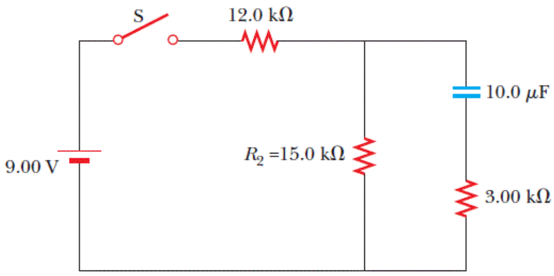
Typical Problems of direct RC and RL circuits

Quite often, the problem likes to ask you the asymptotic behavior of the RC or RL circuits with several resistors. In those cases, you can not naively apply the simple formula of RC or RL circuits if those resistors are not just in series with the capacitor or the inductor. However, physical understanding of capacitance and inductance should make you solve those problems easily. In this article, I give you two typical examples, one on the RC circuit, and the other on the RL circuit. Normally, the problem will just ask you one part of them.

1. For the RC circuit in the figure, $R_1 = 12.0k\Omega$ and $R_3 = 3.00k\Omega$. The currents in R_1, R_2, and R_3 are denoted as I_1, I_2, and I_3, respectively. The charge on the capacitor is denoted as Q, and the voltage across the capacitor is denoted as V_c. Suppose that initially there is no charge on the capacitor, and the switch is open.
   (a) Close the switch, and find I_1, I_2, I_3, Q, and V_c immediately after the switch is closed.
   (b) After the switch is closed for a length of time sufficiently long for the capacitor to become fully charged, find I_1, I_2, I_3, Q, and V_c.
   (c) The switch is then reopened. Find I_1, I_2, I_3, Q, and V_c immediately after the switch is reopened.
   (d) After the switch is reopened for a length of time sufficiently long for the capacitor to become fully discharged, find I_1, I_2, I_3, Q, and V_c.



Solution:

(a) i. The capacitor just starts to be charged $\Rightarrow Q = 0$.

ii. $V_c = Q/C = 0$.

iii. $V_c = 0$ (no voltage drop accross the capacitor), and therefore, the circuit is $R_1(R_2 \parallel R_3)$ (R_2 and R_3 are in parallel, and then they are in series with R_1). Hence,

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 12 + 2.5 = 14.5(k\Omega).$$

Hence,

$$I_1 = I_{eq} = \frac{9}{R_{eq}} \approx 0.62(mA).$$

iv.

$$V_2 = V_3 = I_{eq}\frac{R_2 R_3}{R_2 + R_3} = I_1 \frac{R_2 R_3}{R_2 + R_3}.$$

Therefore,

$$I_2 = V_2/R_2 = I_1 \frac{R_3}{R_2 + R_3} \approx 0.10(mA),$$

and

$$I_3 = V_3/R_3 = I_1 \frac{R_2}{R_2 + R_3} \approx 0.52(mA).$$

(b) i. After the switch is closed for a length of time sufficiently long, the capacitor becomes fully charged and therefore the current through the capacitor should be zero (Otherwise, it's still charging). Hence,

$$I_3 = 0,$$

because the current through R_3 should be the same as the current through the capacitor (The capacitor and the R_3 are in series).

ii. By the junction rule,

$$I_1 = I_2 + I_3 = I_2.$$

Hence, by the loop rule,

$$9 = I_1(R_1 + R_2) = 27I_1. \quad \Rightarrow \quad I_1 = I_2 = \frac{9}{27} = \frac{1}{3} \approx 0.33(mA).$$

iii. The voltage V_2 across R_2 is given by

$$V_2 = I_2 R_2 = \frac{1}{3} \times 15 = 5(V).$$

It is in parallel with "the capacitor and R_3 in series". Therefore,

$$V_2 = V_c + V_3 = V_c \quad \Rightarrow \quad V_3 = 5(V),$$

where I have used the fact that $V_3 = I_3 R_3 = 0$, since $I_3 = 0$.

iv.

$$Q = CV_c = 10 \times 5 = 50(\mu C).$$

(c) i. Once the switch is open, the left-hand side circuit is open and therefore $I_1 = 0$.

ii. The charge was fully charged to be $50\mu C$ before the switch is reopened. The charge on the capacitor can only change continuously, as being opposed to the current that can suddenly drop to be zero once the circuit is open. Therefore, Q remains $50\mu C$ immediately after the switch is reopened.

iii. $V_c = Q/C$. Therefore, $V_c = 5(V)$, the same as in Part (b).

iv. The right hand side is still a closed circult. It's that R_2, R_3, and C are in series.
It is obvious that $I_2 = I_3$ since $I_1 = 0$. Therefore, by the loop rule,

$$V_c = I_2 R_2 + I_3 R_3 = I_2(R_2 + R_3) = 18I_2.$$

Hence,

$$I_2 = I_3 = V_c/18 = 5/18 \approx 0.28(mA).$$

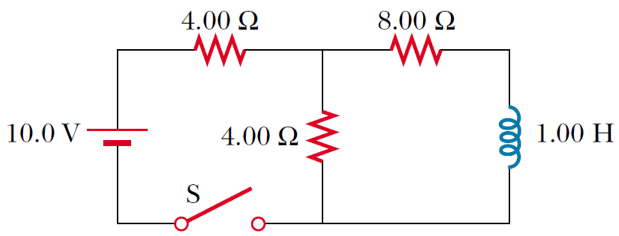
(d) i. $I_1 = 0$ (The same reason as in Part (c)).

ii. After the switch is reopened for a length of time sufficiently long for the capacitor to become fully discharged, $Q = 0$, and therefore $V_c = Q/C = 0$ as well.

iii. $V_c = 0$, and thus $I_2 = I_3 = 0$ (loop rule. Check Part(c)).

2. This problem is almost the same as the first problem except that the capacitor is replaced by the inductor. However, the answers will be totally different.
For the RL circuit in the figure, $R_1 = 4\Omega$ (leftmost one in the figure), $R_2 = 4\Omega$ (middle in the figure), and $R_3 = 8\Omega$. The currents in R_1, R_2, and R_3 are denoted as I_1, I_2, and I_3, respectively. The voltage drops accross R_1, R_2, R_3 and the inductance L are denoted as V_1, V_2, V_3, and V_L, respectively. Suppose that initially there is no current in the inductor, and the switch is open.

(a) Close the switch, and find I_1, I_2, I_3, and V_L immediately after the switch is closed.
(b) After the switch is closed for a very long time, find I_1, I_2, I_3, and V_L.
(c) The switch is then reopened. Find I_1, I_2, I_3, and V_L immediately after the switch is reopened.
(d) After the switch is reopened for a very long time, find I_1, I_2, I_3, and V_L.



Solution:

(a) i. The inductor is against the change of current (Lenz's law) $\Rightarrow I_3 = 0$.
ii. Since $I_3 = 0$, $I_1 = I_2$. Therefore, by the Kirchoff's loop rule,

$$\varepsilon - I_1 R_1 - I_2 R_2 = 0 \Rightarrow 10 - 8I_1 = 0.$$

Hence, $I_1 = I_2 = 10/8 = 1.25$ (A).
iii. L and R_3 are in series, and then they are in parallel with R_2. Therefore,

$$V_L + V_3 = V_2.$$

However, $V_3 = I_3 R_3 = 0$. We get $V_L = V_2 = I_2 R_2 = 1.25 \times 4 = 5$ (V).

(b) i. After the switch is closed for a very long time, it reaches a steady state which means the current is steady, no changing with respect to time. Therefore

$$V_L = L\frac{dI_3}{dt} = 0. \qquad \text{no voltage drop accross the inductor}$$

Or if you prefer, $V_L = L\Delta I_3/\Delta t = 0$.
Therefore, the circuit is $R_1(R_2 \parallel R_3)$ (R_2 and R_3 are in parallel, and then they are in series with R_1). Hence,

$$R_{eq} = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 4 + 4 \times 8/(4+8) = 20/3(\Omega).$$

Hence,

$$I_1 = I_{eq} = \frac{10}{R_{eq}} = 1.5(A).$$

ii. $V_2 = V_3 + V_L = V_3$.

$$V_2 = V_3 = I_{eq}\frac{R_2 R_3}{R_2 + R_3} = I_1 \frac{R_2 R_3}{R_2 + R_3}.$$

Therefore,

$$I_2 = V_2/R_2 = I_1 \frac{R_3}{R_2 + R_3} = 1.0(A),$$

and

$$I_3 = V_3/R_3 = I_1 \frac{R_2}{R_2 + R_3} = 0.5(A).$$

(c)
i. Once the switch is open, the left-hand side circuit is open and therefore $I_1 = 0$.

ii. The current in L is $I_3 = 0.5$ A before the switch is reopened. The current in the inductor can only change continuously in the inductor: the inductor is against the change of current through it. This is opposed to the current I_1 that can suddenly drop to be zero once the circuit is open. Therefore, I_3 remains 0.5 A immediately after the switch is reopened.

Note: Through the capacitor, the voltage or change change continuously, but the current can change discontinuously (Check part(a) of Problem 1 for example. The current suddenly changes from zero to 0.52 mA). However, through the inductor, the current changes continuously but the voltage can change discontinuously (See the following. V_L suddenly changes from zero to 6 V).

iii. The right hand side is still a closed circult. It's that R_2, R_3, and L are in series. It is obvious that $I_2 = I_3$ since $I_1 = 0$. Therefore, by the loop rule,

$$V_L = I_2 R_2 + I_3 R_3 = I_3(R_2 + R_3) = 0.5 \times 12 = 6.$$

(d)
i. $I_1 = 0$ (The same reason as in Part (c)).

ii. After the switch is reopened for a very long time, the current becomes steady. Therefore, $V_L = 0$.

iii. $I_3 = I_2 = 0$ (If it's non-zero, it means there is still power dissipation in the resistors, which will decrease the energy and thus the current of the system.$\Rightarrow$ it has not become a steady state yet.).