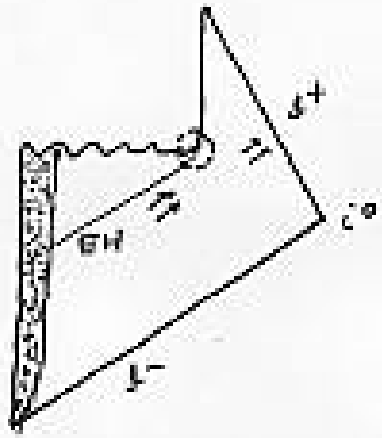
Quantum Geometry & BHs
Going beyond semi-classical approximation

Semi-classical scenario



QFT in CST
+
Heuristics of
back reaction

Limitations overlooked: cannot trust figure
i) Near the <u>entire</u> singularity
ii) role of the EH: <u>global</u>, <u>teleological</u> notion

What really happens? Loop QG <u>paradigm</u>

1. Quantum geometry
2. Isolated horizons & Hawking process
3. Dynamical horizons
4. BH evaporation & Information loss

1. Quantum Geometry

- Gravity ~ geometry in GR, No background metric. Matter & geometry, both "born" quantum mechanically.

- Quantum Riemannian geometry : New arena devoloped systematically by ~2 dozen groups in the mid-90s.

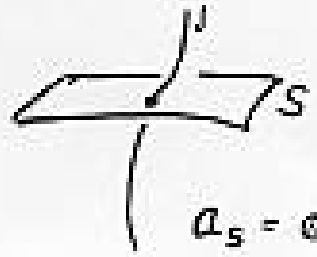
- gravity sector : $\underbrace{A_a^i}_{SU(2)\ \text{spin-conn.}}$; $\underbrace{E_i^a}_{\text{'Triad'/geometry}}$

$\mathcal{H} = L^2(\bar{\mathcal{A}}, d\mu_0)$; $\hat{h}_e$, $\hat{E}_i^a$: Well-defined

grav. holonomy.

$\hat{A}_a^i$: NOT defined
Non-Fock rep.

- Fundamental excitations : 1-dimensional
  'polymer' geometry
  "Flux lines of Area"



Technically : holonomies

$a_S = 8\sqrt{j(j+1)}\,\ell_p^2$

- Eigenvalues of all geometric operators : discrete

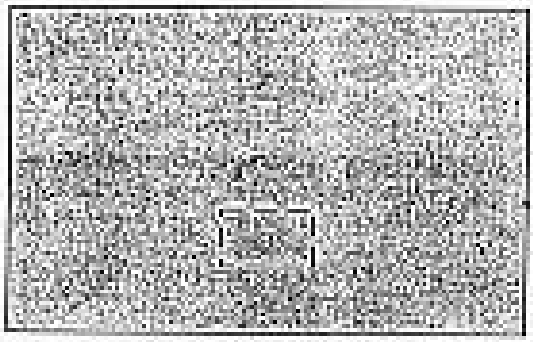
Analogy to H-atom : geometry quantized

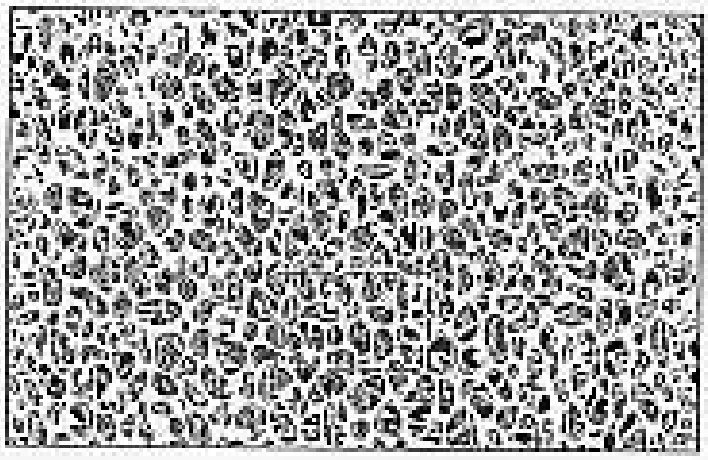
continuum only an approximation

Wheeler's qualitative picture:

Analogy with an ocean

"Space-time Foam"





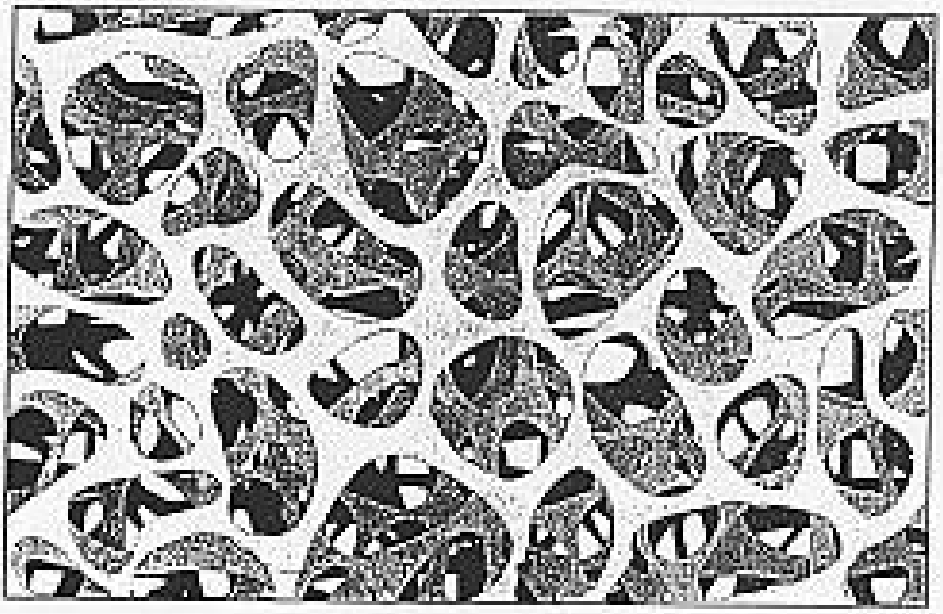
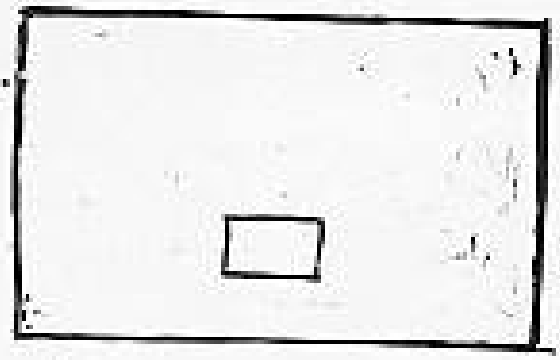


Figure 19. Spacetime is smooth (top) at macroscopic scales but may be foamy at the Planck scale (middle and bottom). Quantum-mechanical fluctuations of the vacuum perpetually rip and join the fabric of spacetime, constantly changing its topology. These topological fluctuations constitute spacetime foam.

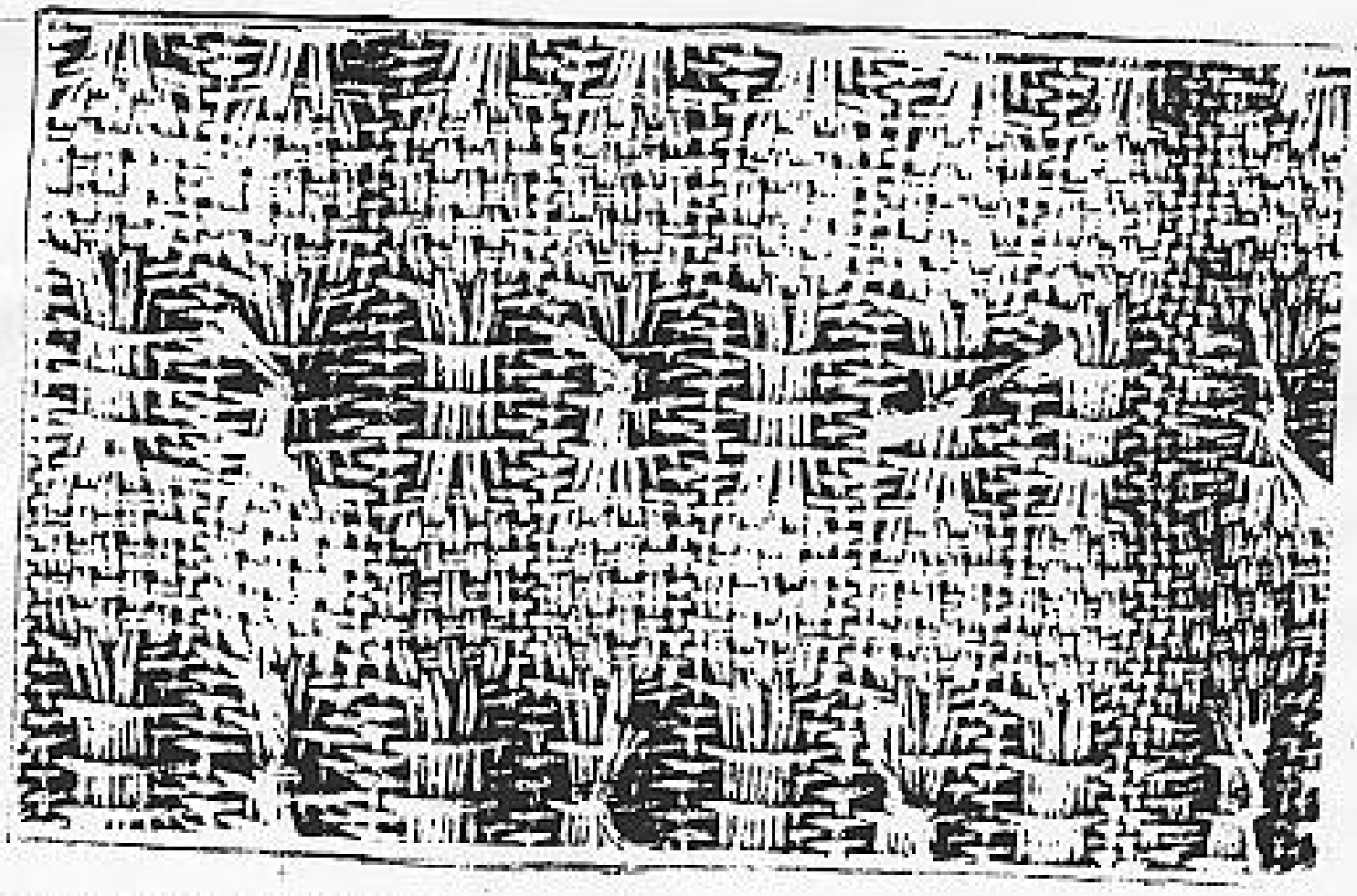
other (unknowable) universe. Consequently, a fundamental theory of everything (such as superstrings) would never be able to predict the values of the coupling constants because wormholes would wash these values away.

Interestingly, wormholes might provide a different mechanism that acts to fix the values of the coupling constants, including the cosmological constant. Coleman shows that the sum-over-histories corresponding to the state vector of the universe is dominated by a single term. The term corresponds to a uni-



Continuum

HUGE
blow-up
$\sim 10^{34}$



Planck-scale
structure

List 1

The first 100 area eigenvalues of the quantum area operator are listed below. These numbers need to be multiplied by the planck length squared to give the actual areas.

1	0.43301269	51	1.69895792
2	0.50000000	52	1.70710677
3	0.66143781	53	1.71391368
4	0.70710677	54	1.72474492
5	0.82915622	55	1.73205078
6	0.86602539	56	1.75116259
7	0.93301269	57	1.75588831
8	0.96824586	58	1.76216891
→ 9	1.00000000	59	1.77947181
10	1.08972478	60	1.78535712
11	1.09445050	61	1.79683155
12	1.11803401	62	1.79740208
13	1.14011946	63	1.79903805
14	1.16143781	64	1.80155727
15	1.19895792	65	1.80277562
16	1.20710677	66	1.82287562
17	1.22474492	67	1.82495376
18	1.26216891	68	1.82514077
19	1.29903805	69	1.82915622
20	1.32287562	70	1.83427125
21	1.32915622	71	1.84722623
22	1.36602539	72	1.85404956
23	1.36854458	73	1.86039573
24	1.39194107	74	1.86602539
25	1.40125856	75	1.86854458
26	1.41421354	76	1.87082875
27	1.43301269	77	1.88618273
28	1.46824586	78	1.89194107
29	1.47902000	79	1.90125856
30	1.49059[illegible]43	80	1.90606469
31	1.50000000	81	1.91303269
32	1.52273947	82	1.91421354
33	1.52746320	83	1.91888100
34	1.53636299	84	1.92028642
35	1.55104670	85	1.92360672
36	1.56124949	86	1.93185169
37	1.57313216	87	1.93301269
38	1.58113885	88	1.93649173
39	1.58972478	89	1.94719023
40	1.59445050	90	1.95575017
41	1.61803401	91	1.96047586
42	1.62968367	92	1.96824586
43	1.63197061	93	1.96927568
44	1.63935959	94	1.97902000
45	1.64011946	95	1.98405939
46	1.65775761	96	1.98431349
47	1.65831244	97	1.99059403
48	1.66143781	98	1.99426219
49	1.67535[illegible]41	→ 99	2.00000000
50	1.69518[illegible]51		

(Steve Fairhurst)

Eigenvalues "crowd"

Geometry is quantized and in a very <u>subtle</u> way.

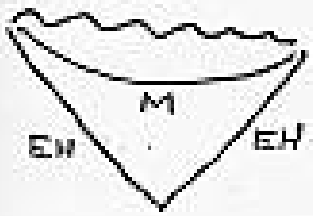
Spectra of all geometric operators : discrete !

(eigenvalues of $\hat{A}_S$)

$$a_S = 2\pi\gamma \ell_P^2 \sum_i \left[2 J_i^d (J_i^d + 1) + 2 J_i^u (J_i^u + 1) - J_i^{u+d} (J_i^{u+d} + 1) \right]^{1/2}$$

J^u J^d : $\frac{1}{2}$ integers ; $J_i^{u+d} \in \{ J_i^u + J_i^d \cdots |J_i^u - J_i^d| \}$

Resolution of BH singularity



Schwartzschild interior
(generic, 'stable' singularity)

- $M = \mathbb{S}^2 \times \mathbb{R}$; 4 KVFs
  KS Minisuperspace.
  Apply methods from QC
  Mimic full, unreduced theory

2 DOF:
τ : Radius of $\mathbb{S}^2$
ϕ : anisotropy.

New quantum mechanics!

$\mathcal{H} \neq L^2(\mathbb{R}^2, d^2x)$; $\mathcal{H} = L^2(\bar{\mathbb{R}}^2_{Bohr}, d\mu_0)$

only holonomies well-defined, not connections

[In QM: $\widehat{\exp ix}$ fine but NOT $\hat{x}$!]

Inequivalence has dramatic consequences

Co-triad diverges classically at the singularity.
Quantum co-triad operator bounded above!
$\Rightarrow$ curv bounded above

Quantum evolution : "WDW" equation

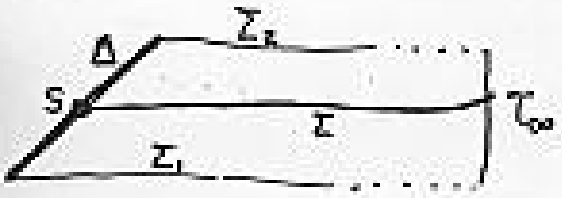
$$|\Psi\rangle = \sum_\tau \psi_\tau(\phi)\, |\tau, \phi\rangle$$

$$= C^+(\tau)\, \hat{O}_+\, \psi_{\tau+4\tau_0}(\phi) + C^0(\tau)\, \hat{O}_0\, \psi_\tau(\phi) + C^-(\tau)\, \hat{O}_-\, \psi_{\tau-4\tau_0}(\phi).$$

'Evolution' through discrete time steps.
Can evolve right through $\tau = 0$!
Classical spacetime !

2. Quasi-local substitutes for event horizon

Time independent phase : Isolated Horizon



Δ : Null, $S^2 \times \mathbb{R}$, Intrinsic geometry time indendent

⇒ cross-sections S are marginally trapped S^2

IH Boundary Conditions ⇒

- Well-defined action principle
- Hamiltonian Framework
- Time evolution Hamiltonian ⇔

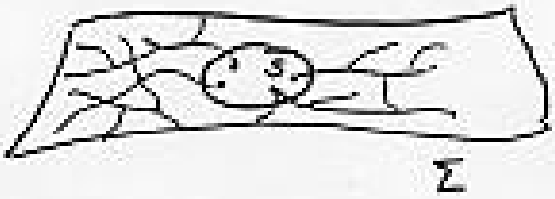
$$\delta E_\Delta^{(t)} = \frac{\kappa_\Delta^{(t)}}{8\pi G}\,\delta a_\Delta + \Omega_\Delta^{(t)}\,\delta J_\Delta + \Phi_\Delta^{(t)}\,\delta Q_\Delta \qquad \text{1st law}$$

Entropy ↔ Area

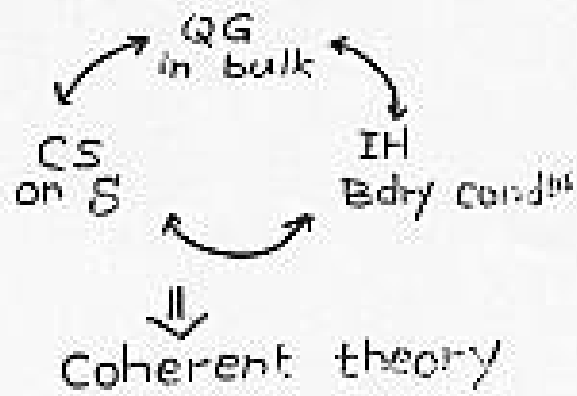
(AA, Beetle, Fairhurst, Krishnan, Lewandowski)

Quantum geometry of IHs

(AA, Baez, Krasnov)



- strategy
- Bulk : Polymer geometry
- Horizon : U(1) chern-Simons theory on punctured S^2
- Quantum-horizon Boundary condition: <u>very strong constraint</u>

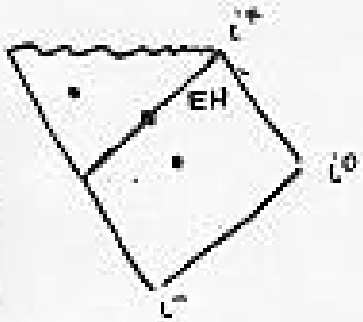


- Surface states account for entropy of all IHs

  Distorted, rotating, colored, ··· non-minimal couplings,

Hayward
AA + Krishnan

• Dynamical horizons

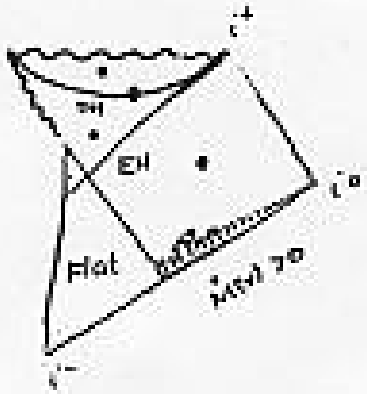


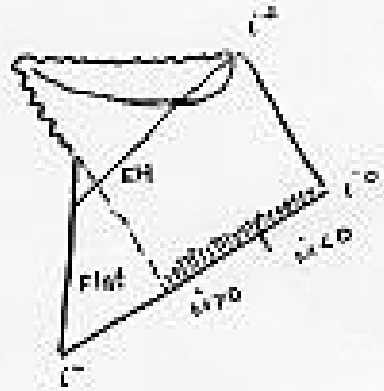
Eaternal black hole

- untrapped
- marginally trapped
- Trapped

Dynamical situations : A DH is $S^2 \times \mathbb{R}$, foliated by MT_s ($\Theta_{(l)} = 0$, $\Theta_{(n)} < 0$).

Ex : Vaidya Solutions (spherical, null fluid)





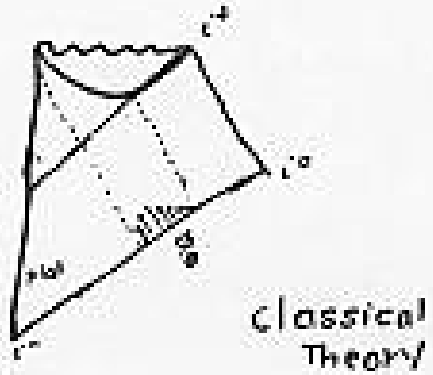
DH :	Space-like	Time-like
Area of MT_s	Increases	Decreases
Energy flux	+ve	−ve

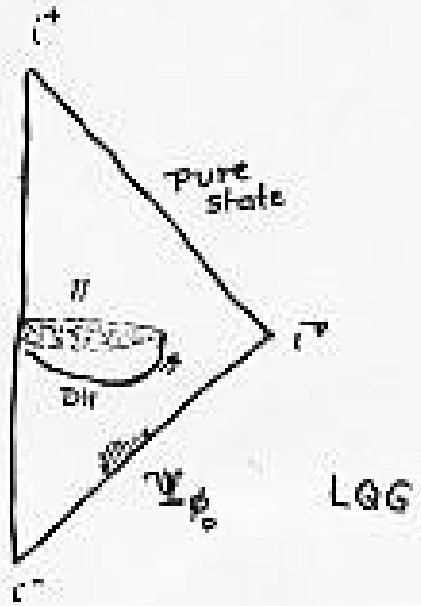
Integral generalization of first law holds on general DH_s

4. BH evaporation: Quantum geometry paradigm

- Gather these precise results to develop a qualitative-heuristic picture within LQG.

- Spherical collapse of a scalar field: <u>Large</u> BH





Deep Planck regime
No classical space-time

Assumptions:

i) Full quantum state becomes semi-classical again

ii) $O(N) \to O(N)$ when $a_{BH} \gg \ell^2$

BH evaporation: Quantum geometry paradigm

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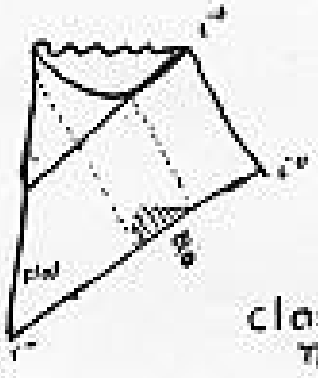
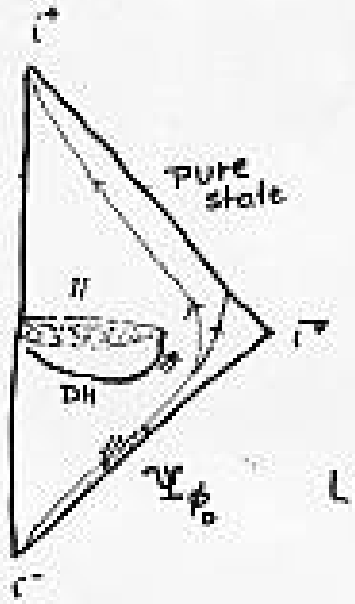


classical Theory

- Space-time does not end in a singularity ⇒ No sink of information.
- pure states on $\mathcal{I}^-$ ⇝ pure states on $\mathcal{I}^+$
- 'Event horizon': No longer a useful notion



LQG

- DHs form in semi-classical region and evaporate.

$$\frac{dR}{dt} = -8\pi G R^2 \cdot T_{ab}\, \ell^a \hat{r}^b$$

Area quanta ⇝ ϕ quanta

- Recovery of correlations (patience!)

※ Deep Planck regime
No classical space-time

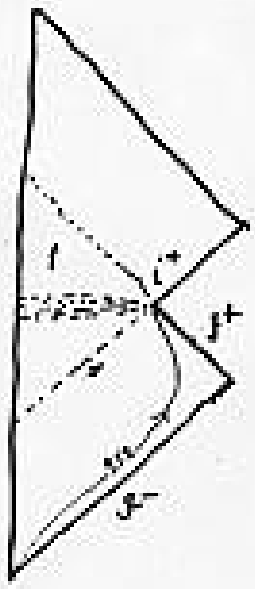
Assumptions:

i) Full quantum state becomes semi-classical again

ii) DHI ⇝ DTH when area ⇝ R^2

Reconciliation with semi-classical result:

Evaporation time for $M_\odot$ BH $> 10^{50} \times$ age of universe
What happens if we take it to be infinite?



Picture changes qualitatively

- Event horizon reappears
- Pure state on $\mathscr{I}^-$ $\rightsquigarrow$ Mixed state on $\mathscr{I}^+$
- Information lost to a second asymptotic region

Precise sense in which the semi-classical scenario is an approximation to the LQG paradigm

Summary.

Quantum geometry, underlying loop quantum gravity introduces qualitatively new elements in BH physics:

- Quantum isolated horizons described by U(1) CS theory on punctured sphere
- BH entropy for physical BHs (distortion, rotation, hair, non-minimal couplings, ...)
- BH singularity resolved.

When combined with classical IH and DH frameworks, one obtains a new paradigm for BH evaporation:

- No information loss
- Evaporation of DHs $\rightsquigarrow$ Hawking radiation
- Elements of semi-classical scenario ok in suitable approximations
- Lorentzian picture of the process.