Three-Charge Supertubes

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Introduction

Much has been learned from relating the gravity and gauge theory descriptions of the D1-D5-P system.

- NS-NS vacuum $\leftrightarrow$ $\mathrm{AdS}_3 \times S^3 \times T^4$ (or $K3$)
- low energy chiral primaries $\leftrightarrow$ sugra perturbations
- Thermal ensemble $\leftrightarrow$ BTZ $\times S^3 \times T^4$

More recently, we have learned (Lunin, Mathur; Lunin, Maldacena, Maoz)

- chiral primaries $\leftrightarrow$ 2-charge supertubes: D1-D5 $\rightarrow$ kk

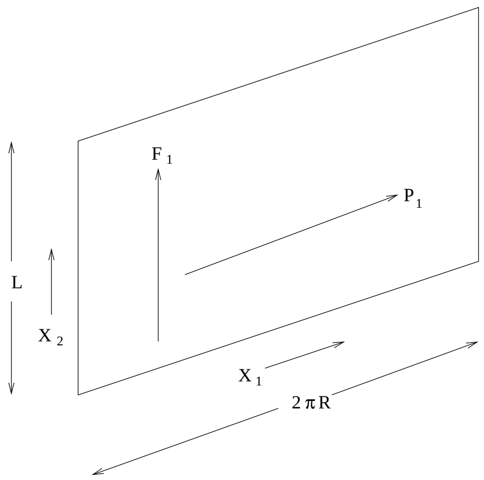
More general 3-charge supertubes exist; where do they fit in the picture?

Review of 2-charge supertubes *(Mateos, Townsend)*

Start with a flat Dp-brane in $x^{0,1,\cdots p}$, and turn on worldvolume electric and magnetic fields

$$2\pi F_{02} = 1, \qquad 2\pi F_{12} = B$$

Induces F1-strings, D(p-2)-branes, and P_1:



$$\begin{array}{rcl} N_{p-2} & \approx & BRL \\ N_{F1} & \approx & RT_p/B \\ P_1 & \approx & RLT_p \end{array}$$

- $N_{p-2}N_{F1} - J = 0, \quad J \equiv P_1 R$

Born-Infeld action gives

$$\mathcal{L}_{BI} = -\sqrt{-\det(\eta_{\mu\nu} + 2\pi F_{\mu\nu})} \approx -B$$

and so the energy is

$$\mathcal{H} = \pi_E F_{02} - \mathcal{L}_{BI} = Q_{F1} + Q_{p-2}$$

- BPS, and no contribution from Dp-brane

Open string quantization

Fluxes described by open string metric:

$$\langle X^\mu(\tau_1) X^\nu(\tau_2)\rangle = -G^{\mu\nu} \ln|\tau_1 - \tau_2|^2 + \frac{i}{2}\theta^{\mu\nu}\epsilon(\tau - \tau')$$

$$G^{\mu\nu} = \begin{pmatrix} -1 + B^{-2} & -B^{-1} & 0 \\ -B^{-1} & 0 & 0 \\ 0 & 0 & B^{-2} \end{pmatrix}$$

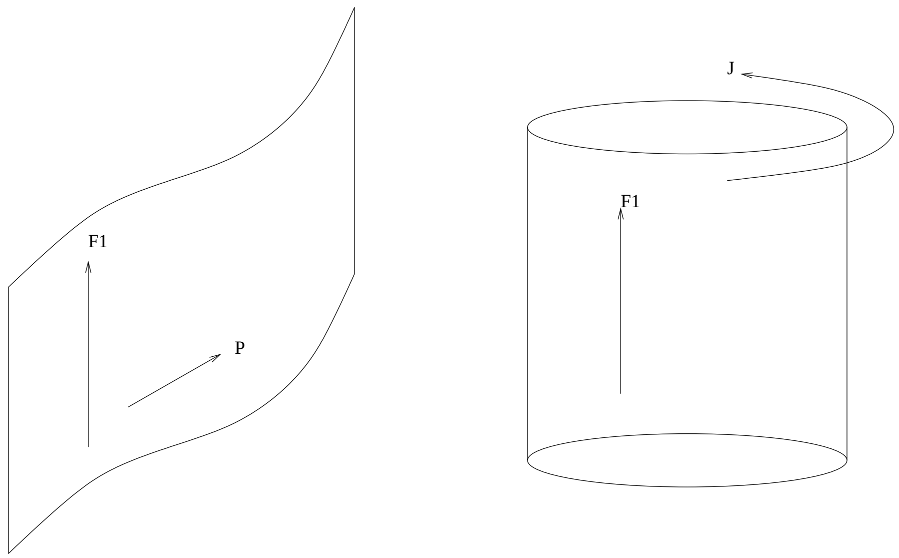
- $G^{11} = 0$! $\quad \Rightarrow \quad \langle X^1(z_1) X^1(z_2)\rangle = 0$

So we can start with a zero momentum vertex operator $\epsilon_\mu \partial_{n,t} X^\mu$ and attach a factor $e^{ip_1 X^1}$ to get a dimension 1 primary (Kruczenski, Myers, Peet, Winters)

$$V = \epsilon_\mu \partial_{n,t} X^\mu e^{ip_1 X^1}, \quad G^{\mu\nu}\epsilon_\mu p_\nu = 0$$

- Adds momentum P_1 but no energy or other charge.
- Multiple such operators can be added, and exponentiated

The Dp-brane can change its shape and local flux density at no cost in energy



In the tubular case J is angular momentum. For a circular tube

$$J = N_{p-2} N_{F1}$$

Adding open string excitations decreases J, and counting is same as for momentum of gas in $1+1$ dim:

$$S \sim \sqrt{c \Delta J} = \sqrt{c(N_{p-2} N_{F1} - J)}$$

- Counting also done by dualizing to FP (Lunin, Mathur) or in Born-Infeld (Marolf, Palmer)

Comments

- Radius formula gives $R^2 \sim g_s$, so at weak coupling the tube structure is lost. Makes counting at weak coupling more subtle.
- But since tubes become large at strong coupling, they are more directly related to finite size gravitational description.
- Entropy of 2-charge tube too small to correspond to classical black hole horizon, but was given a stretched horizon type interpretation

  (Lunin, Mathur)

3-charge supertubes *(Bena, P.K.)*

To compare with black hole physics would like a tube carrying D1-D5-P charges. But more convenient to dualize and take D0-D4-F1 since F1 appears in supertube construction.

Starting from

$$D0 + F1 \quad \to \quad d2$$

and dualizing, we have

$$\begin{aligned} D4 + F1 \quad &\to \quad d6 \\ D0 + D4 \quad &\to \quad ns5 \end{aligned}$$

- So we expect a tube with 3 independent dipole charges: d2, d6, and ns5.
- For now set ns5 dipole to zero, since we can't describe it via flux in Born-Infeld. Can include by T-dualizing ns5 $\to$ kk $\approx A_N$ singularity. Or, work in M-theory (Elvang et. al.)

On a D6-brane turn on fluxes $F_{02}, F_{12}, F_{34}, F_{56}$ to induce charges

$$F_{02} \sim \mathrm{F1-strings}, \quad F_{12} \sim \mathrm{D4-branes}, \quad F_{12}F_{34}F_{56} \sim \mathrm{D0-branes}$$

But also have D2-branes from

$$F_{12}F_{34}, \quad F_{12}F_{56}, \quad F_{34}F_{56}$$

First two are unwanted; last will give wanted d2 dipole.

- Cancel unwanted D2-branes by introducing second D6-brane with flipped signs of F_{34} and F_{56}.
- Generalizing to N_6 such D6-branes, we get a BPS configuration with energy

$$\mathcal{H} = Q_{F1} + Q_{D0} + Q_{D4}$$

and momentum

$$J = P_1 R = \frac{N_{F1} N_{D4}}{N_{D6}}$$

Quantizing the neutral open strings proceeds just as before. Again find BPS fluctuations of shape and flux profiles, and can form circular tube.

Spectrum of charged strings more involved (e.g. Callan et. al.). Need to work with superstring. Zero mode problem in $x^{3,4,5,6}$ like charged particle in magnetic field

$$[P_3, P_4] \approx iF_{34}, \qquad [P_5, P_6] \approx iF_{56}$$

Get a Landau level degeneracy

$$V_{3456}F_{34}F_{56}$$

- Combine these with massless states from R or NS sector.
- Including $X^{0,1,2}$ part, we can again attach $e^{ip_1 X^1}$ factors at no cost in energy.
- With N_6 D6-branes, have number of species

$$N_6^2 V_{3456}F_{34}F_{56} \approx N_6 n_2$$

- Entropy is therefore

$$S \sim \sqrt{N_6 n_2 \left(\frac{N_{F1}N_{D4}}{N_{D6}} - J \right)} = \sqrt{n_2 N_{F1} N_{D4} - N_6 n_2 J}$$

Comments

- Still too small to correspond to black hole area. Need the ns5 dipole!
- Enhancement of entropy compared to 2-charge case came from Landau degeneracy. Corresponds to changes in non-abelian part of flux.
- Since states are described by Landau levels, wavefunctions are inhomogeneous in $x^{3,4,5,6}$.
- So sugra solutions for microstates need to capture non-abelian degrees of freedom, and inhomogeneity on T^4.

Including the ns5 dipole charge

- Including NS5 in the flat case yields a brane carrying charges D2-D6-NS5-P. These are the standard ingredients of the 4d black hole, after compactification on T^6.
- Entropy given by quartic $E_{7(7)}$ invariant:

$$S = 2\pi\sqrt{J_4}$$

$$\begin{aligned} -J_4 &= x^{ij}y_{jk}x^{kl}y_{li} - x^{ij}_{ij}x^{kl}y_{kl}/4 \\ &+ \epsilon^{ijklmnop}(x^{ij}x^{kl}x^{mn}x^{op} + y^{ij}y^{kl}y^{mn}y^{op}) \end{aligned}$$

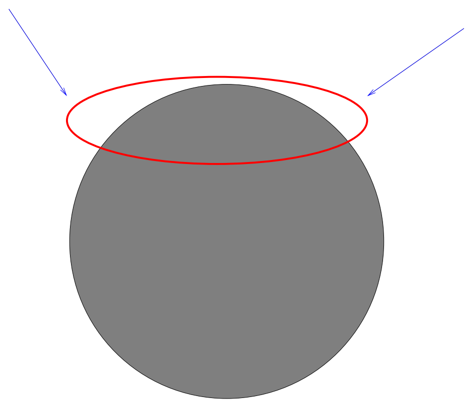
with the charges identified as

$$\begin{aligned} x_{12} &= N_{D0}, \quad x_{34} = N_{D4}, \quad x_{56} = N_{F1}, \quad x_{78} = 0 \\ y^{12} &= n_{d6}, \quad y^{34} = n_{d2}, \quad y^{56} = n_{ns5}, \quad y^{78} = J \end{aligned}$$

- System now has finite size $S^2 \times T^6$ horizon. As before, we can instead curl up one direction into a circle and compactify on T^4. Result should be a horizon of topology $S^1 \times S^2 \times T^4$ — a black ring. Entropy should agree with above.

Supertubes and BMPV

- BMPV has $J_\phi = J_\psi$ with $J_{\phi,\psi} \leq \sqrt{N_1 N_5 N_p}$.
- Supertube (T-dualized) can have $J_\phi \neq J_\psi$ and obeys $-N_1 N_p \leq J_{\phi,\psi} \leq N_1 N_p$.
- Interesting to try to slowly drop supertube through BMPV horizon.



- For general tube profile $r(\sigma), \psi(\sigma)$ find expected BPS no force condition. Shape constrained by

$$(N_p)_{tube} = \int \frac{d\sigma}{B(\sigma)} \, (1 + \frac{(Q_5)_{hole}}{r^2}) \left(\sin^2\theta \, r^2 (\partial_\sigma \psi)^2 + (\partial_\sigma r)^2\right)$$

Horizon at $r = 0$

- For circular case can bring tube to $r = 0$ provided

$$(N_1 N_p)_{tube} \leq (N_5)_{hole} \quad \star$$

So when $\star$ satisfied looks like we can perturb BMPV to $J_\phi \neq J_\psi$. What is the solution?

- Can tube straddle horizon? No: need $\partial_\sigma r|_{r=0} = 0$.
- Can we overspin BMPV and generate CTCs? Probe adds

$$J_{tube} = (N_1 N_p)_{tube}$$

and with $\star$ satisfied find BMPV bound respected:

$$\Big((N_1)_{hole} + (N_1)_{tube}\Big)\Big((N_p)_{hole} + (N_p)_{tube}\Big)(N_5)_{hole} \leq (J_{hole} + J_{tube})^2$$

Microscopic description of black rings (I. Bena, P.K.)

Supergravity solution for 3-charge supertube was found by (Elvang, Emparan, Mateos, Reall) and generalized further by (Bena, Warner; EEMR; Gauntlett, Gutowski)

In IIB frame solutions carries charges

$$N_1 \quad D1(5), \quad N_2 \quad D5(56789), \quad N_3 \quad P(5)$$

and dipole charges

$$n_1 \quad d5(x6789), \quad n_2 \quad d1(x), \quad n3 \quad kk(x56789)$$

- N_i are conserved charges measured at infinity. Due to Chern-Simons terms, these differ from charges $\overline{N}_i$ measured at ring:

$$\overline{N}_1 = N_1 - n_2 n_3, \quad \text{and permutations}$$

- Similarly, “harmonic" functions Z_i are no longer harmonic; have delocalized sources from fluxes.

$$Z_1 = 1 + \frac{\overline{Q}_1}{\Sigma} + \frac{q_2 q_3 \rho^2}{\Sigma^2}$$

with $\Sigma = \sqrt{(\rho^2 - R^2)^2 + 4R^2\rho^2\cos^2\theta}$.

- $1/\Sigma$ is a harmonic function sourced on the ring: $\rho = R, \ \cos\theta = 0$. $R = 0$ gives BMPV.

- Solution carries angular momenta

$$J_\phi = J_{BMPV} = -\frac{1}{2}\sum_i n_i \overline{N}_i - n_1 n_2 n_3, \quad J_\psi = -J_{BMPV} + J_{tube}$$

with

$$J_{tube} = \frac{R_{KK} V_4}{(2\pi)^4 (\alpha')^4 g^2}(q_1 + q_2 + q_3) R^2$$

- Entropy is

$$\begin{aligned} S &= 2\pi\Big[-\frac{1}{4}(n_1^2\overline{N}_1^2 + n_2^2\overline{N}_2^2 + n_3^2\overline{N}_3^2) \\ &+ \frac{1}{2}(n_1 n_2 \overline{N}_1 \overline{N}_2 + n_1 n_3 \overline{N}_1 \overline{N}_3 + n_2 n_3 \overline{N}_2 \overline{N}_3) - n_1 n_2 n_3 (J_\psi + J_\phi)\Big]^{1/2} \\ &= 2\pi\sqrt{J_4} \end{aligned}$$

- Solutions have 7 free parameters, but only 5 conserved charges. So these black objects have “hair". Makes it especially interesting to understand them on gauge theory side.

Decoupling limit

- As with usual D1-D5-P system, we drop the 1 from the D1 and D5 harmonic functions, but keep it in the P harmonic function.
- Solution is then asymptotic to the same AdS$_3 \times S^3 \times T^4$ as for usual D1-D5-P, so we should be able to understand the black rings as states in the usual CFT.
- Work at orbifold point. Have an effective string of length $N_1 N_2$ which can be broken up into any number of integer length components. Each component has 4 bosons and 4 fermions. Fermions are doublets under $SO(4) \approx SU(2)_L \times SU(2)_R$ R-symmetry (rotation) group.
- Diagonal generators are

$$J_L = J_\psi - J_\phi, \quad J_R = J_\psi + J_\phi$$

- Black rings combine properties of BMPV and 2-charge supertubes, and we know how to describe these at orbifold point, so can hope for same with rings.

Review of BMPV and 2-charge tube

- Setting $Q_3 = q_1 = q_2 = 0$ leaves D1-D5 $\rightarrow$ kk tube. Gauge theory description known (Lunin, Mathur) Have

$$J_L = J_R = \frac{N_1 N_2}{n_3}, \quad R = \frac{\sqrt{Q_1 Q_2}}{q_3}$$

Corresponds to breaking up effective string into $\frac{N_1 N_2}{n_3}$ components of length n_3. Each component is in RR vacuum with $J_L = J_R = 1$.

- Setting $R = 0$ gives BMPV with

$$J_L \neq 0, \quad J_R = 0, \quad S = 2\pi\sqrt{N_1 N_2 N_3 - J_L^2/4}$$

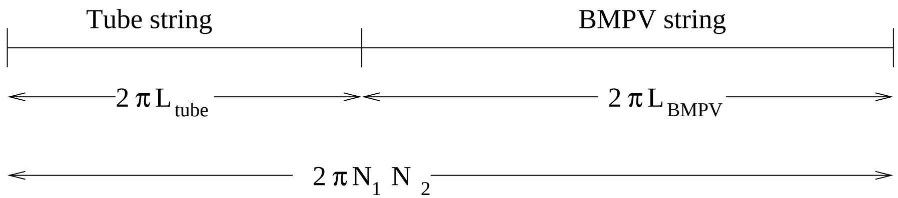
After a coordinate transformation (spectral flow) near horizon geometry becomes BTZ$\times S^3 \times T^4$ (Cvetic, Larsen). Spectral flow invariant version of Cardy formula gives entropy as

$$S = 2\pi\sqrt{\frac{c}{6}(L_0 - 3J_L^2/2c)}, \quad c = 6N_1 N_2$$

- Also recall that BMPV has a single component string (Maldacena, Susskind).

Black ring entropy

- Natural to divide effective string into a tube part and a BMPV part:



- Tube string further breaks up into components of length ℓ_c, and carries J_{tube} but no entropy. BMPV string carries J_{BMPV} and all entropy.
- L_{tube} fixed by $\frac{L_{tube}}{\ell_c} = J_{tube}$. Would like to be able to predict ℓ_c.
- Entropy in this model takes BMPV form

$$S = 2\pi\sqrt{L_{BMPV}N_3 - J_{BMPV}^2}$$

- Convenient to parameterize angular momenta as

$$J_{tube} = \frac{\overline{N}_1\overline{N}_2}{n_3} - \delta, \quad J_{BMPV} = -n_3 N_3 + \gamma$$

which gives

$$S_{ring} = 2\pi\sqrt{n_1 n_2 n_3 \delta - \gamma^2}$$

- 1) $\delta = \gamma = 0$ Take $\ell_c = n_3$, as for 2-charge tube. Then have

$$L_{tube} = \overline{N}_1 \overline{N}_2 \quad \Rightarrow \quad L_{BMPV} = N_1 N_2 - \overline{N}_1 \overline{N}_2 = n_3^2 N_3$$

To account for J_{BMPV} we fill up Fermi sea, which yields correct result

$$J_{BMPV} = -\sqrt{L_{BMPV} N_3} = -n_3 N_3$$

- 2) $\delta \neq 0, \gamma = 0$ Entropy is now nonzero

$$S = 2\pi \sqrt{n_1 n_2 n_3 \delta}$$

Keeping $\ell_c = n_3$ yields wrong result. Correct result obtained from

$$\ell_c = \left(1 + \frac{\overline{N}_3}{N_3} \frac{\delta}{J_{tube}} \right) n_3$$

Might be able to test this via time delay expts.

- 3) $\delta \neq 0, \gamma \neq 0$ After nontrivial cancellations, find that same formula for ℓ_c as in (2) yields correct entropy. Now have additional zero entropy states when $\gamma^2 = n_1 n_2 n_3 \delta$ corresponding to filled Fermi sea.

Near ring geometry

- In the UV (AdS boundary) we have the usual $(4,4)$ CFT with $c_{UV} = 6N_1N_2$.
- In the IR (near the ring) the dipole charges dominate, and we see the CFT of the D1-D5-KK system with $(4,0)$ susy and $c_{IR} = 6n_1n_2n_3$.
- In betweeen have a highly nontrivial RG flow. Note $c_{IR} < c_{UV}$.
- In simplest case $\delta = \gamma = 0$ define

$$\tilde{\psi} = \psi - \frac{1}{q_3}x^+, \quad \tilde{\phi} = \phi + \frac{1}{q_3}x^+, \quad \tilde{x}^+ = q_3\psi$$

to yield near ring

$$AdS_3 \times S^3/Z_{n_3} \times T^4$$

with

$$\ell_{AdS} = \ell_{S^3} = \sqrt{q_1q_2q_3^2}, \quad V_{T^4} \sim \sqrt{\frac{q_1}{q_2}}$$

- Old angular coordinate becomes new coordinate parallel to AdS
- $\tilde{x}^+$ compact and cycle shrinks to zero size: singular.

Remarks

- Can we better understand the states in the UV CFT, and their RG flow to the IR?
- Can we find geometries which cap off smoothly?
- Can we understand microscopic structure of other solutions, e.g. noncircular rings?