1 one-loop corrections to the particle propagator of the Higgs field

I work in unitarity gauge so that I can ignore Goldstone bosons and Fadeev-Popov ghosts.\(^1\) The physical Higgs field, \(h\), appears in a doublet of \(SU(2)_W\) with \(U(1)_Y\) charge of \(+\frac{1}{2}\) in the following form (See Peskin&Schroeder Eqs.20.110,4).

\[
\phi(x) = \left( \frac{m_h}{2\sqrt{\lambda}} + \frac{1}{\sqrt{2}}h \right)
\]

so that

\[
\tau^3\phi(x) = -\frac{1}{2}\phi(x)
\]

All one-loop propagator corrections will appear in the following general form.

\[
h \rightarrow \begin{array}{c} \cdots \arrayrulecolor{gray} \arrayrulewidth 0.8pt \hline \vrule \arrayrulewidth 0.8pt \hline \end{array} \rightarrow h = (\text{propagator})^2 \times \frac{\text{loop integral}}{\text{symm. factor}}
\]

I will ignore the two “propagator” factors, which will be common to all corrections, and concentrate rather on what appears inside the grey circle. However, I will continue to draw the two external propagator lines for clarity.

1.1 Higgs self-interactions (from \(V(\phi)\))

The physical Higgs self-interaction terms in the Lagrangian are (See Peskin&Schroeder Eqs.20.113)

\[
\mathcal{L}_{int,V(\phi)} = -\frac{\sqrt{2}\lambda m_h}{2}h^3 - \frac{\lambda}{4}h^4
\]

This allows two one-loop corrections to the propagator.

\(^1\)In unitarity gauge, the propagators of the Goldstone bosons and ghosts are \(\lim_{\xi \to \infty} -\frac{1}{\xi m^2}\), where \(m\) is the mass of the corresponding gauge boson (See Peskin&Schroeder p.743). This means that the only gauge-dependent field that appears in nonvanishing amplitude expressions is the photon ghost field. The physical Higgs boson is the scalar boson corresponding to the zero-mass photon. Neither the photon field nor the photon ghost field couple to the Higgs field.
1.1.1  trilinear Higgs self-interaction

For this correction, there are two simple (constant) vertex factors each equal to $-i3\sqrt{2\lambda m_h}$, and the symmetry factor is 2.

\[ p \quad h \quad h \quad = \quad (5) \]

1.1.2  quadrilinear Higgs self-interaction

For this correction, there is one simple (constant) vertex factor equal to $-6\lambda$, and the symmetry factor is 2.

\[ p \quad h \quad h \quad = \quad (6) \]

1.2  Higgs fermion-interactions (Yukawa terms)

The physical Higgs fermion-interaction terms in the Lagrangian are of the following form (See Peskin&Schroeder Eqs.20.114,6).

\[ \mathcal{L}_{int,\phi f} = -\sqrt{2\lambda m_f} \frac{m_h}{m_h} h \bar{f} f \quad (7) \]

This allows a single one-loop correction to the propagator for each fermion. For this correction, there are two simple (constant) vertex each factors equal to $-i\sqrt{2\lambda m_f} m_h$, and the symmetry factor is $-1$.\(^2\)

\[ p \quad \bar{f} \quad f \quad = \quad (8) \]

\(^2\)I count the negative sign that is due to the closed fermion loop as part of the symmetry factor, since it also comes about do to the exchange of identical particles.
1.3 Higgs gauge-interactions (from $|D\phi|^2$)

The physical Higgs gauge-interaction terms in the Lagrangian are of the following form (See Peskin&Schroeder Eqs.20.114,5).

$$L_{\text{int, gauge}\phi} = \left( \frac{2\sqrt{2}\lambda}{m_h} h + \frac{2\lambda}{m_h^2} h^2 \right) \left( m_W^2 W^+ \cdot W^- + \frac{1}{2} m_Z^2 (Z^0)^2 \right)$$  \hspace{1cm} (9)

This allows four distinct one-loop corrections to the propagator.

1.3.1 single-Higgs W-interaction

For this correction, there are two simple (constant) vertex factors each equal to $i\frac{2\sqrt{2}\lambda m_W^2 \eta^{\mu\nu}}{m_h}$, and the symmetry factor is 1.

$$p \quad \begin{array}{c} \hline
W^+ \\
\hline
h \rightarrow \bullet --- h
\end{array} \quad W^- =$$ \hspace{1cm} (10)

1.3.2 double-Higgs W-interaction

For this correction, there is one simple (constant) vertex factor equal to $4\frac{\lambda m_W^2 \eta^{\mu\nu}}{m_h}$, and the symmetry factor is 1.\footnote{Note that only one diagram for the two charged bosons, either $W^+$ or $W^-$, should be included in the correction. I have simply chosen $W^+$. The similar diagram with $W^-$ in place of $W^+$ should not be included, since that would count this correction twice.}

$$\begin{array}{c} \hline
W^+ \\
\hline
h \rightarrow \bullet --- h
\end{array} =$$ \hspace{1cm} (11)

1.3.3 single-Higgs Z-interaction

For this correction, there are two simple (constant) vertex factors each equal to $i\frac{2\sqrt{2}\lambda m_Z^2 \eta^{\mu\nu}}{m_h}$, and the symmetry factor is 2.

$$p \quad \begin{array}{c} \hline
Z^0 \\
\hline
h \rightarrow \bullet --- h
\end{array} =$$ \hspace{1cm} (12)
1.3.4 double-Higgs Z-interaction

For this correction, there is one simple (constant) vertex factor equal to \( \frac{4\lambda m_Z^2 \eta^{\mu \nu}}{m_h^4} \), and the symmetry factor is 2.

\[
Z^0 \quad = \\
\begin{array}{c}
h \\
p \\
h
\end{array}
\]

\( \text{(13)} \)