BPS deformations of

$AdS_p \times S^q$

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Outline

- 1/2 BPS states in field theory & Fermi liquid.
- Technique for constructing gravity solutions.
- Solutions in IIB SUGRA and Laplace equation.
- Solutions in 11D SUGRA and Toda equation.
- Summary.

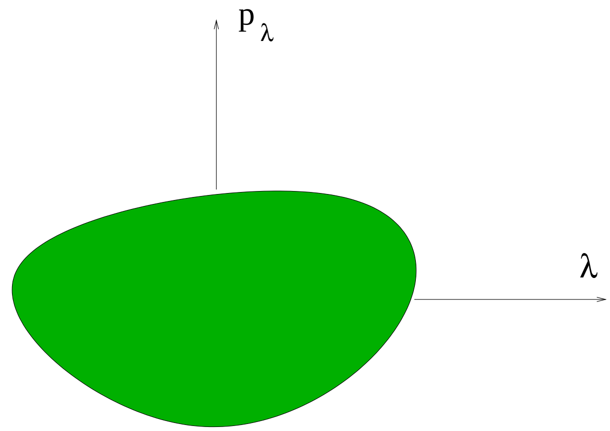
Half–BPS states in $\mathcal{N} = 4$ SYM

- $\mathcal{N} = 4$ SYM on $S^3 \times R$:
  - chiral primaries:

$$\text{Tr}(Z^{n_1}) \ldots \text{Tr}(Z^{n_k}), \qquad Z = \phi_1 + i\phi_2$$

  - symmery: $S^3 \times SO(4)$
- Matrix model description: harmonic oscillator
  - set of harmonic oscillators: $\alpha_n^\dagger = \text{Tr}[(a^\dagger)^n]$
  - eigenvalue basis & Fermi liquid

Berenstein '04



- Brane probe approximation
  - giant gravitons expanding on S^3 or $\tilde{S}^3$.

McGreevy, Susskind, Toumbas '00

  - $AdS_7 \times S^4$: giant gravitons with $S^5 \times S^2$.

Technique for constructing gravity solutions

- Assumptions
  - bosonic symmetries: $SO(4) \times SO(4)$
  - bosonic fields: mertic and $F^{(5)}$
  - existence of Killing spinor:

$$\nabla_M \eta + \frac{i}{480} \Gamma^{M_1 M_2 M_3 M_4 M_5} F^{(5)}_{M_1 M_2 M_3 M_4 M_5} \Gamma_M \eta = 0$$

- Reduction on $S^3 \times S^3$: spinor in 4D interacting with gauge field and 2 scalars
- Using bilinears of Killing spinor

Gauntlett, Gutowski, Martelli, Pakis, Reall, Sparks, Waldrum '02–'04

$$K_\mu = -\bar{\varepsilon}\gamma_\mu \varepsilon \qquad K \cdot L = 0, \quad L^2 = -K^2$$

$$L_\mu = \bar{\varepsilon}\gamma^5 \gamma_\mu \varepsilon$$

  - L is an exact form, K^μ is a Killing vector

$$ds^2 = h^2 dy^2 - h^{-2}(dt + V_i dx^i)^2 + \tilde{h}_{ij} dx^i dx^j$$

1/2 BPS geometries in Type IIB SUGRA

- Explicit geometry and Laplace equation

$$\begin{aligned} ds^2 &= -h^{-2}(dt + V_i dx^i) + h^2(dy^2 + dx^i dx^i) \\ &\quad + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2 \\ F_{(5)} &= F_{\mu\nu} dx^\mu \wedge dx^\nu \wedge d\Omega_3 + \tilde{F}_{\mu\nu} dx^\mu \wedge dx^\nu \wedge d\tilde{\Omega}_3 \end{aligned}$$

$$F = dB_t(dt + V) + B_t dV + d\hat{B}$$

  – functions appearing in the solution:

$$h^{-2} = 2y\cosh G, \qquad ydV = *_3 dz$$

$$B_t = -\frac{1}{4}y^2 e^{2G}, \qquad d\hat{B} = -\frac{1}{4}y^3 *_3 d(\frac{z + \frac{1}{2}}{y^2})$$

$$\tilde{B}_t = -\frac{1}{4}y^2 e^{-2G} \qquad d\hat{\tilde{B}} = -\frac{1}{4}y^3 *_3 d(\frac{z - \frac{1}{2}}{y^2})$$

  – solution is paramaterized by one function z

$$z = \frac{1}{2}\tanh(G), \qquad \partial_i\partial_i z + y\partial_y(\frac{\partial_y z}{y}) = 0$$

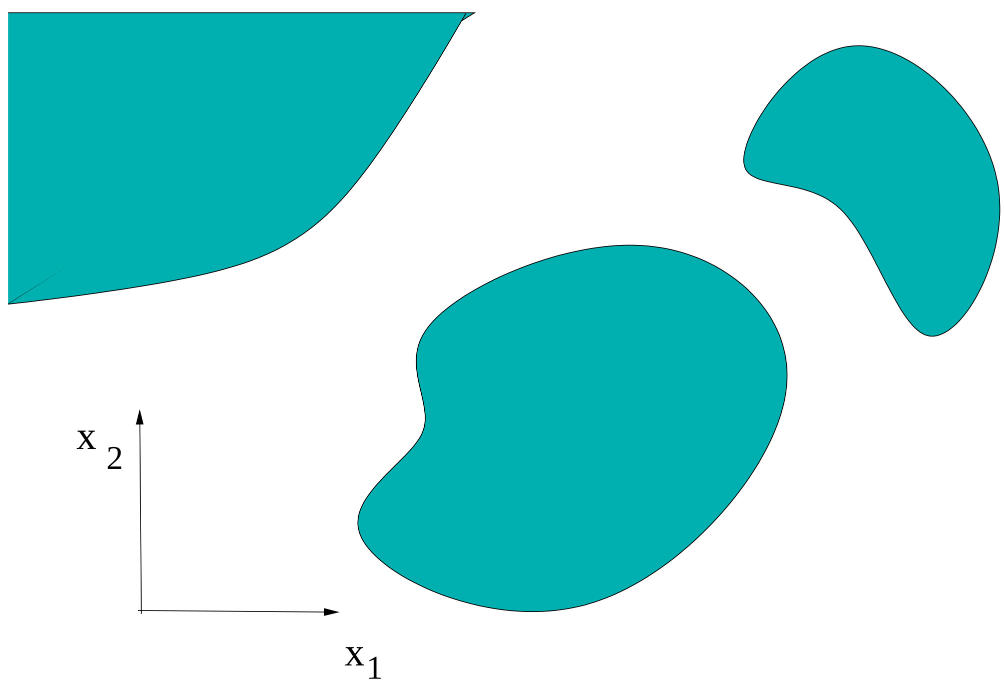
- potential singularities: $R\tilde{R} = y = 0$

Regular solutions:
general description

- Laplace equation and boundary conditions
  - 6D Laplace equation for $\Phi = \frac{z}{y^2}$
  - regularity at $y = 0$: $z = \pm\frac{1}{2}$

$$h^2 dy^2 + y e^{-G} d\tilde{\Omega}_3^2 \sim \frac{1}{c(x)}(dy^2 + y^2 d\tilde{\Omega}_3^2)$$

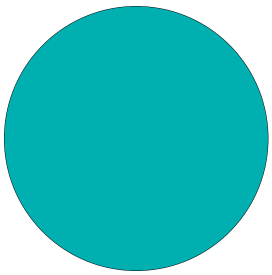
- Boundary condition for a generic state



- Plane $y = 0 \leftrightarrow$ phase space of the oscillator

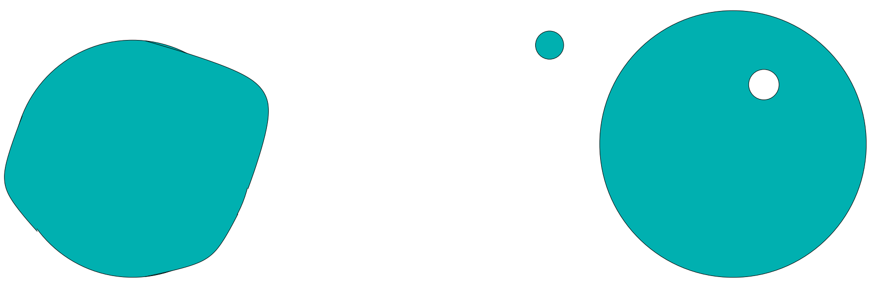
Examples

- $AdS_5 \times S^5$

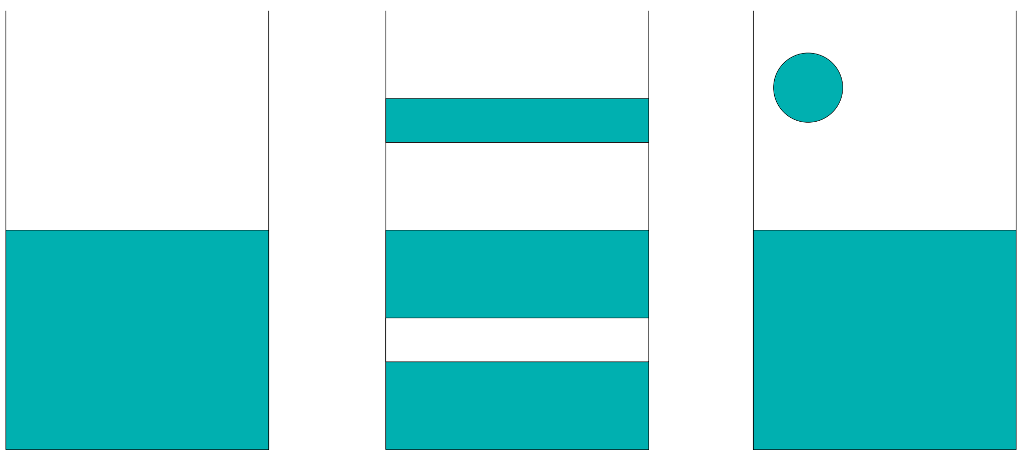


$$z = \frac{r^2 - r_0^2 + y^2}{2\sqrt{(r^2+r_0^2+y^2)^2 - 4r^2 r_0^2}}$$

- (Giant) gravitons



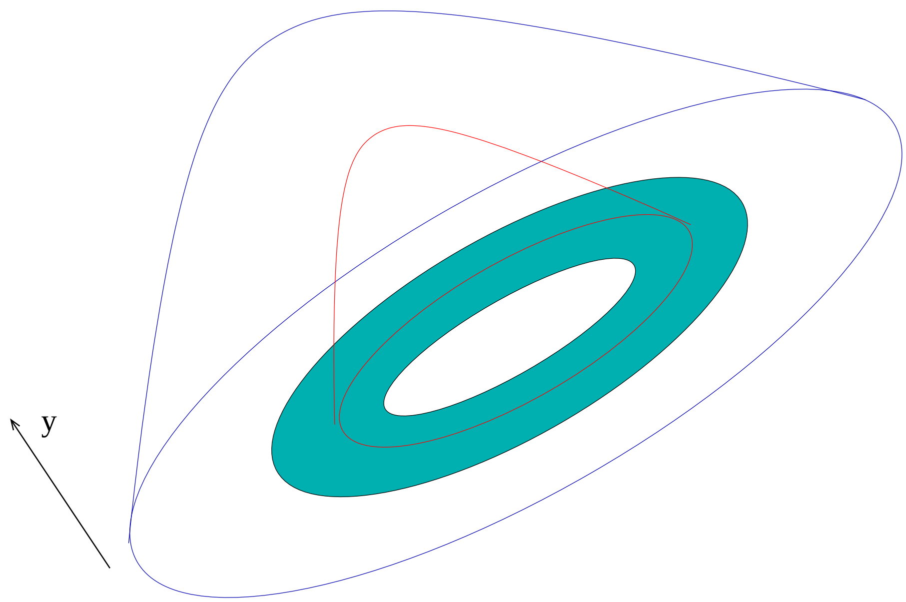
- PP wave and its excitations



$$z = \frac{x_2}{2\sqrt{x_2^2 + y^2}}$$

Topology and fluxes

- Two types of closed five–manifolds



- Different topologies: non–contractible spheres
- Quantization of fluxes:

$$\tilde{N} = -\frac{1}{2\pi^2 l_p^4}\int d\hat{\tilde{B}} = \frac{(\mathrm{Area})_{z=-\frac{1}{2}}}{4\pi^2 l_p^4}$$

- Energy and higher moments

$$\Delta = J = \int \frac{d^2x}{2\pi\hbar}\frac{\frac{1}{2}(x_1^2+x_2^2)}{\hbar} - \frac{1}{2}\left(\int_D \frac{d^2x}{2\pi\hbar}\right)^2$$

$1/2$ BPS geometries in M theory

- Bosonic symmetries: $SO(6) \times SO(2)$

$$
\begin{aligned}
ds^2_{11} &= \frac{e^{2\lambda}}{m^2} d\Omega_5^2 + \frac{y^2 e^{-4\lambda}}{4m^2} d\tilde{\Omega}_2^2 \\
&- \frac{e^{2\lambda} h^2}{m^2} (dt + V_i dx^i)^2 + \frac{e^{-4\lambda}}{4m^2 h^2} \left(dy^2 + e^D d\mathbf{x}^2 \right)
\end{aligned}
$$

$$
h = 1 + y^2 e^{-6\lambda}, \qquad e^{-6\lambda} = \frac{\partial_y D}{y(1 - y\partial_y D)}
$$

- Solution is parameterized by one function D

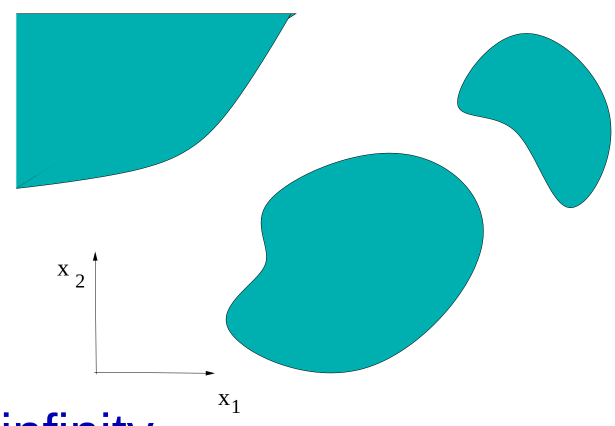
  which satisfies 3D Toda equation:

$$
\Delta D + \partial_y^2 e^D = 0
$$

- Boundary conditions at $y = 0$

$$
\partial_y D = 0, \quad R_2 \to 0
$$

$$
D \sim \log y, \quad R_5 \to 0
$$



- Boundary condition at infinity

Solutions of Toda equation

- $AdS_7 \times S^4$

$$e^D = \frac{r^2 L^{-6}}{4+r^2}, \quad x = (1+\frac{r^2}{4})\cos\theta, \quad 4y = L^{-3} r^2 \sin\theta$$

- $AdS_4 \times S^7$

$$e^D = 4L^{-6}\sqrt{1+\frac{r^2}{4}\sin^2\theta}$$

$$x = \left(1+\frac{r^2}{4}\right)^{1/4}\cos\theta, \quad 2y = L^{-3} r \sin^2\theta$$

- PP wave

$$e^D = \frac{r_5^2}{2}, \quad y = \frac{1}{4} r_5^2 r_2, \quad x_2 = \frac{r_5^2}{4} - \frac{r_2^2}{2}$$

- Translational invariance in x_1: linear equation

$$e^D = \rho^2, \quad \rho\partial_\rho V = y, \quad \partial_\eta V = x_2$$

$$\frac{1}{\rho}\partial_\rho(\rho\partial_\rho V) + \partial_\eta^2 V = 0$$

Ward '90

- Compactification of x_1: type IIA gravity duals of the BMN matrix model

Solution of gauged SUGRA

- M theory on S^4: gauged SUGRA in 7D
  - field content: $SL(5,R)/SO(5)$ coset, $SO(5)$ gauge field, five 3–forms

    Perini, Pilch, van Nieuwenhuizen '84
  - 1/2 BPS black hole: symmetry group $SO(6) \times SO(3) \times SO(2) \times U(1)$

    Liu, Minasian '99
- Our goal: regular supersymmetric solution
  - symmetry $SO(6) \times SO(3) \times U(1)$
  - excited fields:

$$V_I{}^i = \begin{bmatrix} e^{-3\chi} g & \mathbf{0}_{2\times 3} \\ \mathbf{0}_{3\times 2} & e^{2\chi}\mathbf{1}_{3\times 3} \end{bmatrix}, \quad A_{\mu I}{}^J = \begin{bmatrix} iA_\mu \sigma_2 & \mathbf{0}_{2\times 3} \\ \mathbf{0}_{3\times 2} & 0_{3\times 3} \end{bmatrix}$$

$$g = \exp(i\theta\sigma_2)\exp(-\rho\sigma_3) \in SL(2,R)/U(1)$$

- Regular solution exists & gives e^D

Relation to known solutions

- $\mathcal{N} = 2$ superconformal field theories
  - 16 supercharges, $SO(4,2) \times SU(2) \times U(1)$
  - double analytic continuation of 11D solutions:

$$d\Omega_5^2 \to -ds^2_{AdS_5}, \qquad t \to \psi$$

  - different boundary conditions
  - example of a solution

$$e^D = \frac{1}{x_2^2}\left(\frac{1}{4} - y^2\right)$$

Maldacena, Nunez '00

  - new feature: space ends at $y = 1/2$:

$$\psi \sim \psi + 2\pi$$

- M2 brane with mass deformation
  - IIB with x_1 isometry $\longrightarrow$ IIA $\longrightarrow$ M theory
  - Bena & Warner solutions with regular BC

Summary

- Geometries dual to chiral primaries: no singularities & horizons

- All $1/2$ BPS gravity solutions for type IIB
  - reduction to 3D Laplace equation
  - boundary conditions and free fermions
  - explicit solutions in terms of integrals
  - fluxes and topologies

- All $1/2$ BPS solutions of 11D SUGRA
  - reduction to 3D Toda equation
  - specific boundary conditions
  - examples: AdS, pp wave
  - new regular solution of gauged SUGRA

- Future directions
  - properties of the new geometries
  - $1/4$ BPS states