Two observers agree to discuss a number of Physics problems in relation to four particular events: **A, B, C, D**. One observer is stationary, and he labels the $(x, c*t)$ coordinates of these events as follows (in meters):

$$\mathbf{A}\ (600, 0),$$
$$\mathbf{B}\ (1000, 0),$$
$$\mathbf{C}\ (1666\tfrac{2}{3}, 0),\ \text{and}$$
$$\mathbf{D}\ (1666\tfrac{2}{3}, 1333\tfrac{1}{3}).$$

The other observer is moving with respect to the first with velocity $v_x = 0.8c$ and she sets her clock to zero as she passes the first observer at O.

a) Draw a diagram, showing all these events, indicating the temporal and spatial relationship between them.

(1 point)

b) What are the coordinates of the event **D** in a system moving with the second observer?

(2 points)

c) What is the velocity, measured in the moving coordinates, for a particle which moves from **A** to **D**?

(2 points)

d) The line **CD** represents a particle not moving with respect to the x-axis. How far does the moving observer say the particle is away from O at the instant that the particle passes through **D**?

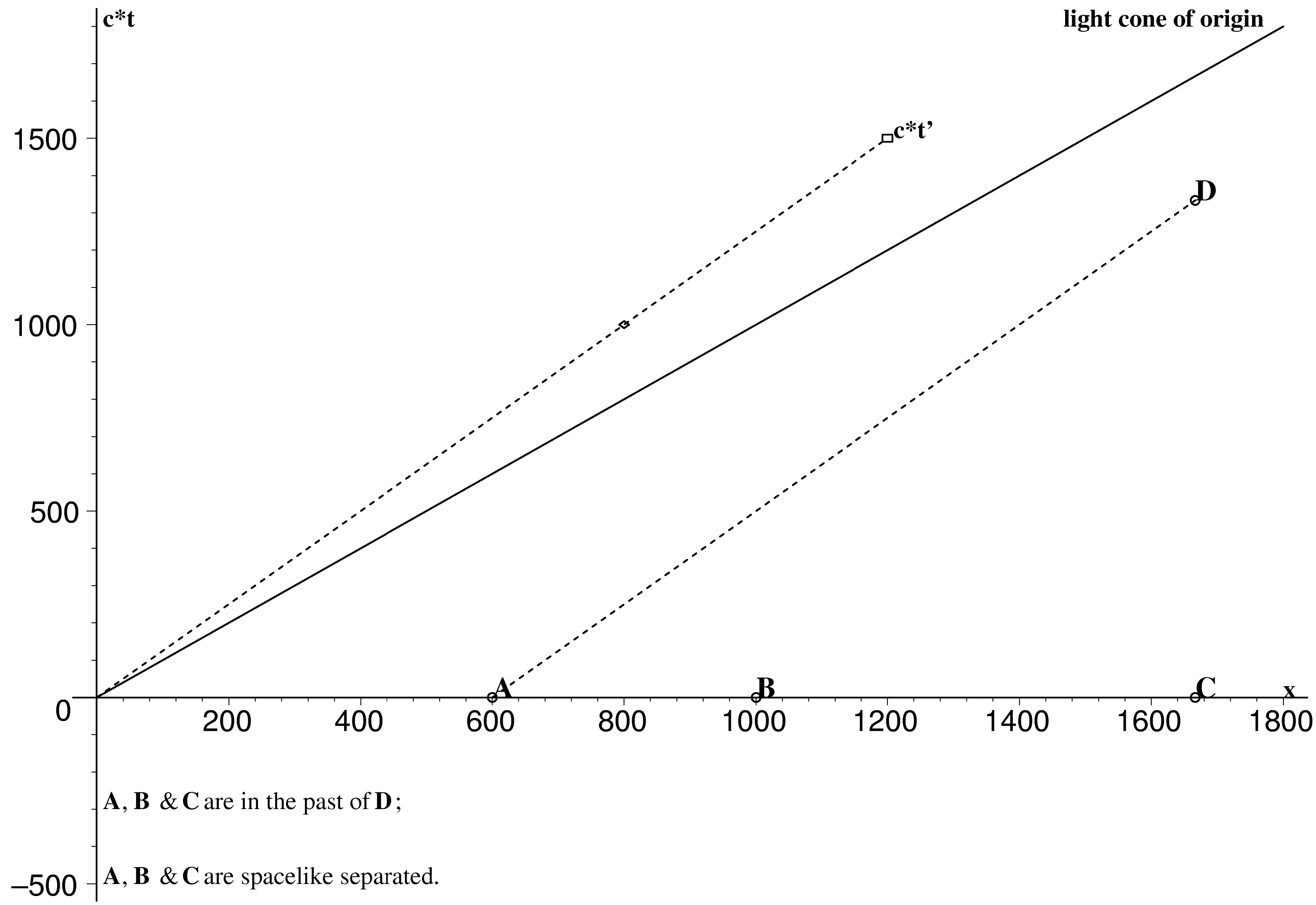
(1 point)

e) A charged particle, of charge Q_1, moves along the straight line passing through **A** and **D**. Meanwhile, the moving observer carries another charged particle of charge Q_2. What is the electromagnetic force exerted on the second particle by the first when the second particle is at point $800m$ from the stationary observer?

(2 points)

f) An uncharged particle moves from **B** to **D**. Find its velocity with respect to the stationary observer, and then use the appropriate velocity addition formula to find the velocity of this particle in the moving system.

(2 points)


c*t
light cone of origin
c*t'
D
A
B
C
x
1500
1000
500
0
–500
200
400
600
800
1000
1200
1400
1600
1800
A, B & C are in the past of D;
A, B & C are spacelike separated.

We will need to make use of the Lorentz transformations for motion in the x-direction.

$$\begin{aligned} x' &= \gamma(x - v_x * t), \text{ and} \\ t' &= \gamma(t - v_x * x/c^2), \text{ where} \\ \gamma &= \frac{1}{\sqrt{1-(v_x/c)^2}}. \end{aligned}$$

For the moving observer we have $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

a) The accompaning diagram shows:
i) the path of the moving observer – in the direction labelled $c * t'$,
ii) part of the future light cone of the origin, and
iii) the path for motion between **A** and **D**.

It is clear that **A**, **B** and **C** are all in the past of **D**, and that **A**, **B** and **C** are all space-like separated from each other, with **A** being closest to the origin and **C** being furtherest from the origin.

b) We find:

$$\begin{aligned} x'_{\mathbf{D}} &= \gamma(x_{\mathbf{D}} - v_x * t_{\mathbf{D}}) \\ &= (5/3) * (1666\frac{2}{3} - 0.8 * 1333\frac{1}{3}) \\ &= 1000m, \text{ and} \\ c * t'_{\mathbf{D}} &= \gamma(c * t_{\mathbf{D}} - v_x * x_{\mathbf{D}}/c) \\ &= (5/3) * (1333\frac{1}{3} - 0.8 * 1666\frac{2}{3}) \\ &= 0. \end{aligned}$$

Thus $(x'_{\mathbf{D}}, c * t'_{\mathbf{D}}) = (1000, 0)$.

c) Similarly, we find:

$$\begin{aligned} x'_{\mathbf{A}} &= \gamma(x_{\mathbf{A}} - v_x * t_{\mathbf{A}}) \\ &= (5/3) * (600 - 0.8 * 0) \\ &= 1000m, \text{ and} \\ c * t'_{\mathbf{A}} &= \gamma(c * t_{\mathbf{A}} - v_x * x_{\mathbf{A}}/c) \\ &= (5/3) * (0 - 0.8 * 600) \\ &= -800m. \end{aligned}$$

Thus $(x'_{\mathbf{D}}, c * t'_{\mathbf{D}}) = (1000, -800)$, and

$$
\begin{aligned}
v'_{\mathbf{AD}}/c &= \frac{x'_{\mathbf{D}} - x'_{\mathbf{A}}}{c * t'_{\mathbf{D}} - c * t'_{\mathbf{A}}} \\
&= \frac{(1000 - 1000)}{(0 - (-800))} \\
&= 0,
\end{aligned}
$$

i.e., the particle remains at a fixed distance from the moving observer.

d) This really involves being able to interpret the result in **b)** above. At **C**, the particle is $1666\frac{2}{3}m$ away from the origin. As time evolves, the particle remains a fixed distance away from the stationary observer, but not from the event O. At the event **D**, the particle is at $t' = 0$, which is also the time at event O for the moving observer. Thus, at **D**, the particle's distance from the moving observer is given by its x'-coordinate. Hence, it is only $1000m$ away from the moving observer at the event **D**.

e) This involves interpreting the results in both **c)** and **d)** above. Since a particle moving along **AD** is at rest with respect to the moving observer, the charge she carries will experience only an electrostatic (Coulomb) force; *i.e.*, the force will be given by

$$
F = \frac{k_e Q_1 Q_2}{(1000m)^2}.
$$

Note the distance used. When the moving observer is $800m$ from the stationary observer, the charge moving along **AD** will have advanced far beyond the event **D**.

f) First, we can compute:

$$
\begin{aligned}
v_{\mathbf{BD}}/c &= \frac{x_{\mathbf{D}} - x_{\mathbf{B}}}{c * t_{\mathbf{D}} - c * t_{\mathbf{A}}} \\
&= \frac{(1666\frac{2}{3} - 1000)}{(1333\frac{1}{3} - 0)} \\
&= 0.5.
\end{aligned}
$$

Then, we must use the velocity addition formula:

$$
\begin{aligned}
v'_{\mathbf{BD}}/c &= \frac{v_{\mathbf{BD}} - v_{O'O}}{1 - v_{\mathbf{BD}} * v_{O'O}/c^2} \\
&= \frac{(0.5 - 0.8)}{(1 - 0.5 * 0.8)} \\
&= -0.5,
\end{aligned}
$$

where, here, $v_{O'O}$ has been used for the velocity of the moving observer relative to the stationary observer. The signs are especially important. Note from the diagram that, in travelling from **B** to **D**, the particle is heading towards the moving observer, and with a negative velocity.