this I would like to point out that there are better alternatives. FT-MONTE uses a Kalman filter algorithm first used by P. Billoir in track fitting which continuously corrects the track for dE/dx losses and multiple scattering as it traces the track toward the production point, accumulating measurements as it goes. It is described in detail in CBX 92–39. Although the Kalman filter method is not particularly fast and requires special treatment to start the fit, it is "optimal" in the sense of using all the information to obtain the best possible parameters. My tests in FTMONTE show that it does indeed give very good parameters, particularly for slower tracks which have large energy and multiple scattering corrections. A full implementation would require that each track be fit under five hypotheses (each with its own covariance matrix), corresponding to the possibility that the particle is an electron, muon, pion, kaon or proton. In this way, physics analyses would use the correct particle hypothesis when calculating invariant masses or applying kinematic fitting. The potentially large amount of ROAR storage this method would entail (especially for the covariance matrices) could be reduced through clever ways of encoding the information, using the fact that the five sets of track parameters do not differ much.

3. Theoretical treatment

The corrected momentum $p_{\rm cor}$ is calculated from the initial momentum $p_{\rm old}$ by the formula

$$p_{\rm cor} = R p_{\rm old} + \Delta p$$

where R is a B field correction (it is applied in `GETCD`) and Δp is the dE/dx correction. Δp is calculated by integrating the Bethe-Bloch formula

$$\Delta p = \int_0^t \frac{dp}{dx} dx = \int_0^t \frac{1}{\beta}\frac{dE}{dx} dx = \int_0^t \frac{A}{\beta^3}\left[\ln\left(\frac{2m_e c^2 \beta^2\gamma^2}{I_0}\right) - \beta^2\right] dx$$

where t is the thickness of the material to be traversed (including the increased path length due to non-normal tracks), A is a constant that depends on the material, Z is the atomic number, m_e is the electron mass, I_0 is the ionization potential and β and γ are the usual relativistic parameters. I use the approximation $I_0 = Z^{0.9}$eV, and take $Z = 9$ for representative media. A is determined from our knowledge of $(dE/dx)_{\rm min}$, the quantity listed in the Particle Data Book. The minimum occurs at $\beta\gamma = 3.40$, which gives $A = (dE/dx)_{\rm min}/11.528$.

It is easier to write the differential equation in terms of $\tau \equiv \beta\gamma = p/m$ and the normalized thickness $x' = x/t$:

$$\begin{aligned}
\frac{d\tau}{dx'} &= \frac{\Delta E_{\rm min}}{m} F(\tau) \\
F(\tau) &= \frac{(1+\tau^2)^{3/2}}{11.528\tau^3}\left[9.0872 + 2\ln(\tau) - \frac{\tau^2}{1+\tau^2}\right] \\
\tau &= \frac{p}{m}
\end{aligned}$$

where $\Delta E_{\rm min} = (dE/dx)_{\rm min} t$ is the average energy loss suffered by a minimum ionizing particle passing through a distance t of material (including path length corrections from non-normal tracks). The change in momentum can then be calculated from the integral

$$\Delta p = m\Delta\tau = \Delta E_{\rm min} \int_0^1 F(\tau) dx'$$

For moderate velocities ($\beta > 0.7$), $F(\tau)$ is approximately constant giving $\Delta p \simeq \Delta E_{\rm min} F(\tau)$. At low values of β, where $F(\tau)$ changes rapidly, it is probably necessary to integrate the $d\tau/dx'$ differential equation numerically using a method such as the Runge–Kutte technique. The integration is aided by transforming to a new variable $\xi = \tau^4$ which removes the $1/\tau^3$ term from the differential equation and makes the numerical solution more accurate. Using this variable the dE/dx differential equation becomes

$$\begin{aligned}
\frac{d\xi}{dx'} &= \frac{\Delta E_{\rm min}}{m} F'(\xi) \\
F'(\xi) &= \frac{(1+\sqrt{\xi})^{3/2}}{4 \cdot 11.528}\left[9.0872 + \frac{1}{2}\ln(\xi) - \frac{\sqrt{\xi}}{1+\sqrt{\xi}}\right] \\
\xi &= \left(\frac{p}{m}\right)^4
\end{aligned}$$

4. CLEO dE/dx software

The B field correction in CLEO II is applied when you call `GETCD` (the correction is 1.0025 as dicussed in the next section). The dE/dx correction is obtained by calling one of the `DKSxxx` routines as documented in CBX 91–12. They require you to use the KN track list, except for the lowest level functions which accept all the necessary information in the argument list. The routines store $(dE/dx)_{\rm min} \times t$ as a function of run number, where t is the total amount of material up to and including the inner wall of the main drift chamber.

The algorithm is carried out in the function `DKSDEL`, which actually solves the integral equation $\Delta E_{\rm min} = m \int_{\tau_{\rm old}}^{\tau_{\rm cor}} d\tau/F(\tau)$ for $\tau_{\rm cor} = p_{\rm cor}/m$ using lookup tables. The dE/dx correction calculated by these routines is not completely correct because it is assumed that the particle's momentum is determined entirely in the main drift chamber, a poor assumption at low momentum. The routines partly compensate for this by using only the material up and including the inner VD wall when $p < 0.060$ GeV/c, but this causes gaps in the corrected momentum spectrum. A second problem is that the `DKSxxx` routines apply the correction at the origin, with no change in the track's position and direction. The correction should in principle be done at some effective radius and then the track should be swum back to the origin. I will be studying both these issues to come up with a better algorithm, but the results presented here show that the correction results in a major improvement of the mass values.

In contrast, the charged particle tracing and fitting routines in FTMONTE integrate the $d\tau/dx'$ differential equation using the Runge–Kutte method described above. The FTMONTE routine which returns the momentum of the particle before energy loss is

```
FUNCTION FMDPUP(POLD, XMAS, DEDXMN),
```

where `POLD` is the momentum after energy loss, `XMAS` is the mass and `DEDXMN` is $(dE/dx)_{\rm min} \times t$. The function `FMDPDN` is used to subtract energy from the particle rather than add it. Both `FMDPUP` and `FMDPDN` can be found in `FTLIB`.

The table below shows the effect of 1.75 MeV of material (the amount found for CLEO 2) on particles of various momenta. I assume $\sin\theta = 0.8$ to get a more realistic incident angle on the material.

Corrected momenta for π, K and p vs. measured momentum ($\sin\theta = 0.8$)

$p_{\rm old}$	$p_{\rm cor}(\pi)$	$p_{\rm cor}(K)$	$p_{\rm cor}(p)$
0.050	0.071	0.158	0.247
0.100	0.107	0.166	0.252
0.150	0.154	0.188	0.261
0.200	0.203	0.223	0.278
0.250	0.253	0.265	0.304
0.300	0.302	0.311	0.339
0.400	0.402	0.406	0.421
0.500	0.502	0.505	0.514
0.600	0.602	0.604	0.610
0.700	0.702	0.703	0.707
0.800	0.802	0.803	0.806
0.900	0.902	0.903	0.905
1.000	1.002	1.003	1.004

5. CLEO 2 dE/dx results

The initial energy correction for CLEO 2 was determined using the 120 pb^{-1} BB^* data sample using K_s's, Λs and Ds. In this note I will show the results for the recent 1066 pb^{-1} 4S2_5 and 4S3_1 sample. I select events as follows:

1. K_s and Λ. I require DI particle ID consistency (2.5 σ) for all particles and demand that all V^0 vertices have a radius less than 3.4 cm, to force the decay products to pass through the beam pipe. K_s and Λ masses were plotted in 1.0 and 0.5 MeV bins, respectively. I fit the mass peaks to a gaussian plus a quadratic background.

2. $D^0 \to K\pi$, $D^0 \to K_s\pi\pi$ and $D^0 \to K3\pi$ decays from $D^{*+} \to D^0\pi^+$ and $D^+ \to K\pi\pi$ from $D^{*+} \to D^+\pi^0$. Again I demanded 2.5 σ consistency for all particles to clean up the signals. Masses were plotted in 5 MeV bins. I fit the histograms to a gaussian signal plus a quadratic background.

To get the corrections I tried out several values of the two correction constants. My criteria were that the particle masses should be centered as closely as possible on the true positions and that they should be momentum independent. After several trials I obtained best guess values of 1.75 MeV and 1.0025 for the dE/dx material and the B field correction, respectively (The 1.0025 correction is applied in `GETCD`).

The results are shown in Figures 1–6. Figures 1–2 (3–4) show the K_s (Λ) masses as a function of momentum, before and after corrections are applied. Note that the effect of correcting for energy loss is easily discernable for both particles. After correction the K_s mass rises slightly as a function of momentum, whereas the opposite is true for the Λ. Figures 5 and 6 show the $D^0 \to K\pi,\ K_s\pi\pi,\ K3\pi$ and $D^+ \to K\pi\pi$ masses as a function of $x = p/p_{\rm max}$. For amusement, I have plotted the theoretical uncertainty in the D^0 and D^+ masses as a dotted line. The correction seems to make the masses come out about right, although there is a tendency to rise at high momentum. This rise could be cancelled by slightly decreasing the B field and slightly increasing the amount of material.

6. Determining the CLEO 1.5 energy correction

For this study I analyzed 320 pb^{-1} of CLEO 1.5 data using the SB4Hxx.ROR files. The energy correction that has been used by all previous CLEO 1.5 analysis has been based on the `GET4` (charged tracks) and `GET4V` (K_s and Λ) routines. The amount of material and the B field multiplication factors that were used were 1.05 MeV and 1.000, respectively. The dE/dx value contradicts the calculation of approximation of 1.7 MeV done by Thorndike and Sparks (May 1, 1991 e-mail note). Indeed, when I looked at the mass dependence of K_s, Λ and D^0 particles I discovered a significant momentum dependence. I concluded that the values were incorrect and proceeded to try out new sets of numbers by hand until the masses stabilized. My new values for the dE/dx thickness and R are 1.9 MeV and 0.9990, respectively. The masses as a function of momentum are shown in the table below. As in the CLEO 2 case, I only use V^0s which decay inside the beam pipe.

CLEO 1.5 K_s mass vs. p		
Momentum region	Mass (old correction)	Mass (new correction)
$0.0 < p < 0.5$	496.10 ± 0.07	497.72 ± 0.03
$0.5 < p < 1.0$	496.70 ± 0.06	497.54 ± 0.03
$0.0 < p < 1.5$	497.25 ± 0.10	497.65 ± 0.05
$1.5 < p < 2.0$	497.61 ± 0.16	497.62 ± 0.08
$2.0 < p < 2.5$	497.85 ± 0.12	497.72 ± 0.16
$2.5 < p < 3.0$	497.89 ± 0.22	497.79 ± 0.26

CLEO 1.5 Λ mass vs. p		
Momentum region	Mass (old correction)	Mass (new correction)
$0.0 < p < 0.5$	1115.1 ± 0.15	1115.95 ± 0.09
$0.5 < p < 1.0$	1115.4 ± 0.03	1115.93 ± 0.03
$1.0 < p < 1.5$	1115.5 ± 0.04	1115.73 ± 0.04
$1.5 < p < 2.0$	1115.8 ± 0.10	1115.88 ± 0.09
$2.0 < p < 2.5$	1115.8 ± 0.14	1115.88 ± 0.13

Figures

1. K_s mass as a function of momentum. Open circles have no B field or dE/dx correction applied. Open and filled triangles refer to 4s2_4 and 4s2_5 data, respectively.
2. K_s mass as a function of momentum on an expanded vertical scale. Open and filled triangles refer to 4s2_4 and 4s2_5 data, respectively.
3. Λ mass as a function of momentum. Open circles have no B field or dE/dx correction applied. Open and filled triangles refer to 4s2_4 and 4s2_5 data, respectively.
4. Λ mass as a function of momentum on an expanded vertical scale. Open and filled triangles refer to 4s2_4 and 4s2_5 data, respectively.

5. D^0 mass as a function of $x = p/p_{\max}$.

6. D^+ mass as a function of $x = p/p_{\max}$.