Brane-antibrane systems and neutral black holes

Based on hep-th/0403170

Joint work with * **Omid Saremi** *

Amanda Peet

University of Toronto

Department of Physics

and

Canadian Institute for Advanced Research

Cosmology and Gravity Programme

Ohio Center for Theoretical Science

Quantum theory of Black Holes meeting

via Ryerson Access Grid node

19 September 2004 @11:40-12:20EDT

Others' work in similar vein, around/since ours

Guijosa-Hernandez-(Morales-Tecotl): analytics of rotating D3 case, for one j_i, just before our work. (0402158)

Bergman-Lifschytz: subset of our analytic work, just after us. (0403189)

Lifschytz: charged case. (0405042)

Lifschytz: thermalization timescales. (0406203)

* Guijosa-Garcia *: absorption/annihilation, giving better understanding of why energies not temperatures on brane gases equal ($\rightarrow$). (0407075)

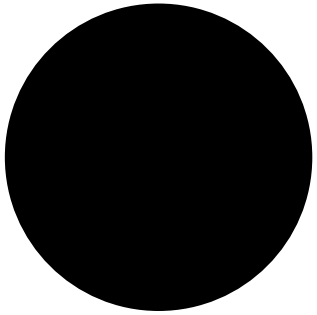
Also, work I don't understand so well by Takahashi, Kalyana Rama, Halyo, Cadoni-Serra (interesting d=2 reduction idea)...

Outline

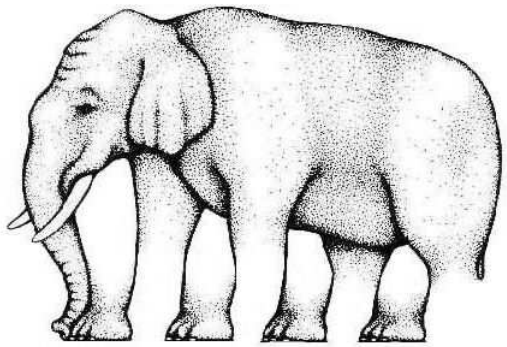
Introduction	(5)
Our analytics	(8)
Our numerics	(5)
Update	(2)
Total	(20)

Introduction

Neutral black holes



= ??



String/M theoretically, neutral black holes most tricky to model.
Apparently, no symmetry or small parameter in which to usefully perturb.

- States maximally far from BPS such as famed Strominger-Vafa BHs, = marginally bound objects with components saturating BPS bound

$$m = |q| c(g_s)$$

- In addition, spacelike singularities, generically hard to resolve.

Question: can we understand *neutral* black holes
($\sim$ condensate of closed-string animals)
purely in terms of open-string animals
(e.g. D-branes and their excitations)?

Open strings *vs.* closed strings

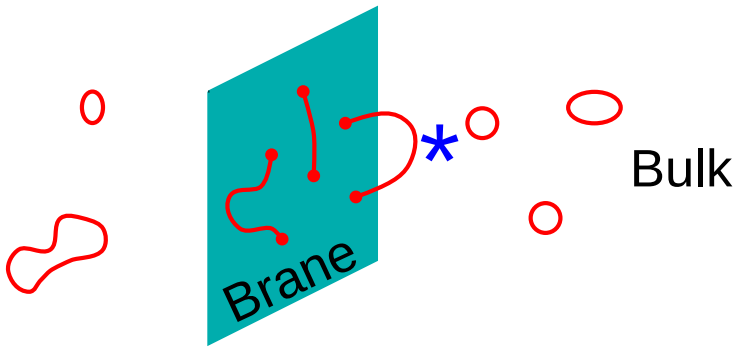
Does open/closed string duality work in general?

- Perturbatively: In unitary theory can take intermediate states, cut Feynman diagram $\Rightarrow$ can put those states external, on-shell.

Is field theory of open strings the "whole enchilada"?

- Promising: OSFT includes D-branes.
- Nontrivial indications re: closed strings e.g. from recent work of Ashoke Sen and others here and elsewhere.
- Will OSFT also handle nonperturbative *closed*-string physics, including calculationally useful machinery? (e.g. study BHs using OSFT?)

"Stripped-down", solid duality:
AdS/CFT correspondence.
Constructed first via decoupling limit applied to set of N D3-branes.



Same decoupling limit on Dp-branes gives IMSY duality.

DGK ('01,'02)

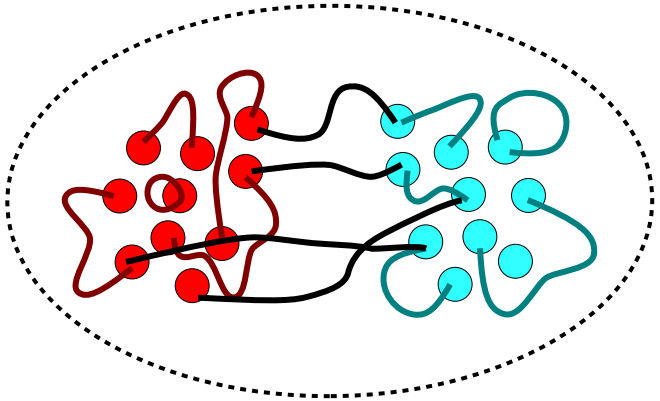
Motivation from odd additivity of far-from-BPS entropy in famed $D = 5$ 3-charge and $D = 4$ 4-charge systems.

Consider conformal branes such as D3.
Geometry: neutral black D3 in $D = 10$ or BH in $D = 10 - 3 = 7$.

What happens to neutral BH at endpoint of evolution? Hawking radiates to long hot string which eventually cascades and decays to closed-string massless modes.

$\Rightarrow$ Look for rendition in microscopic theory which has a vacuum possessing no d.o.f. other than maybe closed strings. SFT?
$\longrightarrow$ Led to brane-antibrane systems.

Model weakly curved BH via
open-string d.o.f., at strong coupling:-
- branes with own SYM (cyan),
- antibranes with own SYM (red),
- $U(N) \times U(\overline{N})$ $\mathbb{C}$ tachyon (black).



DGK ('01,'02) (cont'd)

Why would this model make sense?

QFT intuition [perturbative.]: finite β generates V_{eff}. Rolling uphill costs energy, but it also lowers m_T^2 thereby increasing entropy. So T partially uncondenses at finite-β.

SFT: closed string vacuum = a tachyon condensate $\Rightarrow$ at finite-β, open-string modes reappear.

N D-$\overline{\text{D}}$. *Perturbatively*, diagonal tachyons condense. Energy conserved; ensure all energy flows into open-string modes via $g_s \to 0$ ($g_s N$ fixed).

At $g_s N \ll 1$, endpoint is open-string gas on N partially condensed branes. Minimum of $V_{\text{tot}} = V(T) + V_{\text{eff}}(\beta)$ shifts a bit toward $T = 0$.

Claim: at $g_s N \gg 1$, and finite-β, tachyon T develops large thermal mass, stabilizes around $T = 0$.

Production of closed strings from D-brane annihilation $\propto g_s$. $g_s \ll 1$ suppresses this Hawking radiation, but $g_s N \gg 1$ gives SUGRA regime; then system very long-lived.

DGK ('01,'02) (cont'd)

Truncate and concentrate on lowest modes [-implicitly they believed they could take a decoupling limit like for supersymmetric D-branes].

Strongly coupled brane-antibrane SYM. T makes insignificant contribution to energy or entropy of system because of large thermal mass.

U(N) and ($\overline{N}$) theories on branes and antibranes decouple from each other. Use AdS/CFT to provide correct strong-coupling # d.o.f. in equation of state of SYM.

[Picture reasonable, *if* sub-Hagedorn. For D3, $\beta \gg l_s$ in SUGRA regime.]

Subject to constraint $N - \overline{N} = q$, thermodynamically optimize $N, \overline{N}$.

Work in microcanonical ensemble: in canonical, D3-$\overline{\text{D3}}$ pairs' creation catastrophically sucks energy from reservoir $\leftrightarrow C < 0$.

Taking $m_{\text{SG}} = m_{\text{FT}}$, obtain black hole/brane entropy with right scaling,

$$s_{\text{SG}} \propto m_{\text{SG}}^{5/4} ;$$

only numerical coefficient off by $2^{3/4}$.
Very same factor as Gubser-Klebanov-Peet.

Our Analytics

Our motivations re: neutral BH (in various D)

Analytics:-

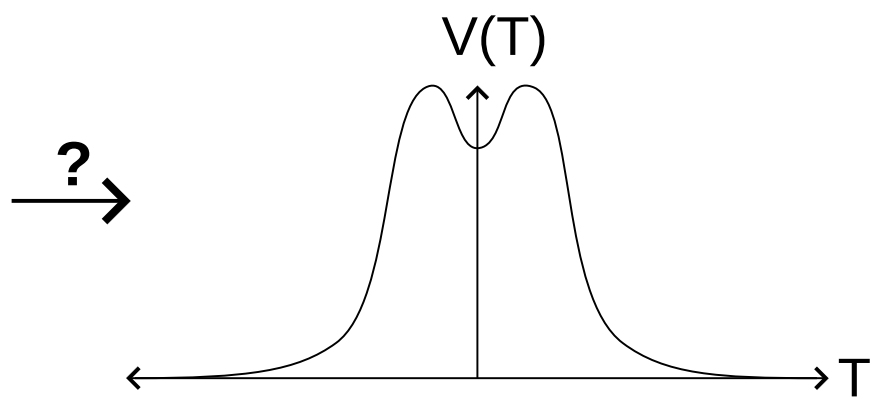
DGK studied only the conformal branes. For general p ($D = 10 - p$), will model give entropy to $\mathcal{O}(1)$, or will scaling fail i.e. miss by a mile? Seat-of-the-pants reasoning: if N is determined thermodynamically in microcanonical ensemble, but m is sole scale in problem, maybe mass scaling of entropy $s(m)$ "had to" come out right? Certainly, equation of state for brane gas is important ingredient of DGK model. Stronger check can be obtained if turn on more independent parameters (j_i).

Numerics:-

DGK claim of

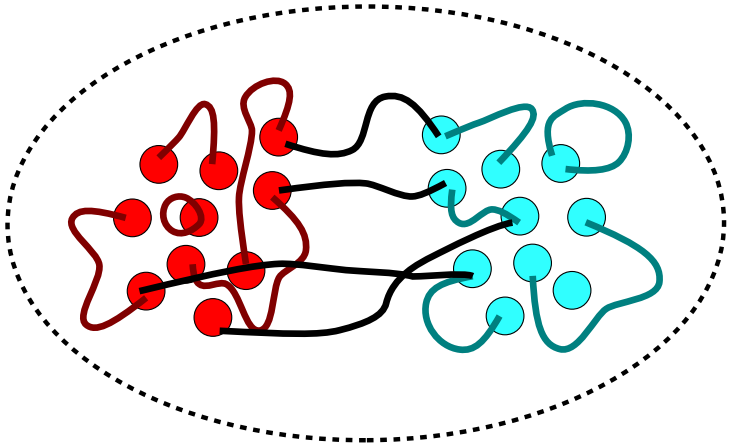
- thermal (but $\beta > \beta_{\text{Hag}}$!)
- strong-coupling

tachyon stabilization hard to check in any strongly coupled QFT.



For $p = 0$, Kabat et al ('99-'00) adapted VPT techniques to numerically simulate $\mathcal{N} = 16$ $U(N)$ SQM at strong coupling. Can we check DGK claim re: tachyon for $p = 0$ using VPT directly on $U(N) \times U(\overline{N})$ theory?

Setup; SUGRA entropy



Mass m, angular momenta j_i,
$i = 1 \ldots [\frac{d-1}{2}]$ (per unit volume).
"Mock up" via NDp, $N\overline{\text{D}p}$ branes.
$U(N) \times U(N)$ SYM$_{p+1}$ + $\mathbb{C}$ tachyon.

Defining $\delta_p \equiv (8-p)/(7-p)$, we have in SUGRA

$$s_{\text{SG}}(m_{\text{SG}}) = \frac{4\pi}{G}(7-p)^{-\delta_p}\delta_p^{-\delta_p}\left(\frac{16\pi}{\Omega_{8-p}}\right)^{\delta_p-1}(Gm_{\text{SG}})^{\delta_p} \times \rho\left(\frac{Gj_{i,\text{SG}}}{(Gm_{\text{SG}})^{\delta_p}}\right),$$

$$\rho(\lambda_i) = \left[\prod_{i=1}^{n}(1+\lambda_i^2)\right]^{-\frac{1}{(7-p)}} \in (0,1],$$

where $\lambda_i \equiv \ell_i/r_H$ obtained by solving $(7-p)^{\text{th}}$ order polynomial

$$\lambda_i\left[\prod_{i=1}^{n}(1+\lambda_i^2)\right]^{-\frac{1}{(7-p)}} = a_p\frac{Gj_{i,\text{SG}}}{(Gm_{\text{SG}})^{\delta_p}} \qquad (*)$$

Strongly coupled SYM rendition of neutral system

Thermodynamic properties of strongly coupled SYM in $D = 10$ SUGRA regime given by taking near-extremal limit of SUGRA formulæ . Focus: equation of state of SYM. (Similar to Klebanov-Tseytlin ('96).)

For open-string gas on (say) set of Dp-branes,

$$s_{\text{branes}} = \frac{2\pi}{b_p}\rho(\lambda_i)\sqrt{N}\, g_{\text{YM}}^{\frac{(p-3)}{(7-p)}}\,(\Delta m)^{\gamma_p}\ ,$$

where

$$b_p \equiv \frac{(7-p)}{2}\gamma_p^{\gamma_p}(2\pi)^{\frac{(p-4)}{(7-p)}}\Big((7-p)\Omega_{8-p}\Big)^{\frac{1}{(7-p)}}\ , \qquad \gamma_p \equiv \frac{(9-p)}{2(7-p)}\,.$$

In brane-antibrane field theory,

$$m_{\text{FT}} = (2)N\tau_p + (2)\Delta m(T_{\text{FT}})\,, \qquad s_{\text{FT}}(T_{\text{FT}}) = (2)s(T_{\text{FT}})$$

Find for strongly coupled SYM$_{p+1}$

$$s_{\text{FT}} = (2)\frac{2\pi}{b_p}\rho(\lambda_i)\sqrt{N}\, g_{\text{YM}}^{\frac{(p-3)}{(7-p)}}\left(\frac{m_{\text{FT}}}{(2)} - N\tau_p\right)^{\gamma_p}.$$

Strongly coupled SYM: entropy result

For *arbitrary* angular momenta, $p \leq 4$, optimizing w.r.t. N yields

$$N = \frac{m_{\mathsf{FT}}}{2\tau_p(1+2\gamma_p)} \quad \text{i.e.} \quad m_{\mathsf{FT}} = (2N\tau_p)2\delta_p\,.$$

Energy density in the brane gases is $\mathcal{O}(2N\tau_p)$. Locked-down proportion of brane gas energy to brane tension energy. Then *either*

$$m_{\mathsf{FT}} = m_{\mathsf{SG}} \quad \text{and} \quad s_{\mathsf{FT}} = 2^{-\frac{(9-p)}{2(7-p)}}\, s_{\mathsf{SG}}$$

or

$$s_{\mathsf{FT}} = s_{\mathsf{SG}} \quad \text{and} \quad m_{\mathsf{FT}} = 2^{\frac{(9-p)}{2(8-p)}}\, m_{\mathsf{SG}}\,.$$

Angular momenta matching requires $j_{i\,,\mathsf{FT}} = 2j_{i\,,\mathsf{SG}}$. Model at this level of sophistication cannot distinguish between renormalization options... Closed-string modes - physics apparently not included in brane-antibrane model - might carry both mass and angular momenta. Binding energy

$$\frac{|m_{\mathsf{FT}} - m_{\mathsf{SG}}|}{m_{\mathsf{SG}}} \equiv \frac{\left|m_{\mathsf{binding}}\right|}{m_{\mathsf{SG}}} = 2^{\frac{(9-p)}{2(8-p)}} - 1\,, \qquad \frac{\left|j_{i,\mathsf{binding}}\right|}{j_{\mathsf{i}\,,\mathsf{SG}}} = 1\,.$$

natural because cleanliness of decoupling limit dodgy!

Heat capacity

How on earth can a system based on (the equation of state of) ordinary SYM yield a system with a *negative* heat capacity?

Previously, found that energy in brane tension $\sim$ energy in brane gases:

$$\frac{m_{\text{gases}}}{m_{\text{tension}}} = \frac{(9-p)}{(7-p)}$$

Now imagine adding more energy to the open string gases. Forced to create more Dp-$\overline{\text{D}p}$ pairs to maintain locked-down proportionality. But making Dp-$\overline{\text{D}p}$ pairs costs energy, so cools system. Therefore, more massive black branes are colder.

Moral: "normal" SYM field theories on worldvolume of branes+antibranes behave 'abnormally', because total number of SYM degrees of freedom controlled by N is not constant but given thermodynamically by entropy maximization scheme.

What else can the strongly coupled SYM properly represent?

[w.l.o.g. turn off j_i, chase scaling analysis not coefficients...]

Horizon extent

Start with brane-antibrane model info

$$m_{\mathsf{FT}} = m_{\mathsf{branes}} + m_{\mathsf{gases}} \sim m_{\mathsf{gases}} \,.$$

IMSY: spatial extent of bunch of strongly coupled Dp-brane modes is

$$\frac{r_0}{\ell_s^2} \sim (g_{\mathsf{YM}}^2 N)^{\frac{1}{(5-p)}} \beta^{-\frac{2}{(5-p)}} \,.$$

Adding in brane-antibrane model info ex optimization of N,

$$N \sim m_{\mathsf{FT}}/\tau_p \,,$$

get

$$r_0^{5-p} \sim G m_{\mathsf{FT}} \beta^{-\frac{2}{(5-p)}} \,.$$

Lastly, use IMSY duality for strongly coupled SYM relationship $\beta(m)$:

$$\beta \sim (G m_{\mathsf{FT}})^{\frac{1}{(7-p)}} \,,$$

giving

$$r_0 \sim (G m_{\mathsf{FT}})^{\frac{1}{(7-p)}} \,.$$

Since $m_{\mathsf{FT}} \sim m_{\mathsf{SG}}$, this is the same radius as that of the horizon in the *neutral* geometry of interest. [... no, this was not a content-free trick!]

Validity of $D = 10$ SUGRA?

For neutral black Dp-branes, if bona fide supergravity entities, require corrections small (at least, at horizon and outside it). For α'

$$(Gm)^{1/(7-p)} \gg \ell_s \,,$$

while for g_s obviously

$$g_s \ll 1 \,.$$

Using optimized value of N, α' condition translates to

$$(g_s^2 \ell_s^8 N \tau_p)^{1/(7-p)} \gg \ell_s \,, \qquad \text{i.e.} \quad g_s N \gg 1 \,.$$

Now, brane-antibrane model of neutral BH used IMSY duality (worked to $\mathcal{O}(1)$). Want to check validity of $D = 10$ near-extremal SUGRA geometry used to get strongly-coupled SYM equation of state. Need

$$1 \ll g_{\text{eff}}^2(U) \ll N^{\frac{4}{(7-p)}} \,,$$

where effective coupling at energy scale U is

$$g_{\text{eff}}^2(U) \sim g_{\text{YM}}^2 N U^{p-3} \,.$$

(validity of $D = 10$ SUGRA, cont'd...)

Near-extremal $D = 10$ geometry also gives relation between energy above extremality m_{gas} and inverse temperature β

$$U_0 \sim (g_{\text{YM}}^4 m_{\text{gas}})^{\frac{1}{(7-p)}} \sim (g_{\text{YM}}^2 N)^{\frac{1}{(5-p)}} \beta^{-\frac{2}{(5-p)}} .$$

Big neutral black brane has low Hawking temperature, especially in 't Hooft units. $p \leq 4$ so no danger of violating α' Ⓛ end of validity bound.

But! Working at low temperature may worry about other Ⓡ end, e.g. for $p = 0$ potential concerned about being driven to $D = 11$ M theory (and/or a connection to BFSS matrix theory!).

Savior: in brane-antibrane model, N not independent; rather, N thermodynamically determined as $N \sim m_{\text{gas}}/\tau_p$. IMSY $D = 10$ validity region translates into neutral BH language as

$$\frac{1}{N} \ll g_s \ll 1 .$$

I.e. open strings strongly coupled, but closed strings weakly coupled.

Our Numerics

Numerical strategy

Idea of direct numerical simulation of strongly coupled $\mathcal{N} = 16$ SUSY QM not new! Kabat et al. found highly nontrivial agreement with SUGRA, in $\beta F \propto T^{1.8}$, by adapting VPT method. First (approximate) nonperturbative check of any gauge theory/gravity duality.

Two different dimensionful coupling constants in our problem: $g_{\rm YM}^2 N = g_s (2\pi)^{-2} \sqrt{\alpha'}^{-3} N$ (SYM) and $1/\alpha'$ (- tachyon mass^2).

Strategy: compute free energy as function of the tachyon background field, plus other quantities, at finite inverse temperature β. Use VPT for open-string massless modes. Read off from βF

- Sign and magnitude of the Tachyon dynamical mass. Want large dynamical mass for negligible tachyon contribution to entropy in brane-antibrane model.
- Phase Portrait – as function of β and $g_s N$.

N.B.: utmost care taken to stay below Hagedorn temperature.

VPT benefits and drawbacks

VPT: approximate theory by Gaussian theory with variational coeff. Strong-coupling expansion. Practically, must truncate at finite order; find solutions to equations variationally. Minimizing F equivalent to requiring trial Gaussian action to satisfy Schwinger-Dyson eqns. Yields set of algebraic coupled equations (∞; $\Rightarrow$ cut off): "gap equations". Solve for variational param's; substitute back to obtain free energy. VPT checked explicitly for [d=0+0 &] d=0+1 e.g.s where can compute in full interacting theory. Expansion *very* fast at strong coupling.

+ VPT automatically respects 't Hooft counting (large-N, etc).
+ Cures IR problems ex flat directions for D0's; that scales as $\mathcal{O}(N)$ in F so is distinguishable from $\mathcal{O}(N^2)$ contributions of interest.
– VPT not so hot at SUSY and gauge symmetry. Tough to maintain symmetries of S in trial free action S_0. But for SUSY QM's, problem can be solved by working with auxiliary fields. For gauge theory in $d = 0 + 1$, can use one-plaquette action for trial S_0.

Clearly, cannot gauge away Polyakov loop (Wilson loop in τ_{Eucl}.) Gaussian starting point $\Leftrightarrow$ clumped eigenvalues in Gregory-Laflamme language. Here, $d = 10$; no compact dimensions except τ_{Eucl}.

Tachyon action

Most stripped-down version is to ignore fermions (anti-periodic BCs at finite-β). Expect capture of main desired features. For this boson-only toy model, relevant terms quadratic in tachyon fluctuations t are

$$
\begin{aligned}
&S_E = \\
&\frac{1}{g_{\rm YM}^2}\int_0^\beta d\tau(\frac{1}{2}{\rm Tr}D_\tau X^i D_\tau X^i - \frac{1}{4}{\rm Tr}[X^i,X^j][X^i,X^j]) \\
&+\frac{1}{g_{\rm YM}^2}\int_0^\beta d\tau(\frac{1}{2}{\rm Tr}D_\tau \overline{X}^i D_\tau \overline{X}^i - \frac{1}{4}{\rm Tr}[\overline{X}^i,\overline{X}^j][\overline{X}^i,\overline{X}^j]) \\
&+\int_0^\beta d\tau {\rm Tr}[\frac{1}{2g_{\rm YM}^2}(\partial_\tau t^\dagger \partial_\tau t + A^0 t^\dagger t A^0 + t^\dagger \overline{A}^0 \overline{A}^0 t + X^i t^\dagger t X^i + t^\dagger \overline{X}^i \overline{X}^i t \\
&-\partial_0 t^\dagger(\overline{A}^0 t - tA^0) + (A^0 t^\dagger - t^\dagger \overline{A}^0)\partial_0 t) + 2\tau_0 \mathbb{I}_{N\times N} - \frac{2\tau_0 \pi^2 \alpha'}{4\ln 2} t^\dagger t]\,.
\end{aligned}
$$

Obviously, always possible to stabilize tachyon at weak couplings – but temperatures needed is well above Hagedorn. Perturbatively, i.e. for $g_s N \ll 1$, to stabilize tachyon sufficient to require

$$\beta < (g_s N)^{4/3}\ell_s \ll \ell_s.$$

Degree of difficulty

Earlier, saw that $g_s N \gg 1$. Is the wrong-sign mass term for tachyon then suppressed? No! Must stay below Hagedorn. In dimensionless units, $\beta \gg (g_s N)^{1/3} \gg 1$. At such low temp, nontrivial to overcome wrong-sign mass term through tachyon couplings to $X^i, \overline{X}^i$ and $A_0, \overline{A}_0$.

Situation even more tricky than that! All variational parameters are *implicit* functions of tachyon, when studying cutoff gap equations. So even trying to read off effective tachyon mass from coefficients of quadratic tachyon fluctuations in free energy is inadvisable.

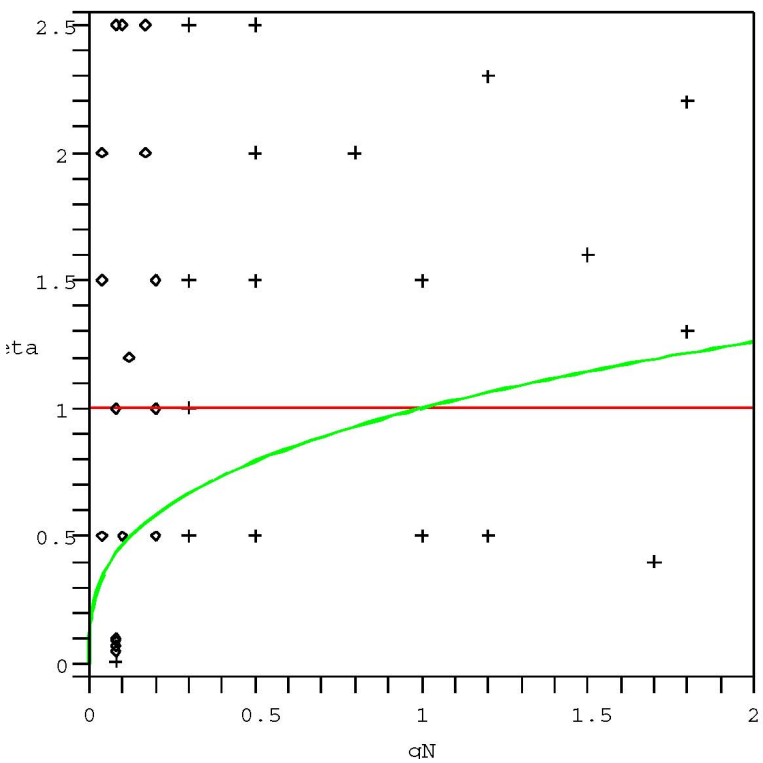
Numerical simulation of this action (approximation to system of D0-branes, anti-D0-branes and tachyon) computationally very expensive – need to find solution to hundreds of nonlinear gap equations.

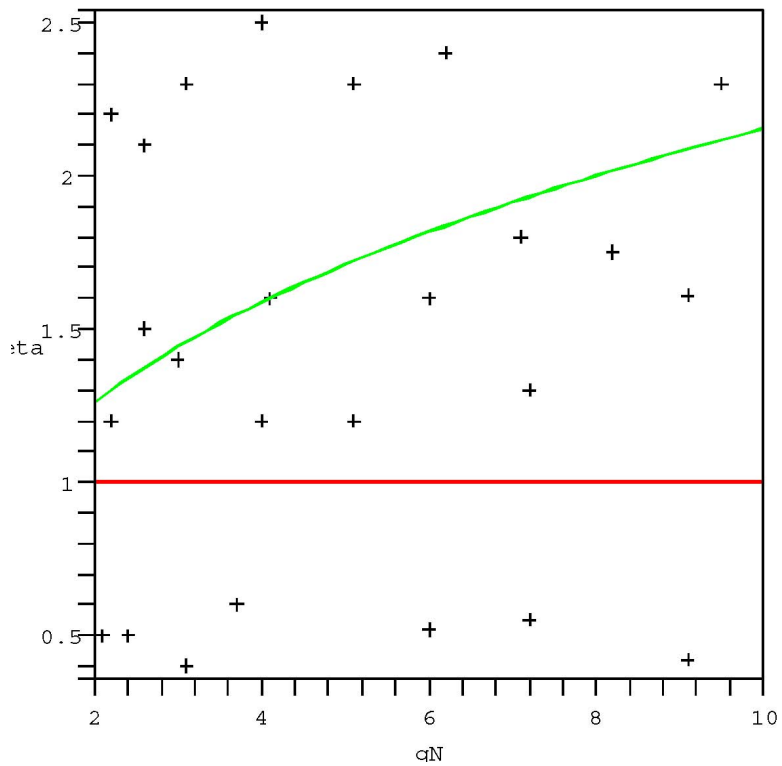
In addition, want to map out phase portrait as function of β, $g_s N$.

Use Metropolis algorithm (Monte Carlo local update method); recast problem of finding common solution to set $\mathcal{G} = \{f_i = 0, i = 1 \ldots n\}$ of nonlinear coupled algebraic equations as finding global minimum for $\mathcal{H} = \sum_i f_i^2$. Metropolis famous for *not* getting stuck in local minima.

Results! (preliminary)

x-axis: open-string coupling.
y-axis: inverse temperature, in dimensionless 't Hooft units.
crosses: positive tachyon mass-squared; circles where negative.
green: above it, Hagedorn not reached.
red: above it, QM strongly coupled.





At small g_sN: need high temp. for stabilization. At large g_sN, even for low temperatures compared to Hagedorn, indications of stabilization.

Update

Guijosa-Garcia

Two main puzzles yet to be understood:-

• Why does entropy not match exactly?

• Why, for charged cases, must brane and antibrane SYM carry same energy, not temperature?

DGK: If energies on brane, antibrane gases same, unphysical in near-extremal limit where temperature of antibrane gas becomes infinite. Reach correspondence point before that! Antibrane energy will go into long hot string, whose contribution to entropy is smaller than that of brane SYM open-string modes.

GG: Taking IMSY lead on strong-coupling field theory behavior, find

• $\ell = 0$ cross-section yields functional match. Coefficient misses by well-known factor $2^{3/4}$ ($\ell = 0$ cross-section *is* entropy, up to $4G$, so by [Gubser-Klebanov-Peet] this must agree with DGK "miss-by" factor.)

• $\ell > 0$ partial waves $\forall \ell$ yield functional match. Coefficient mismatch of $2^{3/4+\ell/2}$. Coefficient mismatch *disappears* if make our assumption of (ℓ-*in*dependent) binding energy of factor of two.

Guijosa-Garcia (cont'd)

• At lowest order in frequency ω, separate D3 and anti-D3 stacks radiate at *same* rate despite having different T!

$$\frac{2}{T_H} = \frac{1}{T_D} + \frac{1}{T_{\overline{D}}}$$

[But recall: in SUGRA limit, based on D1-D5 intuition and c.f. Kabat et al fast/slow mode intuition, don't expect to probe separate $T_D, T_{\overline{D}}$.]

Difficult to do higher-order, because (for one thing) brane and antibrane stacks might absorb each other's Hawking radiation...

Need for having same energy also means horizon radii according to brane stack and antibrane stack are equal. Consistency check.

What about factor of 2? If include α' corrections, apparently [Kruczenski] one can go through β, m, s computations and find by dimensional analysis just a coefficient renormalization in s, m (similar to moral from study of α' corrections in AdS/CFT, but more subtle). Then, after maximization, modified $s(m)$ relationship. This could give factor of 2.