METRIC PERTURBATIONS IN BLACK HOLE SPACE TIMES & THE RADIATION CONDITION IN VACUUM SPACETIMES.

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OUTLINE

RADIATION REACTION ...

- SCHWARZSCHILD

1. VACUUM SITUATION
   (FOR METRIC POTENTIALS)
2. POINT SOURCES
   (WORK IN PROGRESS)

- KERR BLACK HOLE

3. VACUUM SITUATION
   (PROPOSED WORK)

- RADIATION CONDITIONS

4. FLAT SPACE
5. SCHWARZSCHILD

} ONGOING DISCUSSION

- FUTURE EXTENSIONS

6. EXTENDED SOURCES
   (LARGELY UNFORMULATED)

APPLICATIONS

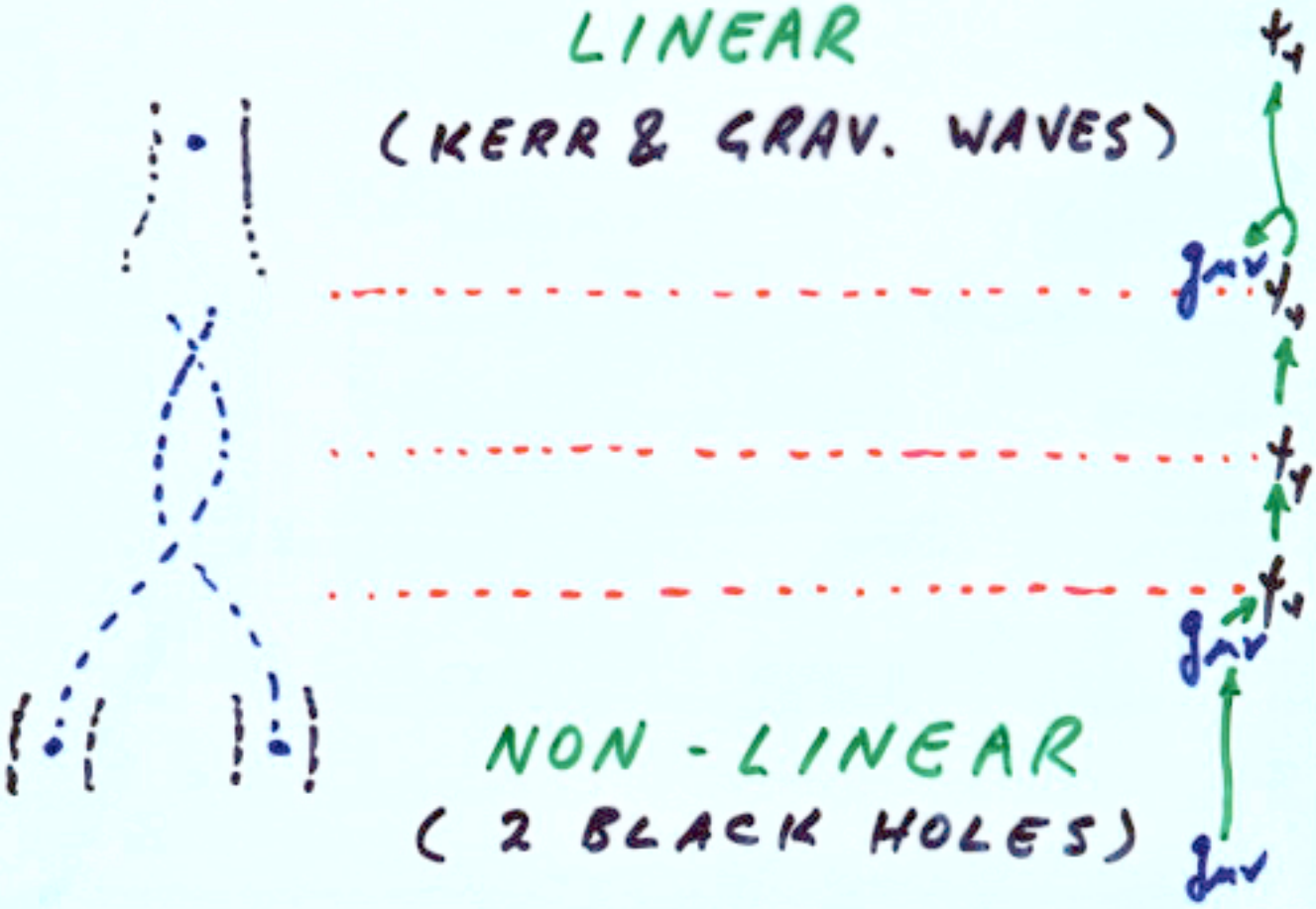
- RADIATION REACTION IN KERR

FIND MOVEMENT OF PARTICLE ORBIT AWAY FROM GEODESIC DUE TO RADIATION

- GRAVITATIONAL COLLAPSE

LINEAR

(KERR & GRAV. WAVES)



NON-LINEAR

(2 BLACK HOLES)

CHRZANOWSKI +

COHEN & KEGELES

GIVE (IN VACUUM):

- I.R. CONDITION

a) $h_{ab} l^b = 0 \quad , \quad h_a{}^a = 0$

b) $h_{ab} = \mathcal{D}_2 (\psi_{IRG})$

c) $\Delta^2 \psi_0 = \Delta^2 D^4 \bar{\psi}_{IRG}$

d) $\bar{\rho}^4 \psi_4 = \bar{L}^4 \bar{\psi}_{IRG} - 3M \dot{\psi}_{IRG}$

- O.R. CONDITION

a) $h_{ab} n^b = 0 \quad , \quad h_a{}^a = 0$

b) $h_{ab} = \tilde{\mathcal{D}}_2 (\psi_{ORG})$

c) $\Delta^2 \cdot \psi_0 = L^4 \Delta^2 \bar{\psi}_{ORG} + 3M \Delta^2 \dot{\psi}_{ORG}$

d) $\bar{\rho}^4 \psi_4 = \frac{1}{16} \Delta^2 d^4 (\Delta^2 \bar{\psi}_{ORG})$

WHERE: i) $J_{-2} (\psi_4, \psi_{IRG}) = 0$

ii) $J_{+2} (\psi_0, \psi_{ORG}) = 0$.

A ROLE FOR ALGEBRAICALLY SPECIAL PERTURBATIONS

① SCHW. VAC. (MODES).

$$\rho^{-4}\psi_4 = c\,.\,\bar{\psi}_{IRG} - 3M\,\dot{\psi}_{IRG} \qquad (=\psi)$$

$$c = (\ell-1).\ell.(\ell+1)(\ell+2)/4 \quad , \quad \Omega_{AS} = 2c/3 .$$

$$J_{-2}(\Omega_{AS})\,\psi_{IRG} = \underbrace{-\frac{1}{3M}\left\{\frac{r^4}{\Delta}\cdot\left(\dot{\psi} + \frac{\Omega}{2M}\bar{\psi}\right) - \frac{4r^2(r-3M)}{\Delta}\psi\right\}}_{F(r)}$$

WRITE $\psi_{IRG} = \psi_{INT} + \psi_s$

- $$\mathrm{Re}(\psi_{INT}) = -y_1(r)\int_{2M}^{r}\frac{z_1(r')\,\mathrm{Re}(F(r'))}{\Delta^2(r')}\,dr' - z_1(r)\int_{r}^{\infty}\frac{y_1(r')\,\mathrm{Re}(F(r'))}{\Delta^2(r')}\,dr'$$

- $$\mathrm{Im}(\psi_{INT}) = z_2(r)\int_{2M}^{r}\frac{y_2(r')\,\mathrm{Im}(F(r'))}{\Delta^2(r')}\,dr' + y_2(r)\int_{r}^{\infty}\frac{z_1(r')\,\mathrm{Im}(F(r'))}{\Delta^2}\,dr'$$

- $$\psi_s = \left(a\,y_1(r) + b\,z_1(r)\right)e^{\Omega t/2M} + i\left(c\,y_2(r) + d\,z_2(r)\right)e^{-\Omega t/2M} .$$

② POINT SOURCES (RADIAL)

- $\overset{Re}{\Lambda}(\psi_s) = \alpha(t)\, y_1(r)\, \Theta(r-r_p) + \beta(t)\, z_1(r)\big(1-\Theta(r-r_p)\big).$

- $\dot{\psi}_{INT} = \frac{1}{3M}\left(c\,\bar{\psi}_{INT} - e^{-4}\psi_4\right) + \underline{\dot{\psi}_T}$

- $\dot{\psi}_{IRG} = \frac{1}{3M}\left(c\,\bar{\psi}_{IRG} - e^{-4}\psi_4\right)$

$$+ \dot{\psi}_T + \dot{\psi}_s - \frac{c}{3M}\bar{\psi}_s$$

- SOLVE $\dot{\psi}_T + \dot{\psi}_{s,\theta} - \frac{c}{3M}\bar{\psi}_s = 0$ for $\alpha(t), \beta(t)$

- $\dot{\psi}_{IRG} = \frac{1}{3M}\left(c\,\bar{\psi}_{IRG} - e^{-4}\psi_4\right) + \underline{\dot{\psi}_{s,\delta}}$

- $\mathcal{J}_{-2}(\psi_{IRG}) = \underline{A\delta + B\delta'}.$

- $h_{ab} \neq h_{ab}(\psi_{IRG}) = h_{ab}(\psi_{INT}) + (\theta, \delta, \delta' \ldots)$

- <u>PROPOSE</u>: $h_{ab}\ell^b,\ h_a{}^a \sim \delta, \delta'$, etc.

- $\alpha(t) = e^{\Omega t/2M}\left(\int_0^t e^{-\Omega t'/2M}\left(-\frac{1}{3M}\cdot\frac{z_1(r_p')\,\tau(r_p')}{\Delta^2(r_p')}\right)dt' + a_0\right)$

5.

③ KERR VACUUM (PROPOSED)

- SCHW: $\rho^{-4}\psi_4 = c\,\bar{\psi}_{IRG} - 3M\dot{\psi}_{IRG}$

- LAPLACE TRANSFORM (RaL I)

$$\rho^{-4}\tilde{\psi}_4 = (c - 3Ms)\,\tilde{\psi}_{IRG} - 3M\,\psi^{(0)}_{IRG}$$

- TEUKOLSKY OPERATOR

$$-3M.\ F(s) \equiv \tilde{\eth}_{-2}(\rho^{-4}\tilde{\psi}_4) = (c-3Ms)\,\tilde{\eth}_{-2}\,\psi_{IRG} - 3M\,\tilde{\eth}_{-2}(s).\psi^{(0)}_{IRG}$$

SO, FOR $s_{AS} = c/3M \equiv \Omega/2M$, HAVE

$$\underline{\tilde{\eth}_{-2}(\Omega/2M).\,\psi^{(0)}_{IRG} = F(s)}$$

- KERR: $\rho^{-4}\psi_4 = \bar{L}^4\,\bar{\psi}_{IRG} - 3M\dot{\psi}_{IRG}$

- LT: $\rho^{-4}\tilde{\psi}_4 = \widetilde{\bar{L}^4\,\bar{\psi}}_{IRG} - 3Ms\,\tilde{\psi}_{IRG} - 3M\,\psi^{(0)}_{IRG}$

- T.O.: $\tilde{\eth}_{-2}(\rho^{-4}\tilde{\psi}_4) = \tilde{\eth}_{-2}\left(\widetilde{\bar{L}^4\,\bar{\psi}}_{IRG} - 3Ms\,\tilde{\psi}_{IRG}\right) - 3M\,\eth_{-2}\psi^{(0)}_{IRG}$

- USE: i) $\eth_{-2}\,\bar{L}^4\big|_{+2} = L^4\,\eth_{+2}\big|_{+2}$

ii) ${}_{-2}S_{\ell m}(a\,\omega_{AS}(\Omega), \theta)$ ...

(4.5) GAUGES ?

$$
\begin{tikzcd}
 & h^{WD}_{ab} \arrow[dl, "\xi^{RW}_a(h^{WD})"'] \arrow[dr, "\xi^{HG}_a(h^{WD})"] & \\
h^{RW}_{ab} \arrow[rr, leftrightarrow, "\xi^{HG}_a(h^{RW})", "\xi^{RW}_a(h^{HG})"'] \arrow[dr, "\xi^{RC} = ?"'] & & h^{HG}_{ab} \arrow[dl, "\xi^{RC} = ?"] \\
 & h^{RC}_{ab} &
\end{tikzcd}
$$

- CONSISTENCY OF ξ^{RC} EQUATIONS:

$$\delta G_{ab}(h^{WD}) \equiv 0 \; !$$

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(6) EXTENDED SOURCES

- $h^{IRC}_{ab}(T_+)$, $h^{ORC}_{ab}(T_0)$ LACK

$$\underline{h(l_a, n_b) \; , \; h(m_a, \bar{m}_b)} \; : \text{ cf } g_{ab}$$

- <u>DEMAND</u> : $\delta G_{ab}(h^{ERC}_{\cdot\cdot}) = \frac{1}{8\pi G}\delta T_{ab}$