

RADIATION REACTION AND THE PRINCIPLE OF EQUIVALENCE*

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The principle of equivalence is shown to extend to situations involving radiation reaction. For example, the Lorentz force law governs the motion of an isolated charge undergoing radiation reaction. In the case of an isolated uncharged particle of small mass, it is the geodesic equation which governs the motion, even when radiation reaction is included. For a local observer to understand such motion he must subtract the singular field of the particle from the actual electromagnetic and gravitational fields he measures. The remaining source-free fields are then used in computing the motion of the particle. With only local measurements, the observer has no knowledge of the existence of radiation and sees no effect which he would be compelled to describe as radiation reaction.

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1. Introduction

The principle of equivalence guides us to expect that a particle of small mass, charge and size, when moving freely through a vacuum, follows a worldline that deviates from a geodesic in a manner governed by the Lorentz force law. Moreover, as determined by a local observer, the gravitational and electromagnetic fields responsible for this motion are source-free fields satisfying the Einstein and Maxwell field equations.

To test the Lorentz force law in flat spacetime, a local observer must determine both the worldline of the particle and the actual electromagnetic field in its vicinity. The observer then compares the worldline to an expectation based upon the Lorentz force law. Importantly, the electromagnetic field applied in the Lorentz force is not the actual, measured field. Rather, it is the *remainder* after the particle's own *singular* field is subtracted from the actual field.

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In Dirac's pioneering analysis of radiation reaction¹ in flat spacetime, he precisely identifies the particle's singular field as the "mean of the advanced and retarded fields" of the particle. To do this, he uses the conservation of the stress-energy tensor inside a narrow worldtube surrounding the particle's worldline. In the limit that the radius of the worldtube vanishes, Dirac derives an equation of motion for the particle with radiation reaction effects included. It is implicit that the singular field makes no contribution to the particle's motion.

A remarkable, often overlooked, consequence of Dirac's work is that radiation reaction on a particle is directly associated with motion under a field which, locally, is source-free. If there is no external electromagnetic field, but the particle is caused to accelerate by some mechanical force, the actual measured electromagnetic field will be the particle's retarded field. The actual field and the singular field both solve the same inhomogeneous field equation with the particle as the source. Their difference, the remainder field, which is necessarily a source-free solution of Maxwell's equations, is completely responsible for the electromagnetic force on the particle **and** causes the dissipative effects of radiation reaction.

Two important features of Dirac's characterization of the particle's singular field are required to ensure this satisfactory account of radiation reaction in terms of a locally identifiable source-free field:

- (i) the singular field must exert no force on the particle; and
- (ii) the singular field must be an inhomogeneous solution of Maxwell's equations, with the particle as the only source.

Together, these two important features guarantee that the remainder field is source-free and solely responsible for the electromagnetic force on the particle.

2. Curved Spacetime

The term "self-force" is commonly used to denote all forces arising from the particle's interaction with its own retarded field.

In curved spacetime, new features arise which complicate a straightforward application of Dirac's procedure to determine the self-force. Most importantly, the "mean of the advanced and retarded fields," in fact, exerts a force on the particle in curved spacetime and, therefore, is not the singular field which must be used in determining the self-force. Moreover, while Dirac's method of invoking the conservation of stress energy in a worldtube surrounding the particle does give the correct force,^{2,3} it does not lead to a source-free field as being responsible for the motion. What has been lacking until recently has been the unambiguous identification of the singular field, arising from the particle but having no influence on the particle's motion.

It is important to stress the achievements of DeWitt and Brehme² for scalar and electromagnetic fields, and Mino, Sasaki and Tanaka³ for gravitational fields, while at the same time realizing the shortcomings of their results. This requires some

elaboration. In curved spacetime the Green's function is traditionally decomposed into "direct" and "tail" parts. The consequent "direct" part of the field at a point is determined completely by the source where it intersects the past null cone of the point; in flat spacetime it is the Liénard–Wiechert potential of electromagnetism. The curvature of spacetime causes backscattering of the field, so that the retarded Green's function also has support inside the past null cone. The contribution to the field at a point from sources within the past null cone of the point is called the "tail" part. The self-force on a particle is then composed of two pieces: the first piece comes from the direct part of the field and depends upon the acceleration of the particle's worldline in the background geometry; in flat spacetime this is the Abraham–Lorentz–Dirac (ALD) force. The second piece comes from the tail part of the field, and is present in curved space even if the worldline is a geodesic.

For a particle in free motion, careful analyses^{2,3} thus reveal that the self-force may be described entirely in terms of the "tail" part of the field. However, under general circumstances, the "tail" part of the field, considered as a field in its own right, does not satisfy the source-free equations in the neighborhood of the particle, but appears to arise from a distributed, unphysical source. Furthermore, the "tail" part is generally not differentiable at the location of the particle. In fact, the gradient of the "tail" part must be averaged over different directions before the self-force can be determined.

Consider the specific case of electromagnetism in curved spacetime. To use the tail part, an observer would need to subtract the direct part from the actual field which he measures. He would then be able to relate the motion he observes for a charged particle to a force that he could calculate. However, from Maxwell's equations, the field from which that force arises would lead him to look for charges where, in fact, he sees none. Thus, he would not be able to understand how Maxwell's equations apply to his observations. He should see this as a significant scientific embarrassment.

Recent analyses^{4–6} rectify this situation by the discovery of an expansion for the singular field which, in curved spacetime, plays a role identical to that of Dirac's "mean of the advanced and retarded fields" in flat spacetime. This singular field possesses the two important features which characterize Dirac's identification of the singular field. In the vicinity of the particle,

- (i) it is a singular solution of the field equations with the particle as a source, and
- (ii) it exerts no force on the particle itself (based upon the conservation of stress-energy arguments of Refs. 2 and 3).

After the subtraction of this newly identified singular field from the actual field in the vicinity of the particle, the remainder is a source-free field which is entirely responsible for the self-force, including all effects of radiation reaction.

This specific singular field has eluded discovery for so long because it is not describable in terms of just the advanced and retarded fields. But relatively

straightforward expansions, valid in the neighborhood of the particle, exist for scalar, electromagnetic and gravitational fields.^{4–6} For example, in a very particular locally-inertial coordinate system, first introduced by Thorne and Hartle,⁷ the singular scalar field for a particle with charge q is just the Coulomb-like field q/r up to corrections proportional to r^3 . In the flat space limit, it reduces to the “mean of the advanced and retarded fields.”

3. Gravitation

To lowest order, a particle in a gravitational field moves along a geodesic. What is the equivalent of the Lorentz force law for such a particle? As suggested in Refs. 3–4 and 5, the motion of the particle should be a geodesic in a spacetime determined by the perturbed metric. But, like the application of the Lorentz force law discussed in the beginning, the singular part of the particle’s field must be removed before this geodesic motion can be confirmed. We now describe in detail how this understanding would be tested.

Consider an observer who records the motion of a particle of small mass μ , and measures the gravitational field in its neighborhood. The measured field is singular at the particle itself. Knowledge of the field in a local neighborhood is sufficient for the singular part to be uniquely calculated and subtracted. What remains is a source-free field in the neighborhood of the particle, composed of the background field and an $O(\mu)$ source-free perturbation arising from the presence of the particle. To complete the analysis, i.e., in order to understand the observed motion of the particle, the observer must calculate geodesic motion in this combined source-free field. Only when it has been verified that the motion of the particle is, in fact, along one of these geodesics, can the observer conclude that he understands his measurements and observations. That this is possible has been explicitly proven in Ref. 5.

4. Conclusion

The principle of equivalence tells us that a particle of infinitesimal mass will move along a geodesic. Naively, in the gravitational case, one might have expected that the motion of a small particle undergoing radiation reaction would not be geodesic. It is now known that, through $O(\mu)$, the path of a particle of small but finite mass is a geodesic, in a metric which is source-free in the neighborhood of the particle. This holds even while the particle is radiating and experiencing radiation reaction. Thus the applicability of the principle of equivalence extends to include radiation.

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