

FLAT SPACE PHYSICS FROM HOLOGRAPHY

R.B. hep-th/0402058, 0310223

Does the (conjectural) holographic relation between
spacetime geometry and its information content
imply anything about non-gravitational physics?

THE LORE :

H.P.

vs.

Q.F.T.

$$N \sim e^A$$

$$N \sim e^V$$

Gravity "provides cutoff"^{*)} on QFT by backreaction

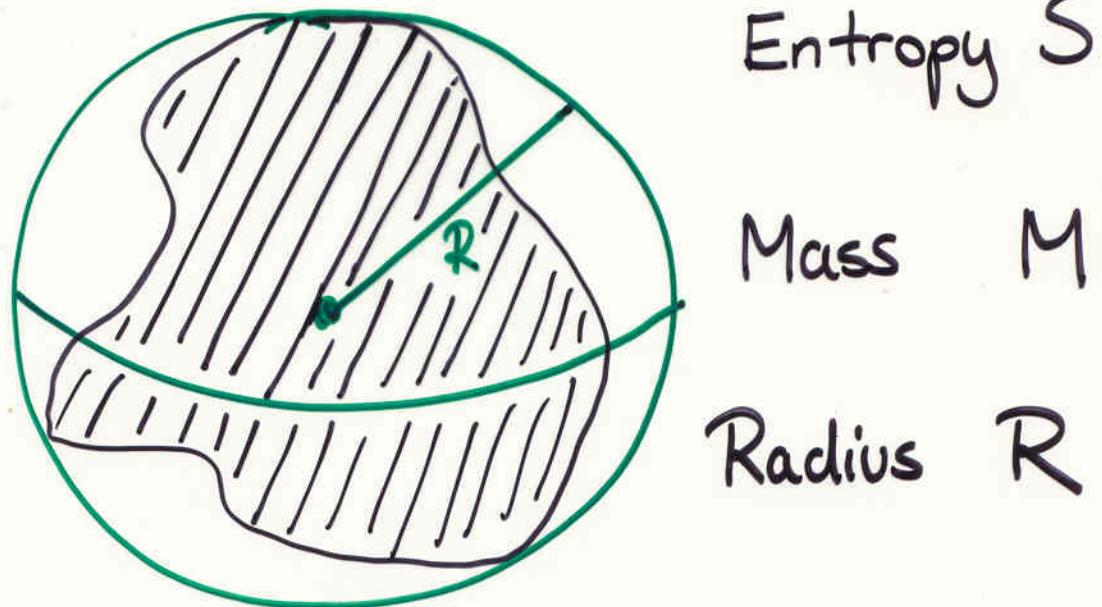
[^{*)} of unspecified type]

In a weakly gravitating region ($GM \ll R$),

holography should restrict N more strongly.

- corresponding entropy bound : Bekenstein bound
- suggests specific cutoff on QFT : DLCQ, finite K

weakly
gravitating
system
 $(GM \ll R)$



$$S \leq \frac{2\pi MR}{\hbar} \ll \frac{\pi R^2}{G\hbar} = \frac{A}{4G\hbar}$$

Bekenstein 1974, 1981

holographic bound

't Hooft 1993

Susskind 1994

tight....

.... lenient

Example :

massive particle (M)

size $R \sim \frac{\hbar}{M}$

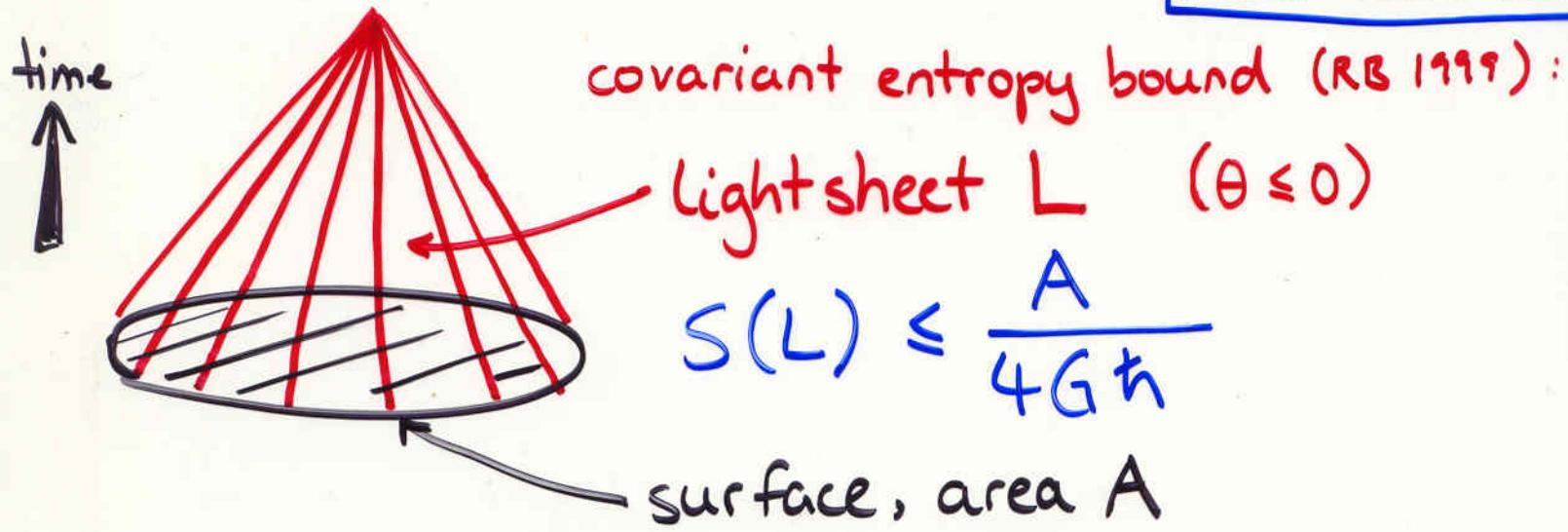
holographic bound $\frac{A}{4G\hbar} \sim \frac{\ell_{Compton}^2}{\ell_{PL}^2} \gg 1$

Bekenstein bound $\frac{2\pi MR}{\hbar} \sim \mathcal{O}(1)$

actual entropy $S \sim \mathcal{O}(1)$

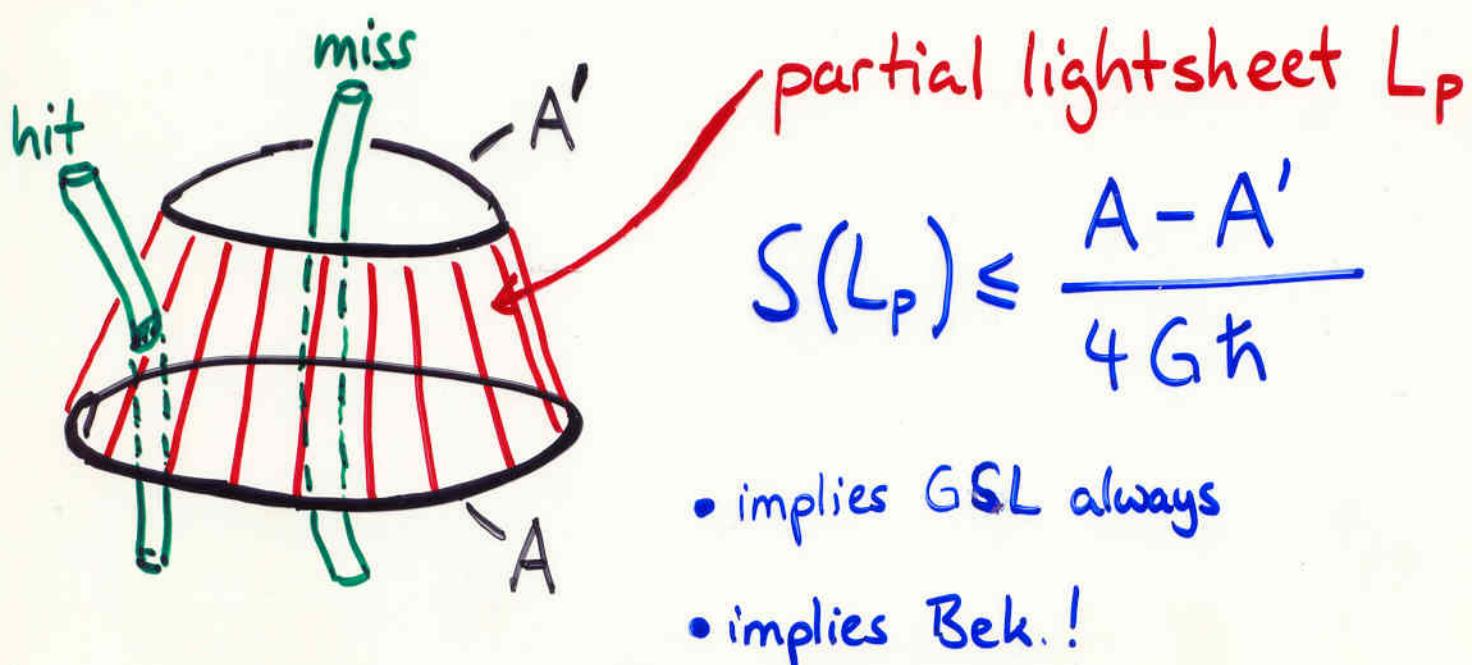
DERIVATION OF BEKENSTEIN'S BOUND:

PRELIMINARIES



- implies $S(V) \leq A/4G\hbar$ if gravity is weak
- implies GSL in black hole formation
- not strong enough to derive Bek.

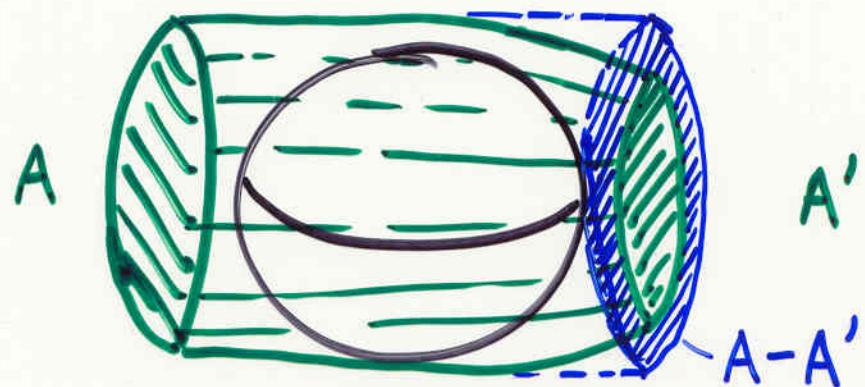
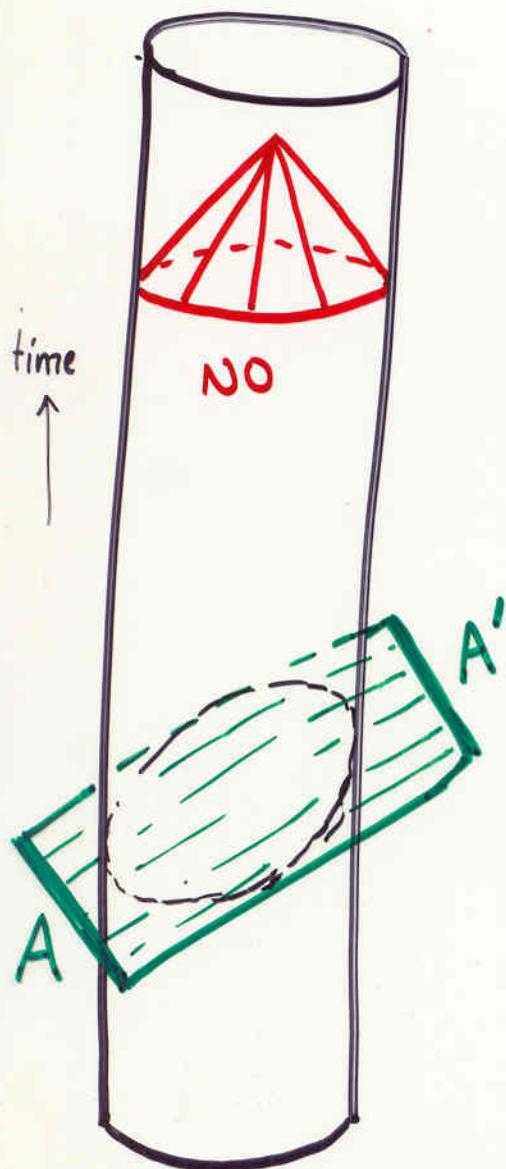
generalization (Flanagan Marolf & Wald, 1999)



DERIVATION OF BEKLENSTEIN'S BOUND:

APPROACH

Choose initial surface astutely: $\theta_0 = 0$



$$\text{angle} \sim \frac{GM}{R}$$

$$\text{annulus width} \sim GM$$

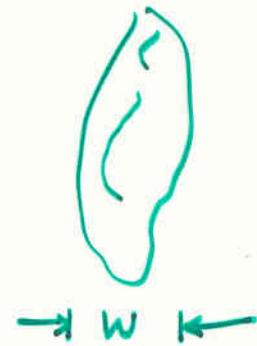
$$\text{annulus area} \sim GMR$$

$$S \leq \frac{A - A'}{4G\hbar} \sim \frac{MR}{\hbar}$$

A more careful calculation yields

$$S \leq \pi M w / \hbar$$

independently of details of $T_{\mu\nu}$.



R.B. hep-th/0210295

One can think of this as an approximate derivation
of the canonical commutation relations of QFT
in the bulk :

- consider holographic relation between information and geometry primary
 - use classical GR
 - derive Bekenstein bound
 - which could be violated if one could have $\Delta x \Delta p \ll \hbar$
 - discover that
- $$S \leq \frac{A - A'}{4G\hbar} \quad (G, \hbar)$$
- $$\underline{G_{ab} = 8\pi G T_{ab}} \quad (G)$$
- $$S \leq \frac{\pi M \omega}{\hbar} \quad (\hbar)$$
- $$\Delta x \Delta p \gtrsim \hbar$$

RB hep-th/0402058

TESTING THE BOUND

- Does Nature obey the Bekenstein bound ?
e.g. Marolf & Sorkin, hep-th/0309218
- ... even with "width" ω instead of $2R$?
- Proof from simple phenomenological assumptions
(not fundamental but common) R.B., Flanagan & Marolf 2003
eliminates most potential counterexamples
- need domination by long wavelength modes,
or violation of energy conditions, to violate bound

The real question is :

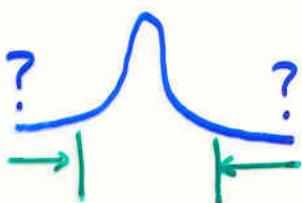
Does there exist an exact formulation of the bound which is true in Nature ?

Attempt : • Specify M, ω .

- Count the number N of Fock space bound states with $E \leq M$, localized to width ω .
- Define $S = \log N$

R.B. 0310148

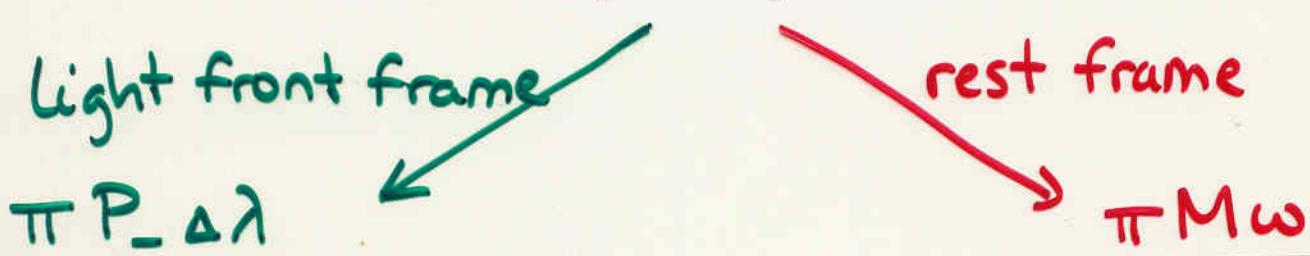
Shortcomings

- Width ambiguity 
- Width unambiguously = 0 for single particle states in free theory 
- Extra macroscopic parameter

cf. also Marolf & Roiban
hep-th/0406037

Observation: full result of derivation is

$$S \leq \pi (P_a k^a)_{\Delta \lambda}$$



- to identify under boosts:
 - set $P_- = \text{const}$
 - $P_\perp = 0$
- \rightarrow only one physical parameter ✓
- LCQ suitable for Hamiltonian methods ✓
- Can instead set $\Delta \lambda = \text{const}$, P_- varies
 - \rightarrow Identify on null circle of length $\Delta \lambda$
 - \rightarrow DLCQ

BEKENSTEIN BOUND IN DLCQ FORM

$$S \leq \pi P_- \Delta \lambda$$

Compactify $\rightarrow P_- \Delta \lambda = 2\pi \hbar K$, K integer

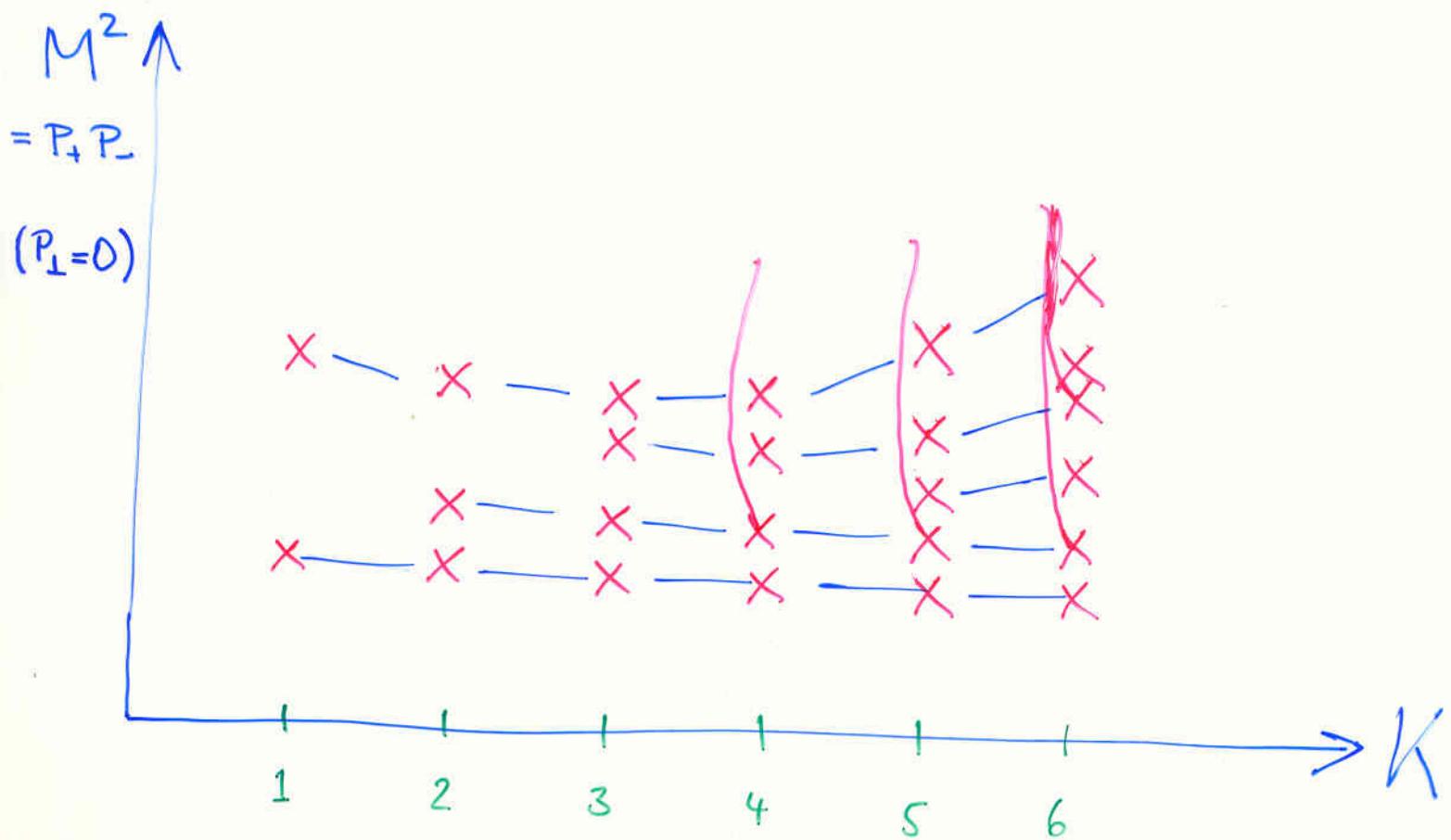
$$S \leq 2\pi^2 K$$

Fock space breaks into sectors labeled by K

$$S = \log N(K)$$

- no width ambiguity, all other problems resolved
- species problem is present

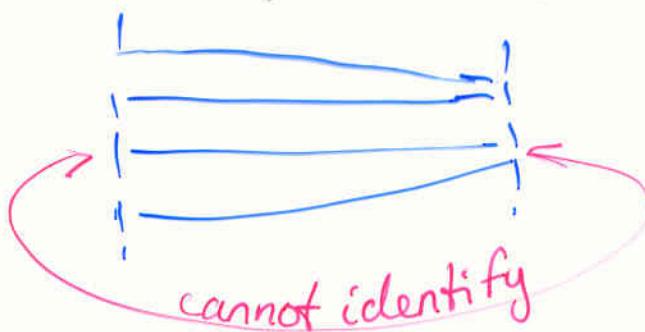
Typical spectrum of DLCQ field theory



- To describe a matter system with mass M , width w well, require $K \gg Mw/\hbar$. (Go to rest frame: want system to be well localized relative to size of circle.)
- $K \rightarrow \infty$ limit recovers full QFT.
- What is the interpretation of finite K ?
Do these spectra have physical meaning?
In the real world, null directions not compact...

- In fact they could not be compact with isometry !

With $G > 0$, focussing them guarantees that for all non-vacuum states, parallel light rays will not remain parallel

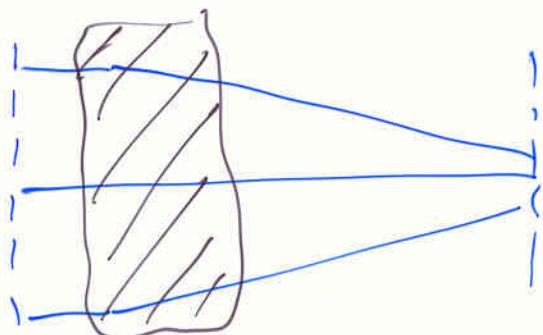


- But fixed background is a good approximation

$$\text{if } \frac{\Delta A}{A_0} \ll 1 \Rightarrow \text{require } K \ll \frac{A_0}{L_p^2}$$

- This provides a "gravity cutoff" limiting the regime of validity of local QFT on fixed background
- For a given system, useful range of K is

$$\frac{M\omega}{\hbar} \ll K \ll \frac{A_0}{L_p^2}$$



Examples :

electron : $1 \ll K \ll 10^{46}$

TeV mass particle : $1 \ll K \ll 10^{34}$

hydrogen atom : $10^5 \ll K \ll 10^{50}$

$$M \rightarrow R/G : \quad \frac{R^2}{\ell_p} \ll K \ll \frac{R^2}{\ell_p} \quad \downarrow$$

QFT as local bulk theory on fixed background
breaks down in this limit. [→ string thy,
Matrix theory]