Standard Deviation of F({f})

Assume that there is a set $\{f_1,\ldots,f_N\}_I$ of values and a set of rules for using this set of values to produce a value $F(\{f\}_I)$. If F can be differentiated with respect to f_i, the difference between F for the $\{f\}_I$ and that for the $\{f\}_J$ is given by

$$F\left(\{f\}_I\right)-F\left(\{f\}_J\right)=\sum_{i=1}^{N}\frac{\partial F}{\partial f_i}\left(f_{i,I}-f_{i,J}\right) \qquad (1.1)$$

The difference squared is

$$\left(F_I-F_J\right)^2=\sum_{i=1,j=1}^{N}\left.\frac{\partial F}{\partial f_i}\right)_{\delta_i=0}\left.\frac{\partial F}{\partial f_j}\right)_{\delta_j=0}\left(\left(f_{i,I}-f_{i,J}\right)\left(f_{j,I}-f_{j,J}\right)\right) \qquad (1.2)$$

The standard deviation in F is the limit that M_I and $M_J \rightarrow$ infinity in

$$\sigma^2_{F,M_I,M_J}=\frac{1}{2}\sum_{\substack{i=1\\j=1}}^{N}\frac{1}{M_I\left(M_J-1\right)}\sum_{\substack{I=1\\J=1\\J\neq I}}^{M_I,M_J}\frac{\partial F}{\partial f_i}\frac{\partial F}{\partial f_j}\left(\left(f_{i,I}-f_{i,J}\right)\left(f_{j,I}-f_{j,J}\right)\right) \qquad (1.3)$$

If f_i and f_j are statistically independent, the terms with $i \neq j$ average to zero so that

$$\sigma^2_{F,M_I,M_J}=\sum_{i=1}^{N}\left(\frac{\partial F}{\partial f_i}\right)^2\left(\frac{1}{2M_I\left(M_J-1\right)}\sum_{\substack{I=1\\J=1\\J\neq I}}^{M_I,M_J}\left(f_{i,I}-f_{i,J}\right)^2\right) \qquad (1.4)$$

The term in parenthesis in the limit that M_I and M_J become infinite is the standard deviation of the values of f_i

$$\delta_i^2=\frac{1}{2M_I\left(M_J-1\right)}\sum_{\substack{I=1\\J=1\\J\neq I}}^{M_I,M_J}\left(f_{i,I}-f_{i,J}\right)^2 \qquad (1.5)$$

Finally the standard deviation in F is

$$\sigma_F^2=\sum_{i=1}^{N}\left(\frac{\partial F}{\partial f_i}\right)^2\delta_i^2 \qquad (1.6)$$

The relationship of σ to the standard deviation in f_i for a Gaussian distribution is derived in Deviations.docx. In the case of Poisson data with more than just a few counts $\delta_i^2=f_i$. The critical step is the assumption in going from (1.3) to (1.4) that the terms for $i\neq j$ sum to zero.